* UNIT-1 *

⇒ Introduction to Engg. Mechanics & Basic Concepts

* **Definition** :- Mechanics is the oldest of physical sciences, which deals with the rest (or) motion of bodies under the action of forces.

→ Here, we shall study mechanics of non deformable (or) rigid bodies. As such, no body is perfectly rigid but does deform under the action of forces. However, when we are concerned only with the external effect of forces on solid bodies, we can generally idealize them to be rigid for analytical purposes.

Classification of Engg Mechanics :-

- Engg. Mechanics (E.M)
  - Mechanics of solids
    - Solid Bodies (or) Rigid bodies
      - Statics
      - Dynamics
        - Kinematics
        - Kinetics
    - Flexible bodies (or) Deformable bodies
      - SOM
      - Mechanics of elastic bodies
      - Mechanics of plastic bodies
  - Mechanics of fluids
    - Ideal fluids
    - Viscous fluid
    - Compressed fluids

⇒ In Engg Mechanics we study about Rigid bodies. In Mechanics of rigid bodies can be classified into Statics & Dynamics

Statics :- Statics is the study of distribution and effect of forces on bodies which are at rest and remain at rest.

Dynamics :- Dynamics is the study of motion of bodies and their correlation with the forces causing them.

Dynamics is further into two parts

Kinematics :- Kinematics is the study of motion of bodies without considering the force causing the motion

It deals with the relationship b/w displacement, velocity, & Acceleration and their variation with time.

Kinetics :- Kinetics is the study of motion of bodies together with the forces causing the motion

It relates the force and the acceleration by the laws of motion

Importance of Mechanics to Engineering :-

* For designing and constructing of dams, roads, sheds, structures, buildings etc.
* For designing and controlling of the fluid flow
* For calculation and estimation of forces of bodies while they are in use.
* For designing and manufacturing of various mechanical ~~tools~~ and equipments.
* For contracting and co-ordinating the various parts of mechanics.
* For designing a fabrication of rockets.

Scalar & vector Quantities :-

All physical quantities are divided into a scalar and vector quantities.

→ A scalar quantity is that physical quantity which has only magnitude and no direction.

Eg: Mass, length, time, density, energy etc.

→ A vector quantity is that physical quantity which has both magnitude and direction.

Eg:- Displacement, velocity, acceleration, momentum, force, etc.

⇒ UNITS & DERIVED UNITS :-

⇒ we must assign units to each physical quantity apart from their numerical values. For instance, to express a measurement of length we may say that it is "n" times a std length. Such types of std measurements are termed units. "Thus unit may be defined as those standards in terms of which the physical quantities are measured" These std also enables comparison of two different measurement of the same category.

⇒ Different system of units are followed for this purpose by different countries.

The most common ones are MKS system of units, cgs system of units, & fps system of units.

Various system of units :-

System	Length	mass	time
C.G.S	Centimetre	gram	second
F.P.S	foot	pound	second
M.K.S	Metre	Kilogram	second.

⇒ **Fundamental units** :- The physical quantities which are independent of other quantities are called fundamental quantities and their units are called fundamental units. The fundamental quantities are length, mass, & time.

⇒ **Derived units** :- The physical quantities which are expressed interms of the fundamental quantities are called derived quantities. The derived quantities are called derived units. For instance, the unit of length is a fundamental unit, while the unit of Area is a derived unit.

S.I System of units (System of International units)

The system of international units (S.I) was formally recognized by the Elevanth General Conference of weights and measurement in 1960. Now this system is being adopted throughout the world.

⇒ Some of standard SI units

Quantity	unit	Symbol	
→ length	metre	L	→ Base units
→ Mass	kilogram	kg	
→ Time	second	s	
→ Area →	$metre^2$ —	m^2	
→ Volume →	$metre^3$ —	m^3	
→ Density →	$\frac{kilogram}{m^3}$ —	kg/m^3	
→ Linear velocity →	metre/second —	m/s	
→ Angular velocity →	radian/second —	rad/s	

→ Linear acceleration →	metre/second^2 →	m/s^2
→ Angular acceleration →	radian/second^2 →	rad/s^2
→ Force →	Newton →	$N (= kg \cdot m/s^2)$
→ Impulse →	Newton-second →	$N \cdot s$
→ moment of force →	Newton-metre →	$N \cdot m$
→ Mass moment of inertia →	kilogram-metre^2 →	$kg \cdot m^2$
→ Linear momentum →	kilogram-metre/second →	$kg \cdot m/s$ $(= N \cdot s)$
→ Angular momentum →	kilogram-$\frac{metre^2}{second}$ →	$kg \cdot m^2/s$ $(= N \cdot m \cdot s)$
→ Pressure, stress →	pascal →	$Pa (= N/m^2)$
→ work, Energy →	joule →	$J (= N \cdot m)$
→ Power →	watt →	$W (= J/s)$
→ Frequency →	Hertz →	Hz
→ spring constant →	Newton/metre →	N/m

Supplimentary units :-

→ plane angle	radian	rad
→ Solid angle	steradian	Sr

→ Standard multipliers in S.I system :-

Factor	Prefix	Factor	Prefix
10^{12}	tera, T	10^{-3}	milli, m
10^{9}	giga, G	10^{-6}	micro, μ
10^{6}	mega, M	10^{-9}	nano, n
10^{3}	kilo, k	10^{-12}	pico, p
10^{-1}	deci, d	10^{-15}	femto, f
10^{-2}	centi, c	10^{-18}	atto, a

Newton's laws of motion :-

Laws of Mechanics :- The mechanics of bodies is governed by two basic laws:

1) The laws of motion &
2) The force laws

⇒ Laws of motion were stated by Issac Newton

First law :- Every body continues in its state of rest (or) of uniform motion in a straight line unless it is compelled to change that state by force acting on it.

Second law :- If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant and in the direction of this resultant force

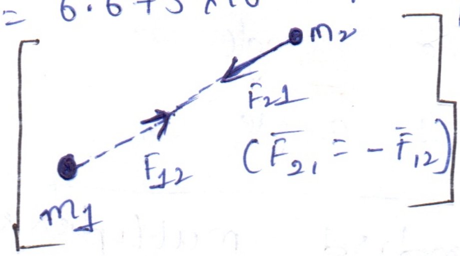
$$\boxed{\vec{F} = m\vec{a}}$$

Third law :- To every action there is always an equal and opposite reaction.

Newton's law of universal Gravitation :-

$$F = \frac{G m_1 m_2}{r^2}$$

$[G = 6.673 \times 10^{-11} \ N \cdot m^2/kg^2]$



r = Distance b/w the two particles

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

⇒ Consider a body of mass "m" located on the surface of the earth, whose mass is "M" and radius "R" Then force of attraction b/w the two bodies is given as

$$F = \frac{GMm}{R^2} = m\left[\frac{GM}{R^2}\right]$$

$$\boxed{F = mg}$$ [∴F is also called as force of gravity]

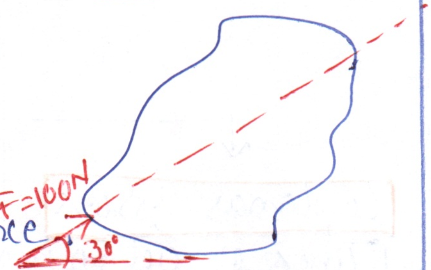
FORCE :- Force may be defined as any action that changes (or) tends to change the position of a body

⇒ It is a vector quantity. The unit of force in SI system is "N" ($kg \cdot m/s^2$)

⇒ The following are the effects of force.

1. It may bring a body under motion to rest (or) retard
2. It may change the motion and direction of a body
3. It may give rise to the internal resistance in a body when acts on it.

Characteristics (or) Specifications of a force :

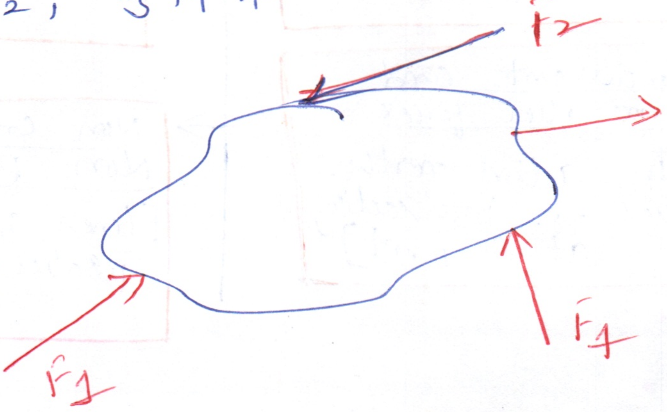


1. Magnitude of the force.
2. point of application of force.
3. position of line of action of force
4. Direction in which the force is acting.
5. Nature of force.

Force System (or) System of forces :-

Whenever a body is subjected to several forces of different magnitude and in different directions, then those forces constitute a system of forces (or) a force system. The figure shows a rigid body containing the forces subjected to a force system

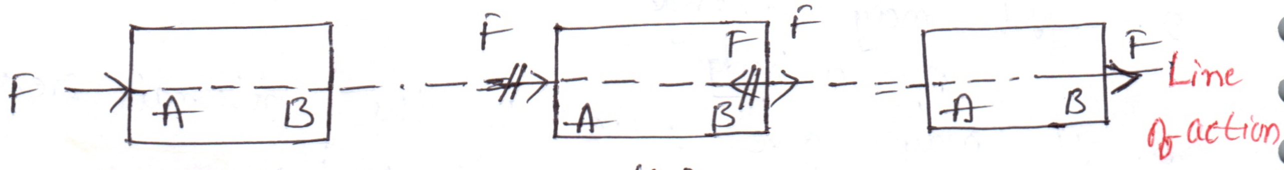
F_1, F_2, F_3, & F_4



Force laws & system of forces (types)

Principle of Transmissibility :- If a force acting at a point on a rigid body is shifted to another point which is in the line of action of force, the net effect of the force on the body remains unchanged.

