* Kinematics * (object in motion but not with forces)

Fundamentals of flowing fluids:

* **Steady & unsteady flow** :-

If all the properties does not change w.r.to time that the flow is said to be steady, Even a single parameter changes w.r.to time. It will be treated as unsteady flow

$\frac{\partial P}{\partial t} = 0 \rightarrow$ steady , $\frac{\partial P}{\partial t} \neq 0 \rightarrow$ unsteady

$P = f[\text{space, time}]$

$f\{x, y, z, t\}$

* **Uniform & Non uniform** :-

w.r.to (space) (velocity also consider) w.r.to time.

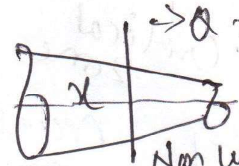
(Length of the direction of flow)

$Q = AV = C$

$\rightarrow Q = |A\, V| = C$

$\frac{\partial P}{\partial S} = 0 \rightarrow$ uniform

$\frac{\partial P}{\partial S} \neq 0 \rightarrow$ Non uniform



* **Compressible flows & Incompressible**

Comp flow $\leftarrow$ Gas

I.C flow $\leftarrow$ liquids

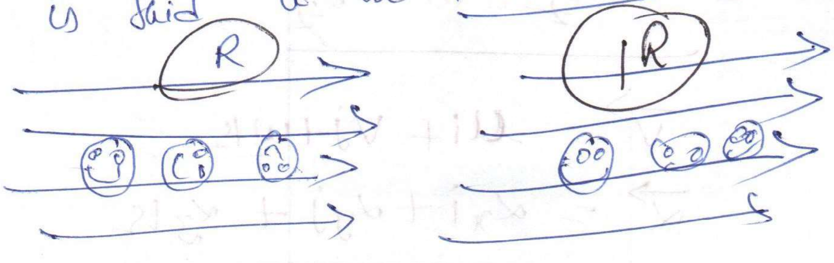
M<(0.4) Gas

M < 0.4

↳ then it is incompressible

* **Rotational & Irrotational** :-

If the fluid particles rotates about their mass centre while moving forward the flow is said to be rotational otherwise irrotational



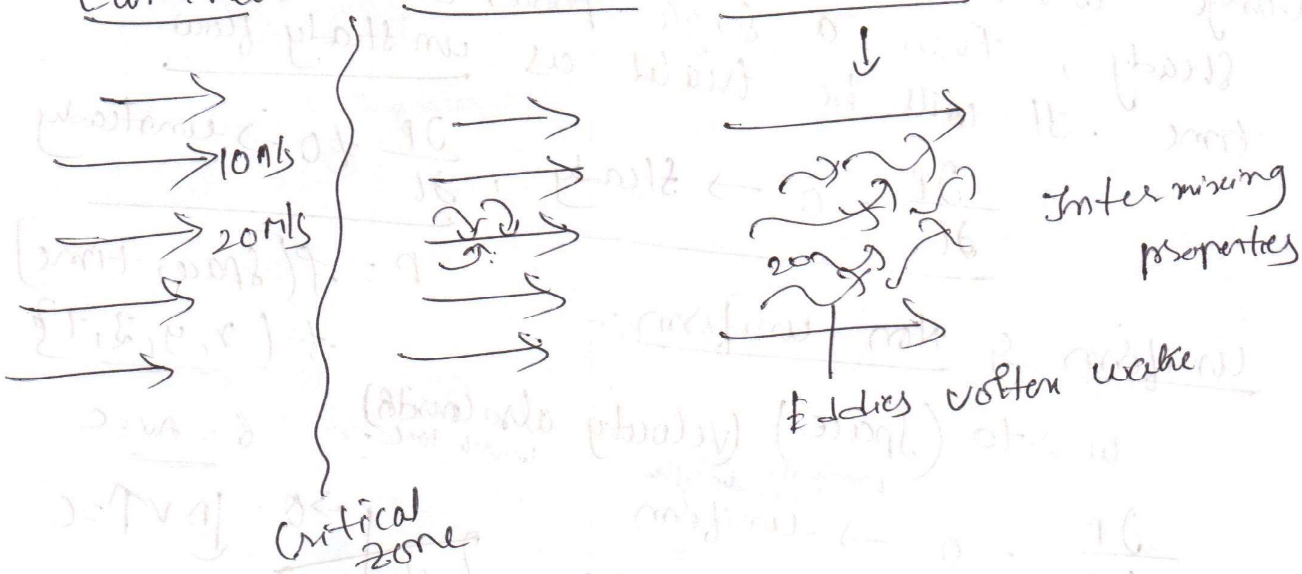
Note:- Rotation is because of variation in shear