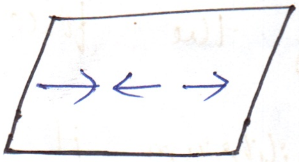
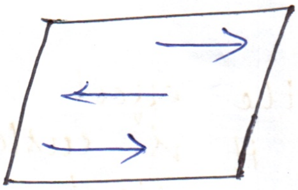
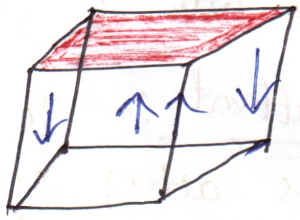
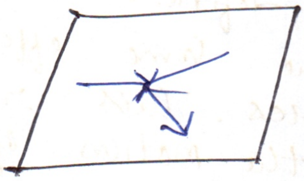
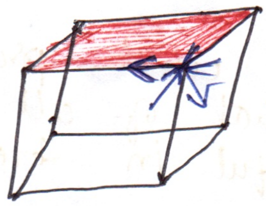
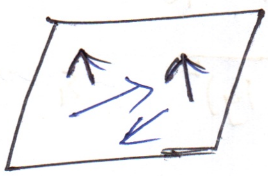
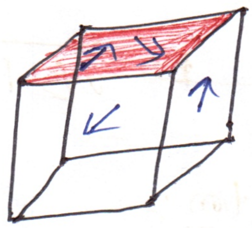
Explanation :-



(a) (b)

System of forces

- **Coplanar forces** [lines of action lying on the same plane]
  - **Collinear forces** [lines of action lying on the same line]
  - **Parallel forces** [lines of action lying on the parallel to each other]
  - **Concurrent forces** [Lines of action intersecting at a point]
  - **Non Concurrent and Non parallel forces** [lines of action neither parallel nor intersecting at a point]
- **Non Coplanar forces** [lines of action lying on different planes or in space]
  - **Collinear forces** (not possible)
  - **Parallel forces** [lines of action parallel to each other]
  - **Concurrent forces** [lines of action intersecting at a point]
  - **Non concurrent and Non parallel forces** [lines of action neither parallel not intersecting at a point]

Description	Coplanar forces	Forces in Space
(i) collinear forces		[Not possible to have]
(ii) parallel forces		
(iii) Concurrent forces		
(iv) Non concurrent non parallel forces		

Resolution and Resultant of Coplanar Concurrent forces(2D) :-

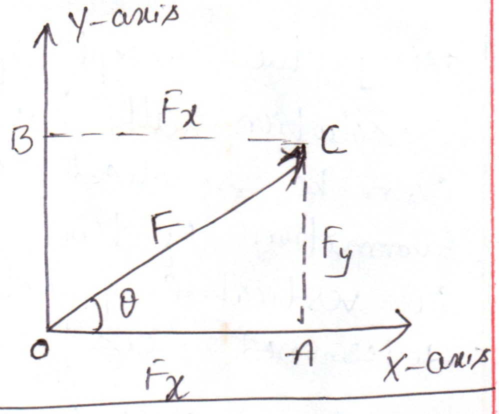
⇒ **Resolution** :- A forces "F" (OC) acting at an angle "θ" with the X-axis can be resolved into two mutually perpendicular components as shown in fig

$$F_x = F\cos\theta \,(OA) \quad \& \quad F_y = F\sin\theta \,(OB)$$

From ΔOAC

$$F = \sqrt{F_x^2 + F_y^2}$$

and $\tan\theta = \dfrac{F_y}{F_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right)$



$\sin\theta = \dfrac{opp \cdot s}{Hyp \cdot s}$ $\qquad$ $\cos\theta = \dfrac{\cancel{Hyp \cdot s}}{\cancel{Adj \cdot s}} = \dfrac{F}{\cancel{OA}}$

$\sin\theta = \dfrac{F_y}{F}$ $\qquad$ $\cos\theta = \dfrac{Adj \cdot S}{Hyp \cdot S} = \dfrac{OA}{F} = \dfrac{F_x}{F}$

$\Rightarrow F_y = F\sin\theta \,(OB)$ $\qquad$ $F\cos\theta = F_x$

Equilibrium :- A body is said to be in equilibrium when the resultant of the force system acting on it is zero

- If a body is in equilibrium it will continue to remain in a state of rest (or) of uniform motion.

Resultant :- To study the effect of a system of forces acting on a body, it is customary to replace the system of forces by its resultant

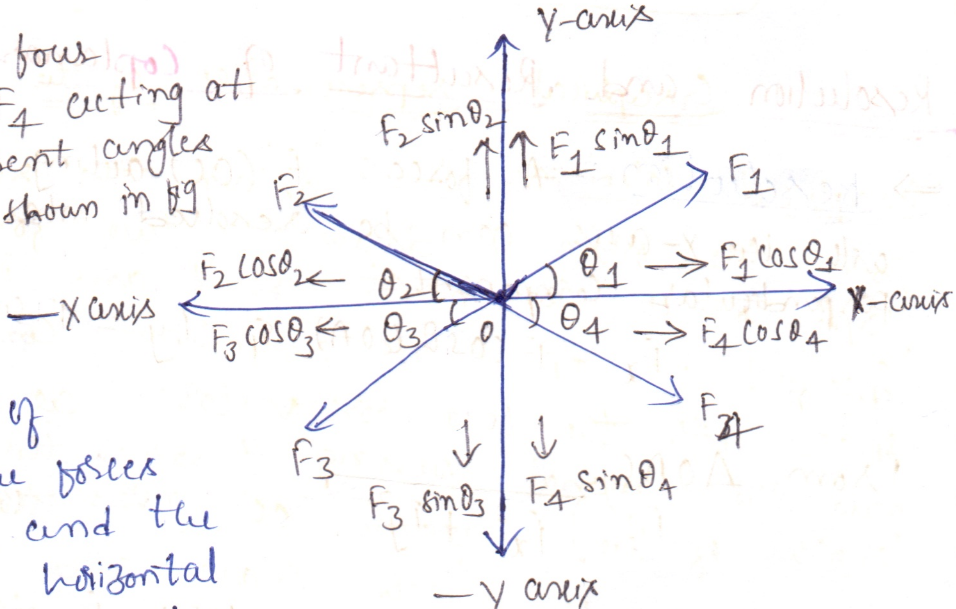
It is defined as single equivalent force which produces the same effect on the body as that of all given forces. This resultant is helpful in determining the motion of the body.

So that the Resultant 'R' is given by

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \quad \& \quad \tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

⇒ Explanation :-

Let us consider four forces F_1, F_2, F_3, F_4 acting at point "O" with different angles $\theta_1, \theta_2, \theta_3, \theta_4$ as shown in fig



Using the concept of resolution all the forces can be resolved and the summation of the horizontal & vertical components can be determined as

$$\Sigma F_x = F_1 \cos\theta_1 - F_2 \cos\theta_2 - F_3 \cos\theta_3 + F_4 \cos\theta_4$$

$$\Sigma F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2 - F_3 \sin\theta_3 - F_4 \sin\theta_4$$

⇒ Then the resultant, $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

& Angle $\tan\theta = \dfrac{\Sigma F_y}{\Sigma F_x}$

Resultant of Coplanar Concurrent forces :-

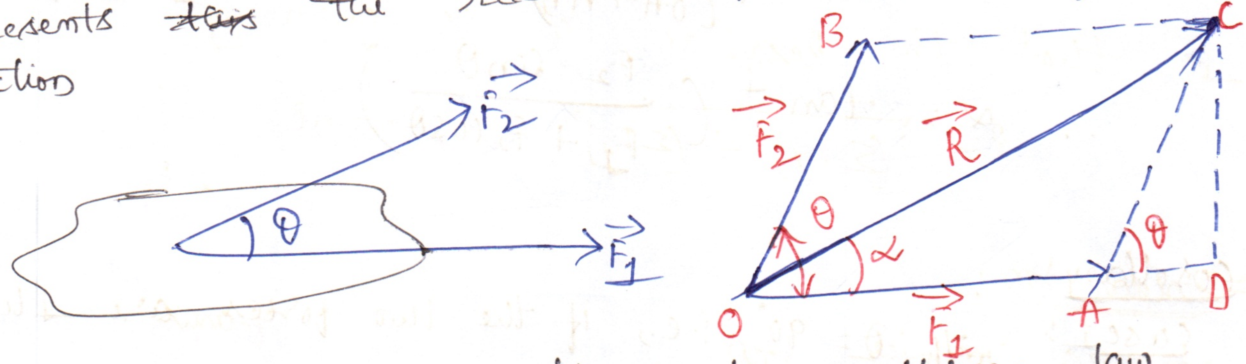
Various methods are employed to determine the resultant of concurrent forces in a plane. They are described below:

(i) Graphical method : parallelogram law, triangle law & polygon law

(ii) Trigonometric method : cosine law & sine law

(iii) Analytical method : vector approach

⇒ **Parallelogram law :-** The parallelogram law states that when two concurrent forces $\vec{F_1}$ & $\vec{F_2}$ acting on a body are represented by two adjacent sides of a parallelogram, the diagonal passing through their point of concurrency represents ~~the~~ the resultant force $\vec{R}$ in magnitude & direction



Graphical representation of the parallelogram law

Graphical solution :- To obtain the resultant graphically from the origin 'O', draw the two force vectors on a graph to a convenient scale and in the directions specified, i.e., OA & OB respectively. Complete the parallelogram OACB with the two forcevectors as adjacent sides. Then the Draw the diagonal passing through the origin. length of the diagonal 'OC' gives the magnitude of the resultant to scale and its inclination 'α' to the reference axis 'OA' gives the direction

⇒ The mathematical statement of the parallelogram law is called the law of cosine

The magnitude and direction of the resultant can also be determined from fig by trigonometry as follows

⇒ From ΔOCD, we know

$$OC^2 = (OA + AD)^2 + (CD)^2$$

⇒ Hence, the magnitude of the resultant $\vec{R}$ is given by

$$R = OC = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$

→ The inclination of $\vec{R}$ with $\vec{F}$ is given by

$$\alpha = \tan^{-1}\left(\frac{CD}{(OA+AD)}\right)$$

$$\therefore \alpha = \tan^{-1}\left(\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}\right)$$

$$\cos\theta = \frac{AD}{F_2}$$

$$\Rightarrow F_2\cos\theta = AD$$

$$\frac{CD}{F_2} = \sin\theta$$

$$F_2\sin\theta = CD$$

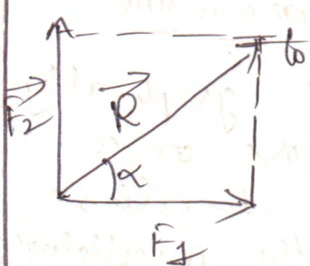
$$F_1^2 + F_2^2\cos^2\theta + 2F_1F_2\cos\theta + F_2^2\sin^2\theta$$

$$F_1^2 + F_2^2(\cos^2\theta + \sin^2\theta) + 2F_1F_2\cos\theta$$

$$\Rightarrow F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

Corollary :-

Case 1 :- If $\theta = 90°$, i.e., if the two forces are ⊥er to each other then

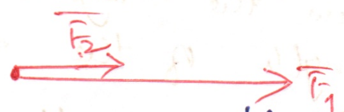


$$R = \sqrt{F_1 + F_2} \quad \& \quad \alpha = \tan^{-1}\left(\frac{F_2}{F_1}\right)$$

∴ Resultant of two concurrent & ~~⊥er~~ ⊥er forces

Case II: If $\theta = 0°$, i.e if the two forces are collinear and act in the same direction then

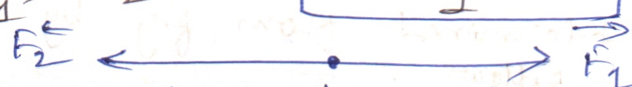
$$\Rightarrow \boxed{R = F_1 + F_2} \quad \& \quad \boxed{\alpha = 0}$$



∴ Resultant of two collinear forces in the same direction

Case III :- If $\theta = 180°$ i.e., if the two forces are collinear but acting in the opposite direction, where $F_1 > F_2$

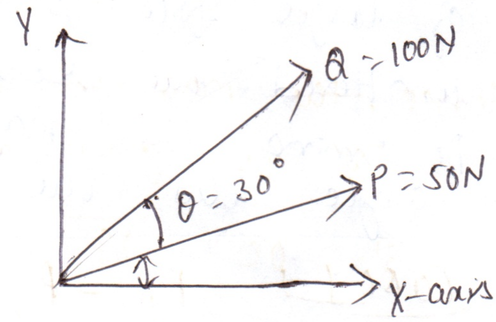
$$\boxed{R = F_1 - F_2} \quad \& \quad \boxed{\alpha = 0}$$



Resultant of two collinear forces in opposite direction

1) Two forces are acting at a point as shown in the figure. Determine the magnitude & direction of the resultant force. By using parallelogram law

Sol: Given data



P = 50N

Q = 100 N

θ = 30°

R = ?

α = ? (Inclination angle with resultant & X-axis)

⇒ Magnitude of resultant force

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$R = \sqrt{(50)^2 + (100)^2 + 2\cdot 50\cdot 100\cdot \cos 30^\circ}$$

$$R = \sqrt{2500 + 10,000 + 2\cdot 50\cdot 100\cdot \frac{\sqrt{3}}{2}}$$

$$\boxed{R = 145.46\,N}$$

Direction of resultant

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) \Rightarrow \boxed{\alpha = \tan^{-1}\left[\frac{F_2\sin\theta}{(F_1 + F_2\cos\theta)}\right]}$$

$$\alpha = \tan^{-1}\left(\frac{(Q\sin\theta)}{(P + Q\cos\theta)}\right)$$

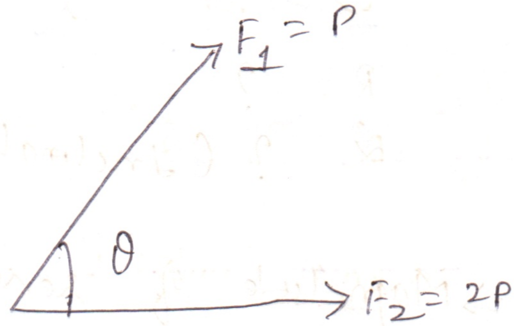
$$\alpha = \tan^{-1}\left(\frac{100\sin 30^\circ}{50 + 100\cos 30^\circ}\right)$$

$$\alpha = \tan^{-1}\left(\frac{100\times 1/2}{50 + 100\times \sqrt{3}/2}\right)$$

$$\alpha = \tan^{-1}\left(\frac{100\times\frac{1}{2}}{50 + 100\times\frac{\sqrt{3}}{2}}\right) \Rightarrow \alpha = 20.10^\circ$$

(2) The resultant of two forces one of which is double than the other is 235N. If the direction of larger force is reversed and the other remains unaltered the resultant forces reduces to 155N. Determine the magnitude of forces and the angle b/w the forces.

Sol: Case I :- $F_1 = P$, $F_2 = 2P$, $R = 235N$



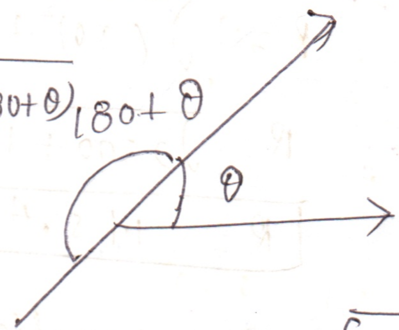
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$235 = \sqrt{P^2 + 2P^2 + 2(P \times 2P)\cos\theta}$$

Squaring on both sides

$$55225 = 5P^2 + 4P^2\cos\theta \quad \text{--- (1)}$$

Case II : $F_1 = P$, $F_2 = 2P$ & $R = 155N$ &



$$R = \sqrt{P^2 + (2P)^2 + 2(P \times 2P)\cos(180+\theta)} \quad 180+\theta$$

$$155 = \sqrt{5P^2 - 4P^2\cos\theta}$$

Squaring on both sides

$$24025 = 5P^2 - 4P^2\cos\theta \quad \text{--- (2)}$$

$\boxed{(\cos 180+\theta)}$

(1) & (2)

$P = 89N$

$F_1 = 89N$ & $F_2 = 178N$

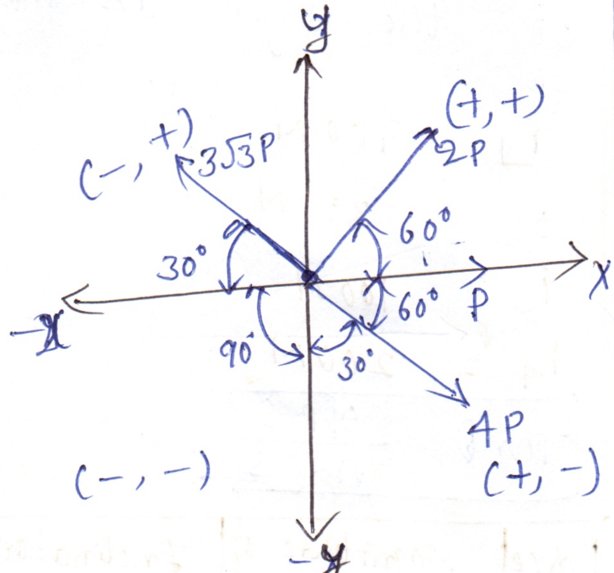
Sub P^2 value in eq (1)

$$5(7925) + 4(7925)\cos\theta = 55225$$

$$\cos\theta = 0.49$$

$$\theta = 60.65^\circ$$

3) Find the magnitude and direction of resultant are of concurrent forces as shown in figure.