Rotation is ~~because~~ of variation in shear

$\omega_x, \omega_y, \omega_z = 0 \, (IR)$

$\omega_x, \omega_y, \omega_z \neq 0 \, (R)$ ✓

Laminar - Transisional & Turbulent :-



Velocity & Accelration :-

$$\boxed{V = \{x, y, z, t\}}$$

→ velocity = $\left(\frac{\text{Distance}}{\text{time}}\right)$

→ Accelration = $\left(\frac{\text{Change in velocity}}{\text{time}}\right)$

→ $\boxed{\text{Jerk} = \frac{\Delta \text{ Acc}}{\text{time}}}$

$V \to x \to V_x = u \to \alpha_x$

$V \to y \to V_y = v \to \alpha_y$

$V \to z \to V_z = w \to \alpha_z$

$V \to x \to V_x = u \to \alpha_x$

$V \to y \to V_y = v \to \alpha_y$

$V \to z \to V_z = w \to \alpha_z$

$$\vec{V} = ui + vj + wk$$

$$\vec{\alpha} = \alpha_x i + \alpha_y j + \alpha_z k$$

$V = f[x, y, z, t]$ $\qquad$ $u, v, w \rightarrow$ velocitys

but it is not functions

$u = \frac{dx}{dt}$, $\quad v = \frac{dy}{dt}$, $\quad w = \frac{dz}{dt}$

Change of velocity in x direction/unit time $\rightarrow$

$$\alpha_x = \frac{du}{dt}$$

$$\alpha_y = \frac{dv}{dt}$$

$$\alpha_z = \frac{dw}{dt}$$

$$\alpha_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\overset{u}{\left(\frac{dx}{dt}\right)} + \frac{\partial u}{\partial y}\overset{v}{\left(\frac{dy}{dt}\right)} + \frac{\partial u}{\partial z}\overset{w}{\left(\frac{dz}{dt}\right)} + \frac{\partial u}{\partial t}$$

$$\alpha_x = u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) + w\left(\frac{\partial u}{\partial z}\right) + \frac{\partial u}{\partial t}$$

$$\alpha_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$\alpha_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

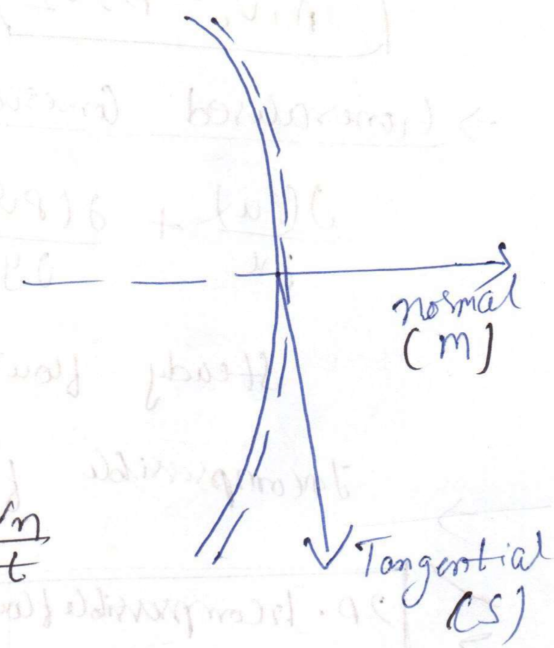
5/7/17

$V = f(x, y, z, t)$

$V = f(s, n, t]$

$V \begin{cases} s \rightarrow V_s \rightarrow \alpha_s \\ n \rightarrow V_n \rightarrow \alpha_n \end{cases}$

u	α_x
v	α_y
w	α_z



$V_s = \frac{ds}{dt}$, $\quad \alpha_s = \frac{dV_s}{dt}$

$V_n = \frac{dn}{dt}$, $\quad \alpha_n = \frac{dV_n}{dt}$

$$a_s = \frac{\partial V_s}{\partial s} \cdot \left(\overset{V_s}{\cancel{\frac{ds}{dt}}}\right) + \frac{\partial V_s}{\partial n}\left(\overset{V_n}{\cancel{\frac{dn}{dt}}}\right) + \frac{\partial V_s}{\partial t}$$

✓ $$a_s = V_s \frac{\partial V_s}{\partial s} + V_n \cdot \frac{\partial V_s}{\partial n} + \frac{\partial V_s}{\partial t}$$

✓ $$a_n = V_s \cdot \frac{\partial V_n}{\partial s} + V_n \cdot \frac{\partial V_n}{\partial n} + \frac{\partial V_n}{\partial t}$$