Sol:-

Given data:-

Forces $F_1 = P\,N$

$F_2 = 2P\,N$

$F_3 = 3\sqrt{3}\,P\,N$

$F_4 = 4P\,N$

Angles

$\theta_1 = 0°,\quad \theta_2 = 60°,\quad \theta_3 = 30°,\quad \theta_4 = 60°$

Horizontal forces:-

$\Rightarrow F_{x_1} = P.\cos\theta_1$

$= P\times\cos 0°$

$\boxed{F_{x_1} = P}$

$\Rightarrow F_{x_2} = 2P\cos\theta°$

$= 2P\cos 60°$

$F_{x_2} = \cancel{2}P\,\frac{1}{\cancel{2}} = P$

$\boxed{F_{x_2} = P}$

$\Rightarrow F_{x_3} = 3\sqrt{3}\cos 30°$

$F_{x_3} = 4.5P$

$\Rightarrow F_{x_4} = 4P\cos\theta_4$

$= 4P\cos 60°$

$F_{x_4} = 2P$

Summation of horizontal forces

$\Sigma H = P + P - 4.5P + 2P$

$\Sigma H = \underline{-0.5PN}$

$\Rightarrow$ Vertical forces

$\Rightarrow F_{y_1} = P.\sin\theta$

$= P.\sin\theta_1$

$F_{y_1} = 0$

$\Rightarrow F_{y_2} = 2P\sin 60°$

$F_{y_2} = 1.73P$

$\Rightarrow F_{y_3} = 3\sqrt{3}\,P\sin 30°$

$\Rightarrow F_{y_3} = 2.59P$

$\Rightarrow F_{y_4} = 4P\sin 60°$

$F_{y_4} = 3.40P$

$\Rightarrow$ Summation of vertical forces

$\Sigma V = 0 + 1.73P + 2.59P - 3.40P$

$= \underline{0.86PN}$

$\Rightarrow$ Magnitude & Direction

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{P^2(\frac{1}{2})^2 + (\sqrt{3}/2)^2}$

$= P\sqrt{1/4 + 3/4} = P\sqrt{4/4} \Rightarrow PN$

$\Rightarrow \boxed{R = PN}$

$\Rightarrow$ Direction $\theta = \tan^{-1}\frac{\Sigma V}{\Sigma H} = \tan^{-1}\left(\frac{0.86P}{-0.5P}\right) \Rightarrow \tan^{-1}\left(\frac{\sqrt{3}/2}{-1/2}\right)$

$\boxed{\theta = 60°}$

(4) Determine the resultant of four forces concurrent at the origin as shown in fig

Sol:

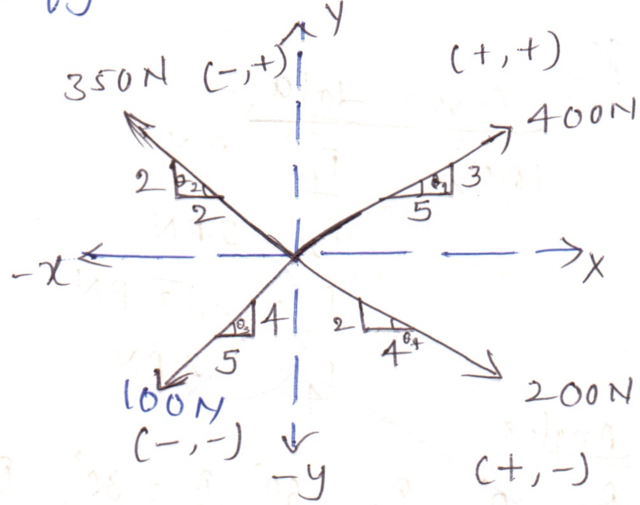
$F_1 = 400\,N$

$F_2 = 350\,N$

$F_3 = 100\,N$

$F_4 = 200\,N$

$$\boxed{\tan\theta = \frac{Opp\text{-}S}{Adj\text{-}S}}$$



Force	Magnitude of force (N)	Inclination of force with x-axis	Horizontal forces in(N)	vertical forces in(N)
F_1	400 N	$\theta = \tan^{-1}(3/5) = 30.96°$	$\Rightarrow F_{x_1} = F_1 \cdot \cos\theta_1 = 400 \times \cos(30.96)$, $F_{x_1} = 343.01\,N$	$\Rightarrow F_{y_1} = F_1 \sin\theta_1 = 400 \sin(30.96)$, $F_{y_1} = 205.78$
F_2	350 N	$\theta = \tan^{-1}(2/2) = 45°$	$\Rightarrow F_{x_2} = F_2 \cos\theta_2 = 350 \cos 45 = 247.49$	$F_{y_2} = F_2 \sin\theta_2 = 350 \sin 45°$, $\Rightarrow F_{y_2} = 247.49$
F_3	100 N	$\theta = \tan^{-1}(4/5) = 38.66°$	$\Rightarrow F_{x_3} = F_3 \cos\theta_3 = 100 \cos 38.66 = 78.09$	$\Rightarrow F_{y_3} = F_3 \sin\theta_3 = 100 \sin 38.66 = 62.47\,N$
F_4	200N	$\theta = \tan^{-1}(2/4) = 26.57°$	$\Rightarrow F_{x_4} = F_4 \cos\theta_4 = 200 \cos 26.57° = 178.88$	$\Rightarrow F_{y_4} = F_4 \sin\theta_4 = 200 \sin 26.57 = 89.46\,N$

$\Rightarrow$ Summation of all Horizontal forces $\Sigma H =$

$$= 343.01 - 247.49 - 78.09 + 178.88 = \underline{196.31\,N}$$

$\Rightarrow$ Summation of all vertical forces $\Sigma V =$

$$= 205.78 + 247.49 - 62.47 - 89.46 = \underline{301.34\,N}$$

$\Rightarrow$ $\therefore$ The magnitude of the resultant is given by

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$R = \sqrt{(196.31)^2 + (301.34)^2}$$

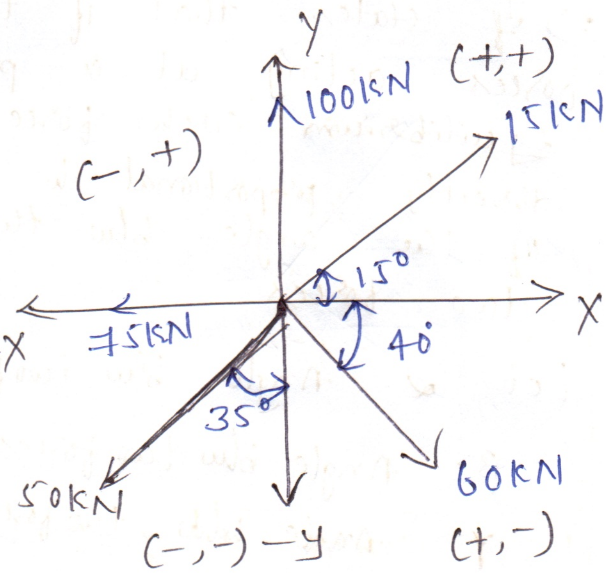
$$R = \underline{359.64\,N}$$

$\Rightarrow$ its inclinations w.r.to x-axis

$$\theta = \tan^{-1}\left[\frac{\Sigma V}{\Sigma H}\right] \Rightarrow \theta = \tan^{-1}\left(\frac{301.34}{196.31}\right)$$

$$\underline{\theta = 56.92°}$$

5) Find the magnitude and direction of resultant 'R' of the five concurrent forces acting as shown in figure



Sol: Given data

$F_1 = 15\,kN$

$F_2 = 100\,kN$

$F_3 = 75\,kN$

$F_4 = 50\,kN$

$F_5 = 60\,kN$

Angles $\theta_1 = 15°$, $\theta_2 = 90°$

$\theta_3 = 0°$, $\theta_4 = 90 - 35° = \underline{55°}$

$\theta_5 = 40°$

⇒ Horizontal forces

⇒ $F_{x_1} = F_1 \cos\theta_1$

$= 15 \cos 15°$

$F_{x_1} = \underline{14.48\,kN}$

⇒ $F_{x_2} = \underline{0}$

⇒ $F_{x_3} = \underline{75\,kN}$

⇒ $F_{x_4} = 50 \times \cos 55°$

$F_{x_4} = \underline{28.67\,kN}$

⇒ $F_{x_5} = 60 \cos 40°$

$= \underline{45.9\,kN}$

⇒ $\Sigma H = 14.48 - 75 - 28.67 + 45.9$

$\Sigma H = -43.29\,kN$

⇒ $\Sigma V = 3.882 + 100 - 40.95 - 38.56$

$\Sigma V = 24.37$

⇒ Vertical forces

⇒ $F_{y_1} = F_1 \sin\theta_1$

$= 15.5 \sin 15°$

$F_{y_1} = \underline{3.882\,kN}$

⇒ $F_{y_2} = F_2 \sin\theta_2$

$= 100 \sin 90°$

$F_{y_2} = \underline{100\,kN}$

⇒ $F_{y_3} = 0$

⇒ $F_{y_4} = F_4 \sin\theta_4$

$= 50 \cdot \sin(55°) = \underline{40.95}$

⇒ $F_{y_5} = F_5 \sin\theta_5$

$= 60 \sin 40$

$F_{y_5} = \underline{38.56\,kN}$

$\boxed{\Sigma H = -43.29\,kN}$ ⇒ $\boxed{\Sigma V = 24.37}$

⇒ $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-43.29)^2 + (24.37)^2}$

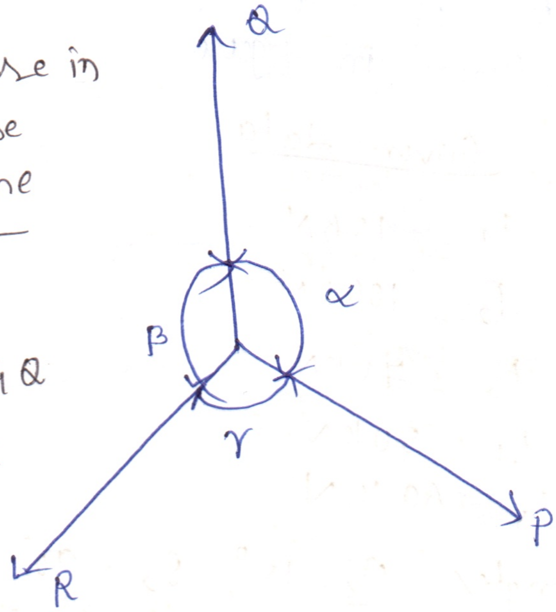
$\boxed{R = 49.67\,kN}$

$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$ ⇒ $\theta = \tan^{-1}\left(\frac{24.37}{43.29}\right)$

$\theta = \underline{29.37}$

Lami's theorem :-

⇒ It states that if three forces acting at a point are in equilibrium each force will be directly proportional to the sine of the angle b/w the other two forces



Let α = Angle b/w two forces P & Q

β = Angle b/w two forces Q & R

γ = Angle b/w two forces R & P

Then according to Lami's theorem

$P \propto \sin\beta$

$\Rightarrow \frac{P}{\sin\beta} = \text{constant} \longrightarrow (1)$

$\Rightarrow Q \propto \sin\gamma$

$\frac{Q}{\sin\gamma} = \text{constant} \longrightarrow (2)$

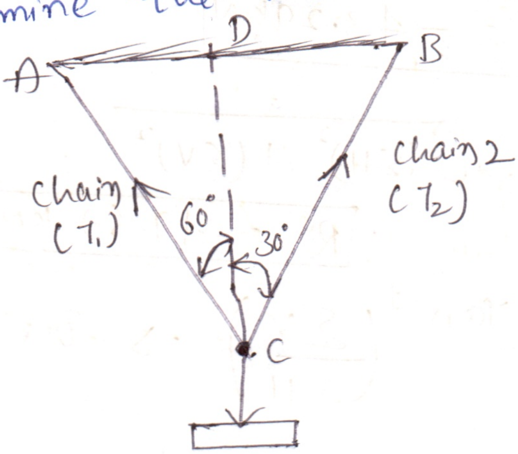
$\Rightarrow R \propto \sin\alpha$

$\frac{R}{\sin\alpha} = \text{constant} \longrightarrow (3)$

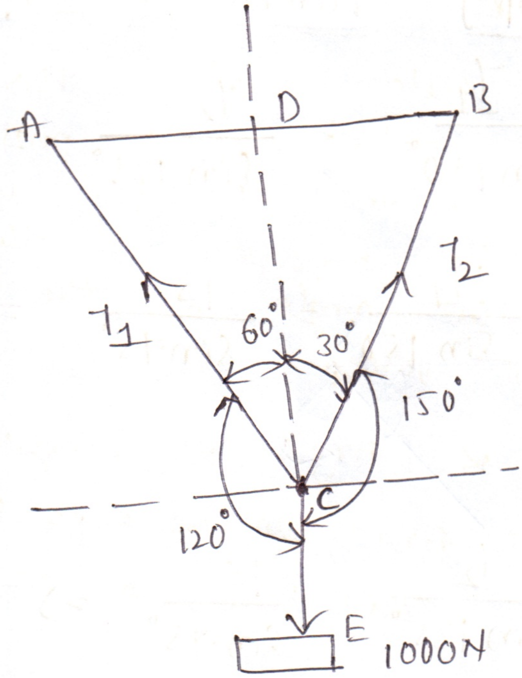
from (1) (2) & (3)

$$\boxed{\frac{P}{\sin\beta} = \frac{Q}{\sin\gamma} = \frac{R}{\sin\alpha} = \text{Constant}}$$

1) A weight of 1000 N is supported by two chains as shown in figure. Determine the tension in each chain?



Free bar diagram :-



Given data :-

weight at $E = 1000N$

T_1 = tension in chain 1

T_2 = tension in chain 2

From figure :-

$\angle ACE = 120°$

$\angle BCE = 150°$

$\angle ACB = 90°$

Applying Lami's theorem :-

$$\Rightarrow \frac{T_1}{\sin 150°} = \frac{T_2}{\sin 120°} = \frac{1000}{\sin 90°}$$

$$\frac{T_1}{\sin 150°} = \frac{1000}{\sin 90} = 1000N \Rightarrow \frac{T_1}{\sin 150} = 1000$$

$$\Rightarrow T_1 = \underline{500N}$$

$$\Rightarrow \frac{T_2}{\sin 120} = 1000 \Rightarrow T_2 = \underline{866 \cdot 02 N}$$

② An electric light weighing 15N hangs from a point "C" by two strings AC & BC. AC is inclined as 60° to the horizontal and BC at 45° to the vertical as shown in figure, using Lami's theorem. Determine the forces in the strings AC & BC

Sol:- Given data

$\angle AC = 60°$

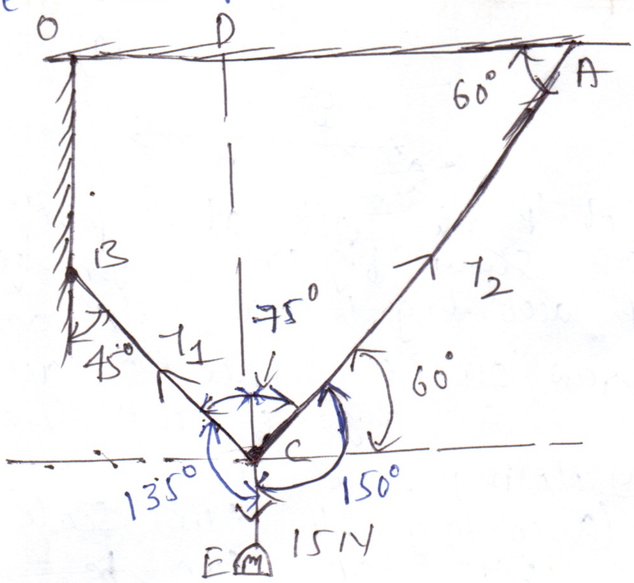
$\angle BC = 45°$

weight = 15N

From figure $\angle BCE = 135°$

$\angle ACE = 150°$

$\angle ACB = 75°$



Apply Lami's theorem :-

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{15}{\sin 75^\circ}$$

$$\frac{T_1}{\sin 150^\circ} = \frac{15}{\sin 75^\circ} \Rightarrow T_1 = \frac{15}{\sin 75} \times \sin 150^\circ$$

$$\Rightarrow T_1 = 7.76 N$$

$$\Rightarrow \frac{T_2}{\sin 135^\circ} = \frac{15}{\sin 75^\circ} \Rightarrow T_2 = \frac{15}{\sin 75^\circ} \times \sin 135^\circ$$

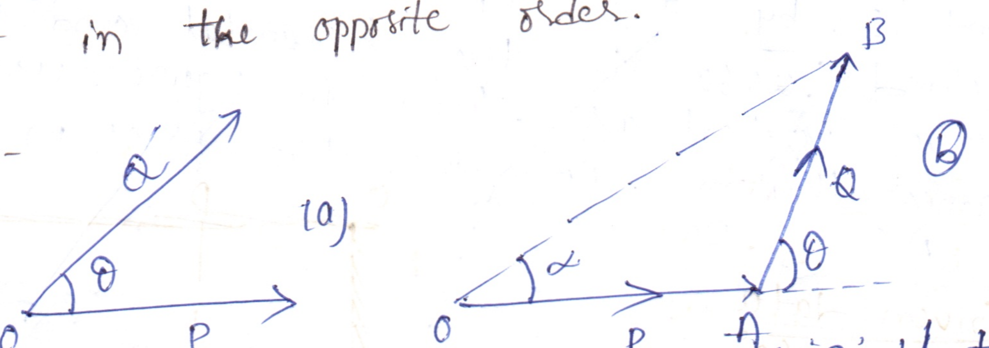
$$\Rightarrow T_2 = 10.98 N$$

$\boxed{T_1 = 7.76 N}$ $\boxed{T_2 = 10.98 N}$

⇒ **Triangle law :-**

The resultant can also be determined by the triangle law which states that if two forces $\vec{F_1}$ and $\vec{F_2}$ acting simultaneously on a body can be represented by the two sides of a triangle (in magnitude and direction) taken in order then the third ~~scale~~ side (closing side) represent the resultant in the opposite order.

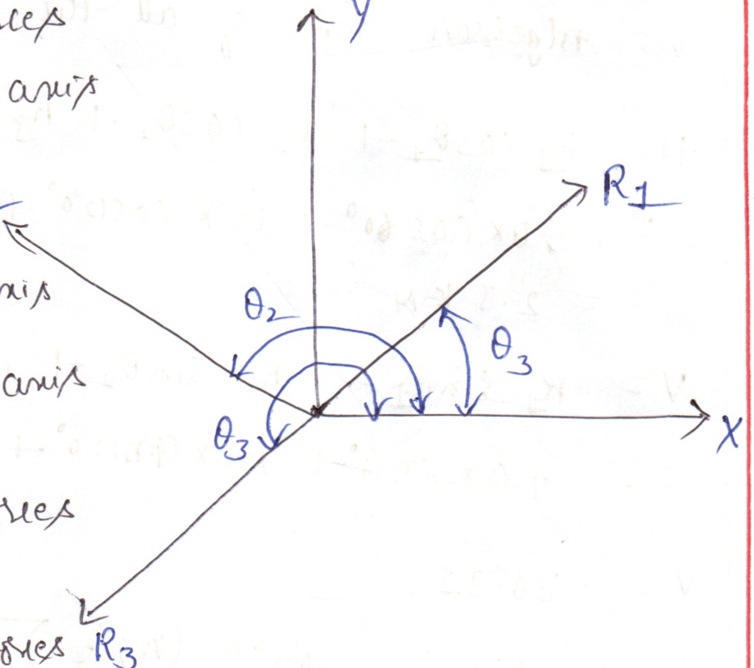
Explanation :-



⇒ Let 'P' & 'Q' act at a point 'O' with an angle 'θ' b/w them as shown fig (a) & In fig (b) draw OA i.e., force 'P' according to a scale and extend its line of action. Draw "OB' i.e., force "Q" according to the scale at an angle "θ" with line of action of 'P'.
By closing side "OB" the resultant magnitude 'R" (according to the scale taken) and the direction (angle α) with OA can be determined.