<u>Continuity equation:</u> $\Rightarrow [A_1 V_1 = A_2 V_2]$

[Conservation of Mass]

→ Mass flow rate is constant

$$\dot{m} = \frac{\text{mass}}{\text{time}} = \frac{\text{kg}}{\text{sec}} = \frac{C \times m^3}{m^3}$$

$$\dot{m} = \overset{\rho}{\frac{\text{kg}}{m^3}} \times \overset{Q}{\frac{m^3}{\text{sec}}} = \overset{\rho}{\frac{\text{kg}}{m^3}} \times \overset{A}{m^2} \frac{m}{\text{sec}}$$ ✓

$\nabla \cdot \vec{V} = 0$

$\therefore \quad m = \rho A V = C$

For I.C → ρ = const $\Leftarrow$ For incompressible flow → ρ = Constant

$$\boxed{A_1 V_1 = A_2 V_2}$$

→ <u>Generalised Conservation equation:</u>

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

Steady flow → $\frac{\partial \rho}{\partial t} = 0$ ✓

Incompressible flow ρ const → $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

$$\boxed{\text{2D. Incompressible flow } (x\, y) \Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$ ✓

(1) [7-03] Velocity field (vector) is given by $\vec{V} = 2yi + 3xj$

Then the convective acceleration in x-direction at point [1, 1)

Sol. $V = (2y)\underline{i} + (3x)j$ ✓

($u = 2y$, $v = 3x$)

α_{conv}^{x} [1, 1] ?

$$\alpha x_{conv} = u \frac{\partial u}{\partial x} + v \cdot \frac{\partial v}{\partial y}$$

$$\alpha_x = (2y) \frac{\partial}{\partial x} (\cancel{2y})^{0} + 3x \cdot \frac{\partial}{\partial y} (2y)$$

$$= 0 + 3x(2)$$

$$\alpha x_{(1,1)} = 6x = 6 \times 1 \longrightarrow \text{U. } 6 \text{ Units} = \underline{6LT^{-2}} \quad 6 \text{ m/sec}^2$$

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(2) For 2-D incompressible flow x component of velocity i.e. $u = C \ln(xy)$ then $v = ?$

Sol. 2D IC flow

$u = C \ln[x \cdot y]$

$v = ?$

$$\rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial y} = -\frac{\partial}{\partial x}[C \cdot \ln(x \cdot y)] \quad v = -C[y/x]$$

$$= -C \frac{1}{x\cancel{y}}(\cancel{y}) = \frac{C}{x}$$

$$\rightarrow \int \partial v = \int \frac{-C}{x} \partial y$$

$$v = \frac{-C}{x} \int 1 \cdot (\partial y)$$

$$\rightarrow \frac{-C}{x}(1 \cdot \partial y) = \frac{C}{x}(1 \cdot \partial y)$$

$$= -\frac{C}{x}(y) \cancel{+C}$$

$$\boxed{v = -C\left[\frac{y}{x}\right]}$$

G-16 PI

$\vec{V} = [a_1 x + b_1 y + c_1 z] + [a_2 x + b_2 y + c_2 z] + (a_3 x + b_3 y + c_3 z)$
$\partial/\partial x \qquad \partial/\partial y \qquad \partial/\partial z$

For incompressible flow $a_1 = 2$, $c_3 = 4$ what is value of $b_2 = ?$

Sol

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$a_1 + b_2 + c_3 = 0$

$2 + b_2 - 4 = 0$

$\underline{b_2 = 2}$ ✓

$\vec{V} = \frac{\partial}{\partial x}[a_1 x + b_1 y + c_1 z]i$

$= \frac{\partial}{\partial y}[a_2 x + b_2 y + c_2 z]j$

$= \frac{\partial}{\partial z}[a_3 x + b_3 y + c_3 z]k$

$\frac{\partial}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \Rightarrow \underline{b_2 = 2}$

ES-06) velocity field is given by $\vec{V} = \frac{\partial}{\partial x}(5x + 6y + 7z)i$

A) $\frac{\partial}{\partial y}(3x + 5y + 4z)j \; \frac{\partial}{\partial z}(2x + 3y + \lambda z)k$ is conserved, ρ varies from inorder to mass

as $\rho = \rho_0 e^{-2t}$, $\lambda = ?$

A) -12 B) -10 C) -8 D) -6

Sol.

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

$\frac{\partial}{\partial x}[\rho_0 e^{-2t}(5x + 6y + 7z)] \Rightarrow$ similarly then $\rho_0 e^{-2t}[5 + 5 + \lambda - 2] = 0$

$\Rightarrow \rho_0 e^{-2t}(5)$

$\underline{\lambda = -8}$.