Resolution of Numbers of Coplanar forces

Let R_1, R_2, & R_3 are forces



→ θ_1 = Angle b/w R_1 and x-axis

→ θ_2 = Angle b/w R_2 and x-axis

→ θ_3 = Angle b/w R_3 and x-axis

→ θ = Angle b/w R and x-axis

→ H = Algebraic sum of all forces along x-axis

→ V = Algebraic sum of all forces along x-axis

$\Rightarrow H = R_1 \cos\theta_1 + R_2 \cos\theta_2 + R_3 \cos\theta_3$

$\Rightarrow V = R_1 \sin\theta_1 + R_2 \sin\theta_2 + R_3 \sin\theta_3$

Resultant force $(R) = \sqrt{H^2 + V^2}$

Angle b/w R & x-axis $\Rightarrow \theta = \tan^{-1}\left[\frac{V}{H}\right]$

problem

(1) Three forces of magnitude 40 kN, 15 kN, & 20 kN are acting at a point "O" as shown in figure. The angles made by 40 kN, 15 kN & 20 kN forces are with x-axis are 60°, 120°, 240° respectively. Determine the magnitude and direction of the resultant force

Sol:- Given data

R_1 = 40 kN

R_2 = 15 kN

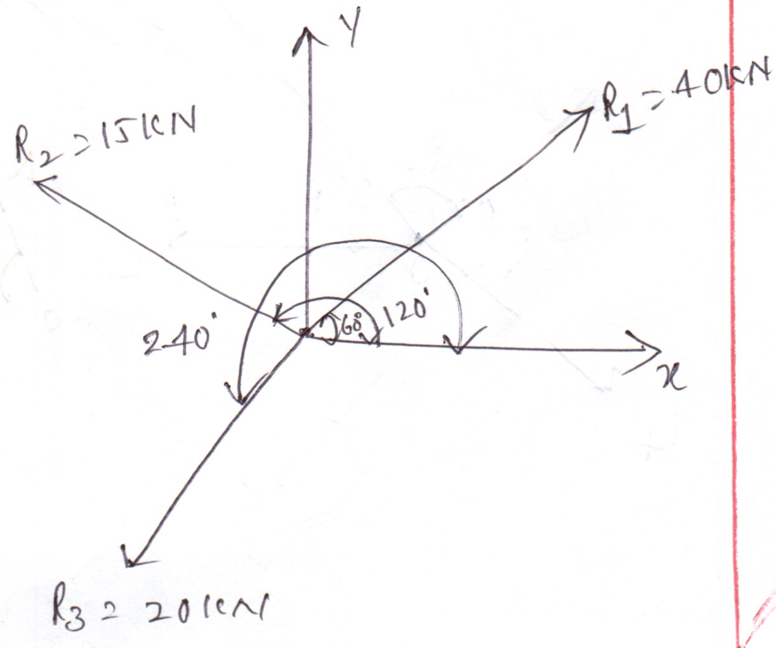
R_3 = 20 kN

Angles (θ)

$\theta_1 = 60°$

$\theta_2 = 120°$

$\theta_3 = 240°$



H = Algebraic Sum of all the forces along X-axis

V = Algebraic Sum of all the forces along Y-axis

$$H = R_1 \cos\theta_1 + R_2 \cos\theta_2 + R_3 \cos\theta_3$$

$$= 40 \times \cos 60^\circ + 15 \times \cos 120^\circ + 20 \times \cos 240^\circ$$

$$= 2.5\ KN$$

$$V = R_1 \sin\theta_1 + R_2 \sin\theta_2 + R_3 \sin\theta_3$$

$$= 40 \times \sin 60^\circ + 15 \times \sin 120^\circ + 20 \sin 240^\circ$$

$$V = 30.31$$

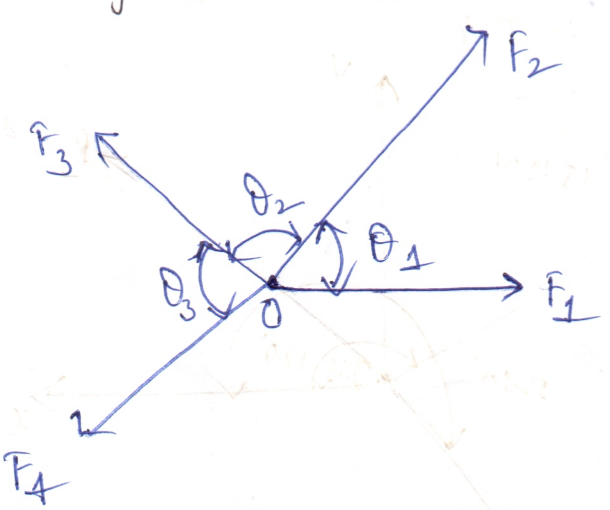
Resultant forces $(R) = \sqrt{H^2 + V^2}$

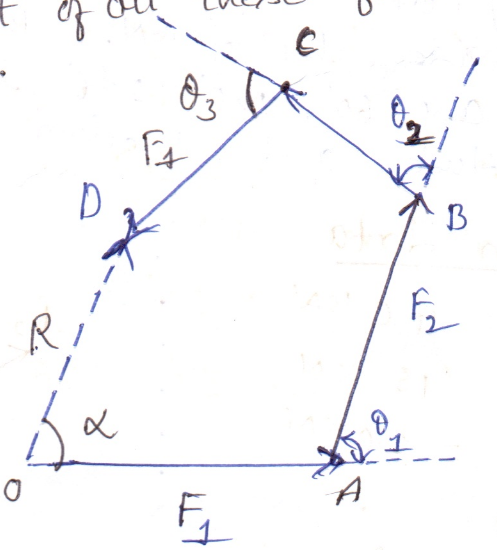
$$= 30.4\ kN$$

$$\theta = \tan^{-1}\left[\frac{30.31}{2.5}\right] = \underline{85.25^\circ}$$

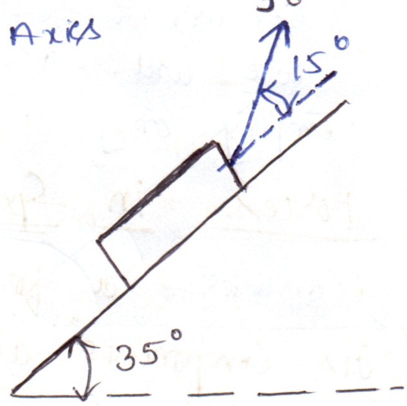
⇒ **Polygon law (Graphical Method) :-**

If the number of forces acting at a point are represented in magnitude and direction by the sides of a polygon taken in order, then the closing side of the polygon taken in opposite order represents the resultant of all these forces in magnitude and direction.





Problem:- 1) Find the components of a force of magnitude 50N acting on a block as shown in figure
(i) ~~parallel~~ Along lines parallel and perpendicular to the inclination plane and
(ii) Along the horizontal and vertical Axis

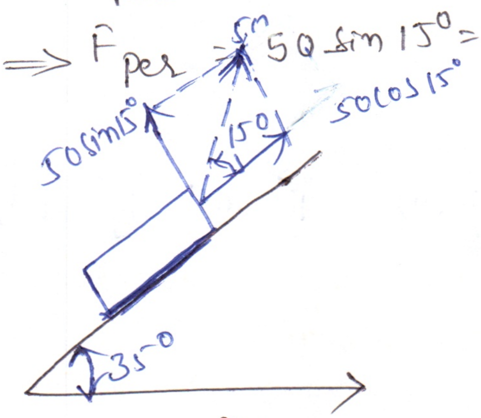


Sol:- (i) Components of the forces parallel and perpendicular to the inclined plane

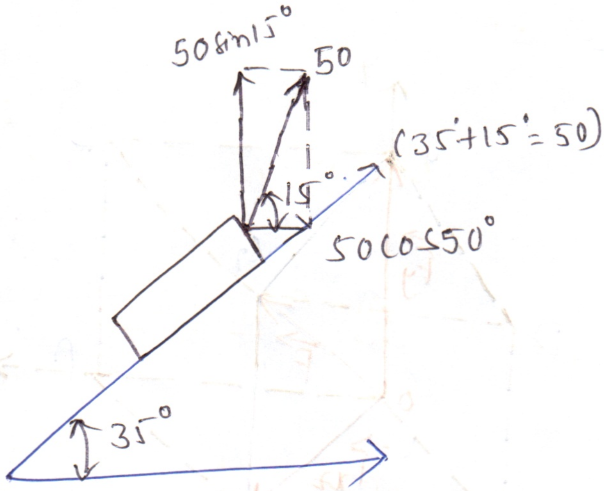
⇒ As the inclination of the force w.r.to the inclined plane is 15°
its components parallel and perpendicular to the inclined plane are

$\Rightarrow F_{par} = 50\cos 15^\circ = 48.3\,N$

$\Rightarrow F_{per} = 50\sin 15^\circ = 12.94\,N$



(a)



(b)

(ii) Components of the force along horizontal & vertical Axes

The inclination of the force w.r.to the horizontal axis is 50°. Hence its components along the horizontal and vertical axes are

$F_x = 50\cos 50^\circ = \underline{32.14\,N}$

$F_y = 50\sin 50^\circ = \underline{38.3\,N}$

Concurrent forces in Space :-

⇒ Previous sections we discussed the coplanar Concurrent forces.
we will discuss concurrent forces in Space. Hence we employ only analytical method to solve forces in Space.

Forces in Space :-

Consider a force $\vec{F}$ in space acting at the origin Its components along mutually ⊥lars X, Y & Z axes can be determined by the method of resolution

⇒ (a) $\vec{F}$ into $\vec{F_y}$ & F_{xz} along OCGE

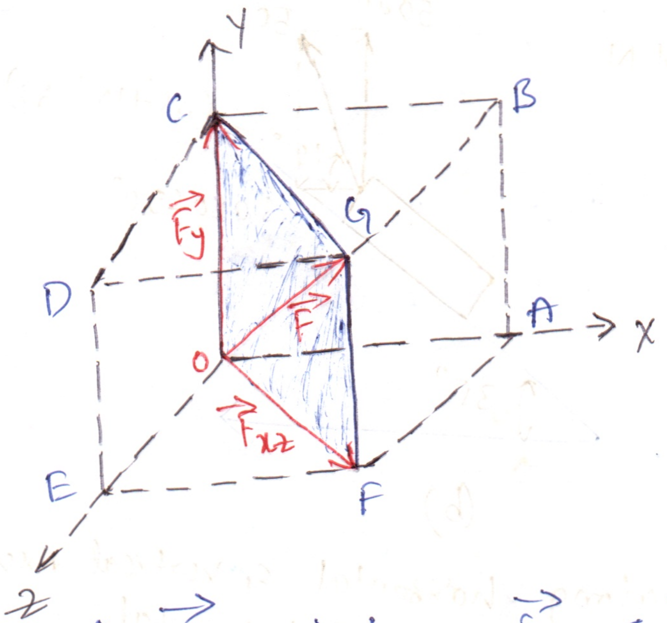
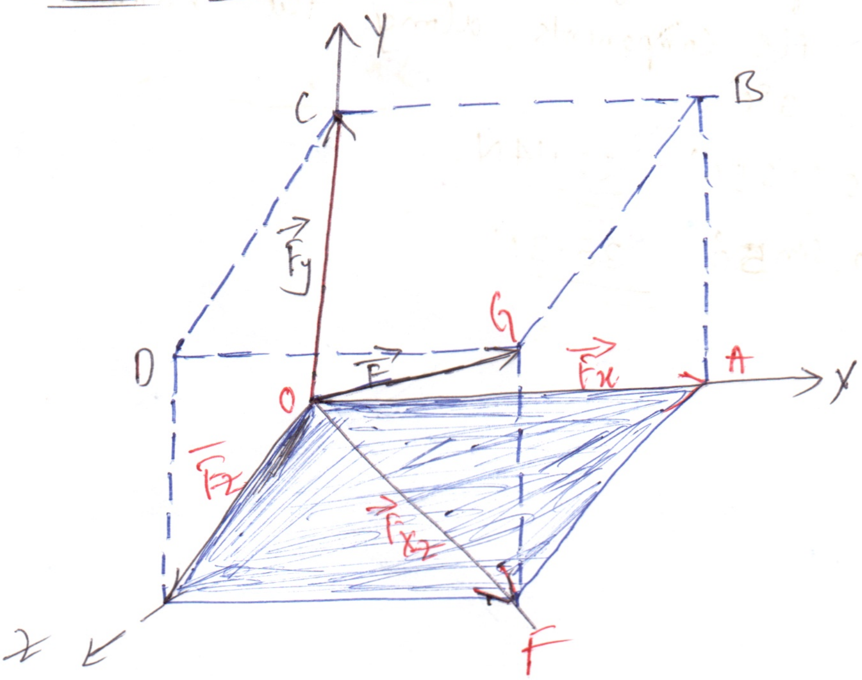


Fig (a) $\vec{F}$ into $\vec{F_y}$ & $\vec{F_{xz}}$

(b) $\vec{F_{xz}}$ into $\vec{F_x}$ & $\vec{F_z}$ on x-z plane



⇒ By vector Addition

$$\vec{F} = \vec{F_y} + \vec{F_{xz}}$$

$$F = \vec{F_x} + \vec{F_y} + \vec{F_z} \quad \left[\because \vec{F_{xz}} = \vec{F_x} + \vec{F_z}\right] \quad \text{—(i)}$$

$$\boxed{\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}} \quad \text{—— (ii)}$$

(or)

⇒ i, j, k are unit vectors represented along x, y & z direction

Magnitude of Force is represented in terms of Components

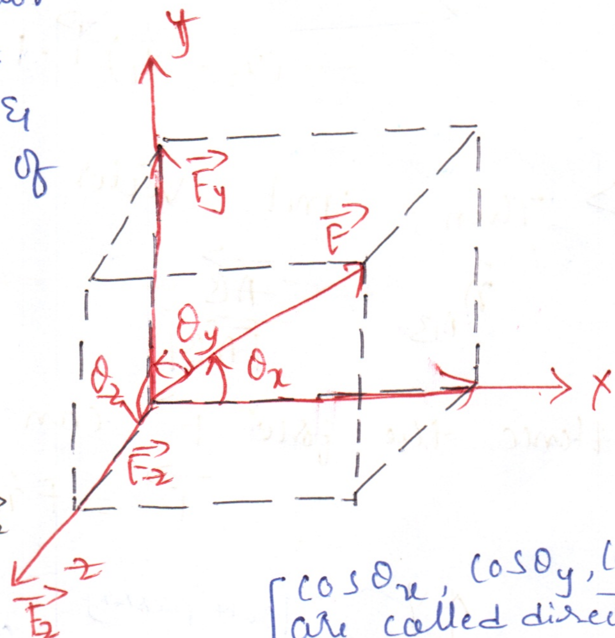
$$\boxed{\vec{F} = F = \sqrt{F_x^2 + F_y^2 + F_z^2}} \quad \text{— (iii)}$$

* ⇒ If θ_x, θ_y & θ_z are angles made by $\vec{F}$ with x, y, z axes respectively

$$\vec{F} = F\cos\theta_x\vec{i} + F\cos\theta_y\vec{j} + F\cos\theta_z\vec{k}$$

$$\Rightarrow \vec{F} = F\left[\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right] \quad \text{— (iv)}$$

⇒ we know that Any vector can be expressed as a ~~of~~ product of its magnitude & unit vector along its line of action, Hence force vector can also written as



$$\boxed{\vec{F} = F\hat{n}}$$

$$\hat{n} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

$\hat{n}$ is unity

⇒ Comparing eqn (iv)

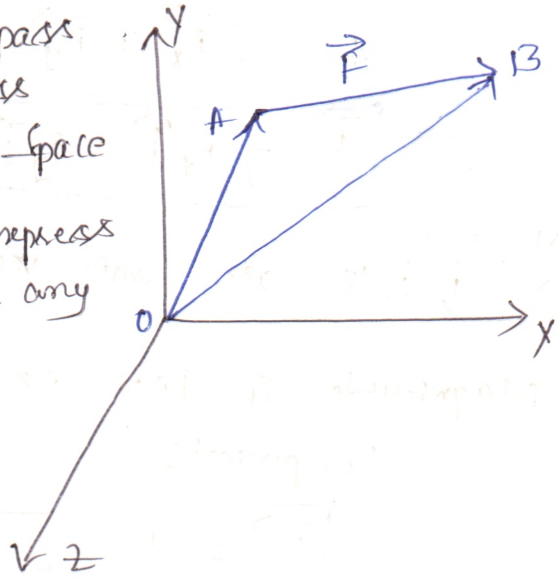
[$\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are called direction cosines]

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

⇒ In direction cosines can be expressed as

$$\boxed{\cos\theta_x = \frac{F_x}{F}}, \quad \boxed{\cos\theta_y = \frac{F_y}{F}}, \quad \& \quad \boxed{\cos\theta_z = \frac{F_z}{F}}$$

Representation of a force passing through any two points in space



In general forces do not pass through the origin, but pass through any two points in space

⇒ Hence, we will see how to express a force vector passing through any two points in space.

⇒ Consider a force $\vec{F}$ passing through two points A & B in space whose respective coordinate are (x_1, y_1, z_1) & (x_2, y_2, z) Then the position vectors of the points A & B w.r.to the origin are

$$\vec{OA} = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$$

$$\vec{OB} = x_2\vec{i} + y_2\vec{j} + z_2\vec{k}$$

Hence $\vec{AB} = \vec{OB} - \vec{OA}$

$$= (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$$

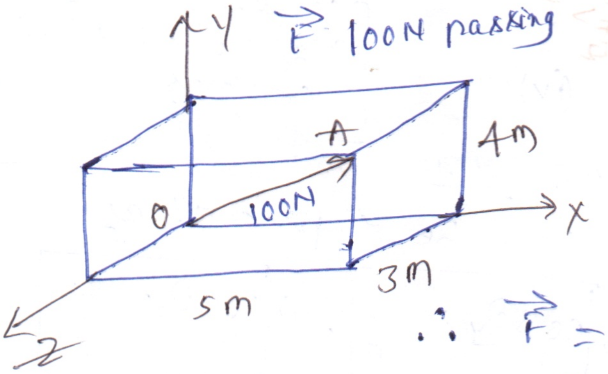
⇒ Then unit vector along AB is given by

$$\hat{n}_{AB} = \frac{\vec{AB}}{|\vec{AB}|}$$

Hence the force $\vec{F}$ can be expressed as

$$\vec{F} = F\hat{n}_{AB}$$

1 problem



$\vec{OA} = 5\vec{i} + 4\vec{j} + 3\vec{k}$

$|\vec{OA}| = \sqrt{50}$ m

$\hat{n} = \frac{\vec{OA}}{|\vec{OA}|} = \frac{5\vec{i} + 4\vec{j} + 3\vec{k}}{\sqrt{50}}$