Q-08

$\nabla \cdot \vec{V} = 0$ to the valid which of the following is necessary conditions

a) steady

b) incompressible

c) unsteady

d) compressible

Ans (b) incompressible

a) steady

b) unsteady

c) compressible

d) Incompressible

(i) only a

(ii) only d

(iii) a & d

(iv) b & d

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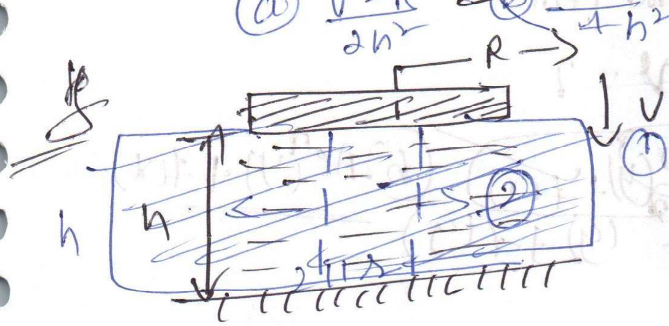
(5) The gap b/w a moving circular plate and stationary surface is being continuously reduced as the circular plate comes down with uniform velocity V as shown in the figure. Assume the fluid in b/w as incompressible and it was flowing out radially.

(1) The radial velocity V_r at any radius radius 'r' when gap width h?

(a) $\frac{V \cdot r}{h}$ (b) $\frac{V r}{2h}$ (c) $\frac{V \cdot h}{r}$ (d) $\frac{2 \cdot V \cdot h}{r}$

(2) The radial acceleration at $r = R$?

(a) $\frac{V^2 R}{2h^2}$ (b) $\frac{V^2 R}{4h^2}$ c) $\frac{3V^2 R}{2h^2}$ d) $\frac{3V^2 R}{4h^2}$



(1) ↓ (2) →

vel V V_r

Area πr^2 $2\pi r h$

By using continuity equation $A_1 V_1 = A_2 V_2$

(1) $A_1 V_1 = A_2 V_2$

$\pi r^2 V = 2\pi r \cdot h \times V_r$

$V_r = \frac{V \cdot r}{2h}$

(2) $\alpha_r = \frac{d}{dt}\left[\frac{v.r}{2h}\right]$

$$\boxed{V_r = \left(\frac{dr}{dt}\right)}$$

h constant $\rightarrow$ $\boxed{\alpha_r = \frac{d(V_r)}{dt}}$ $= \frac{V}{2h}\cdot\left(\frac{dr}{dt}\right)$

$$= \frac{V}{2h}\left[\frac{V.r}{2.h}\right]$$

$$\alpha_r = \frac{V^2 r}{4h^2}\Big|_{r=R} \qquad \alpha_R = \frac{V^2 R}{4h^2}$$

h- variable

$V = f(r, h, t)$

$V \begin{cases} r \to V_r \to \alpha_r \\ h \to V_h \to \alpha_h \end{cases}$

$V_r = \frac{dr}{dt} = \frac{V.r}{2.h}$

$V_h = \frac{dh}{dt} = (-V)$

$$\alpha_r = \frac{\partial(V_r)}{\partial r}\left(\frac{dr}{dt}\right) + \frac{\partial(V_r)}{\partial h}\cdot\left(\frac{dh}{dt}\right) + \frac{\partial V_r}{\partial t}$$

$$= \frac{\partial}{\partial r}\left[\frac{V.r}{2h}\right] \times \left[\frac{V.r}{2.h}\right] + \frac{\partial}{\partial h}\left[\frac{V.r}{2.h}\right] * (-V) + \frac{\partial}{\partial t}\left[\frac{V.r}{2h}\right]$$

$$= \frac{V}{2h}\left(\frac{V.r}{2h}\right) + \frac{V.r}{2}\left[\frac{-1}{h^2}\right](-V) + 0$$

$$= \frac{V^2 r}{4h^2} + \frac{V^2 r}{2h^2} = \frac{3V^2 r}{4h^2}\Big|_{r=R} \qquad \alpha_R = \frac{3V^2 R}{4h^2}$$

7) 2 Dimensional incompressible flow

$u = A.e^x$ then $v = ?$

(a) $-A.e^{x.y}$ (b) $-A.e^x(y) + f(x)$ (b) $2Ae^x(y) + f(x)$

(c) $-A.e^{xy} + f(y)$ (d) $-Ae^x(y) + f(y)$

Sol $\rightarrow$ $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial y} = -\frac{\partial}{\partial x}(A.e^x)$

$$\int \partial v = \int -A.e^x.\partial y.$$