$\therefore \vec{F} = F\hat{n}_{OA} = 100[\hat{n}_{OA}]$

$= 70.71N\vec{i} + 56.57N\vec{j} + 42.43\vec{k}$

Resultant of Several Components of Concurrent forces in Space

$$\left[\begin{array}{l} \vec{F}_1 = F_{1x}\vec{i} + F_{1y}\vec{j} + F_{1z}\vec{k} \\ \vec{F}_2 = F_{2x}\vec{i} + F_{2y}\vec{j} + F_{2z}\vec{k} \\ \vdots \\ \vec{F}_n = F_{nx}\vec{i} + F_{ny}\vec{j} + F_{nz}\vec{k} \end{array}\right]$$

$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

$$\vec{R} = \Sigma(F_x)_i\vec{i} + \Sigma(F_y)_i\vec{j} + \Sigma(F_z)_i\vec{k}$$

$$\Rightarrow R_x\vec{i} + R_y\vec{j} + R_z\vec{k} = \Sigma(F_x)_i\vec{i} + \Sigma(F_y)_i\vec{j} + \Sigma(F_z)_i\vec{k}$$

$$\vec{R_x} = \Sigma(F_x)_i, \quad R_y = \Sigma(F_y)_i \quad \& \quad R_z = \Sigma(F_z)_i$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \sqrt{\Sigma(F_x)_i^2 + \Sigma(F_y)_i^2 + \Sigma(F_z)_i^2}$$

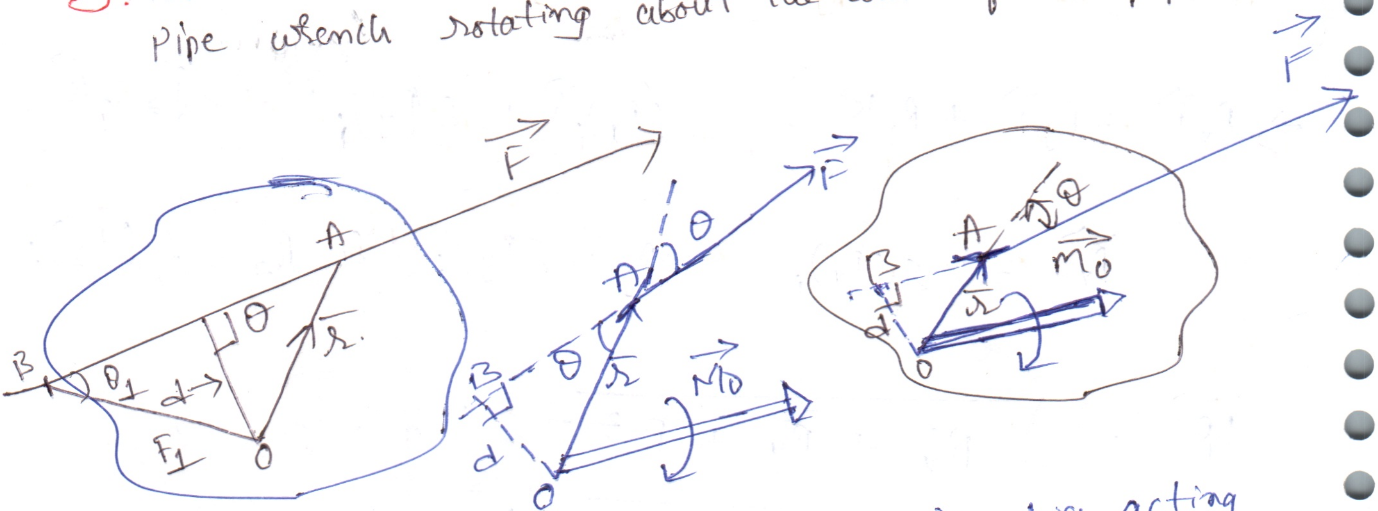
$$\cos\theta_x = \frac{R_x}{R} = \frac{\Sigma(F_x)_i}{R}, \quad \cos\theta_y = \frac{R_y}{R} = \frac{\Sigma(F_y)_i}{R}$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{\Sigma(F_z)_i}{R}$$

⇒ SYSTEM OF FORCES & Resultant - II
(Non Concurrent forces) :-

Moment of a force :- A force acting on a body can translate it. at the same time it can rotate the body also. These rotational effect produced by a force is a measure of moment of a force.

Eg: Door rotating about its hinge
Pipe wrench rotating about the axis of the pipe.



⇒ Consider a force $\vec F$ as shown in fig acting at a point 'A' whose position vector is $\vec r$. The effect of such a force will be to rotate the body about the origin, assuming the body is free to rotate about the origin. This rotational effect of $\vec F$ about the origin is determined by a physical quantity is called moment of a force

$$\boxed{\Rightarrow \sin\theta = \frac{d}{r} \quad [r\sin\theta = d]}$$

$$[\vec{M_0} = \vec r \times \vec F]$$

$$\vec{M_0} = \vec r \times \vec F$$

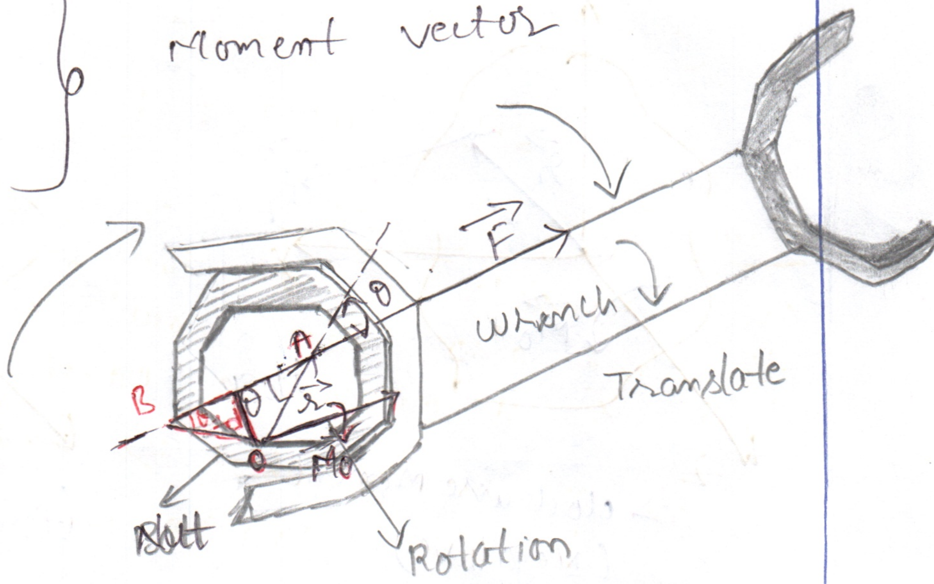
$$= \vec r \times F\sin\theta$$

[Magnitude of force is $F\sin\theta$]

$$\vec{M_0} = (r\sin\theta) \times F$$

$$\boxed{\vec{M_0} = F d}$$

$$\left.\begin{aligned} \vec{r} &= x\vec{i} + y\vec{j} + z\vec{k} \\ \vec{F} &= \vec{F_x} + F_y\vec{j} + F_z\vec{k} \end{aligned}\right\} \text{Moment vector}$$



$$\vec{M_0} = \underbrace{x\vec{i} + y\vec{j} + z\vec{k}}$$

$$\downarrow$$

$$\vec{r} \times \vec{F}$$

$$\downarrow$$

$$(F_x\vec{i} + \vec{F_y} + \vec{F_z}\vec{k})$$

$$\vec{M_0} = \vec{r} \times \vec{F} = (x\vec{i} + y\vec{j} + z\vec{k}) + (F_x\vec{i} + F_y\vec{j} + F_z\vec{k})$$

$$= [xF_y\vec{k} - xF_z\vec{j} - yF_x\vec{k} + yF_z\vec{i} + zF_x\vec{j} - zF_y\vec{i}]$$

$$* \begin{bmatrix} \vec{i} \times \vec{j} = \vec{k} & \vec{j} \times \vec{i} = -\vec{k} \\ \vec{j} \times \vec{k} = \vec{i} & \vec{k} \times \vec{j} = -\vec{i} \\ \vec{k} \times \vec{i} = \vec{j} & \vec{i} \times \vec{k} = -\vec{j} \end{bmatrix}$$

$$[\because \bar{i}.\bar{i} = \bar{j}\,\vec{j} = \vec{k}.\vec{k} = 1]$$

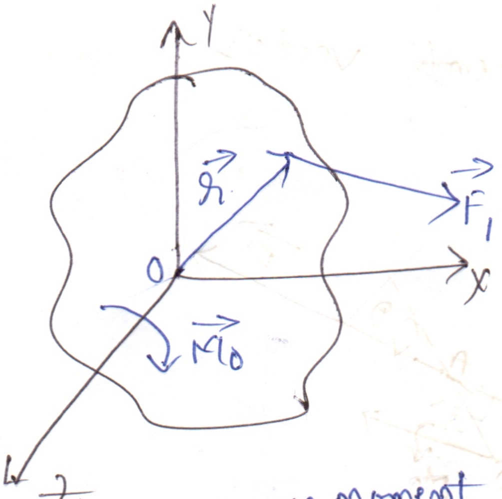
$$[\bar{i}.\bar{j} = \bar{j}.\bar{i} = \bar{j}.\bar{k} = \bar{k}.\bar{j} = \vec{k}.\vec{i} = \vec{i}.\vec{k} = 0]$$

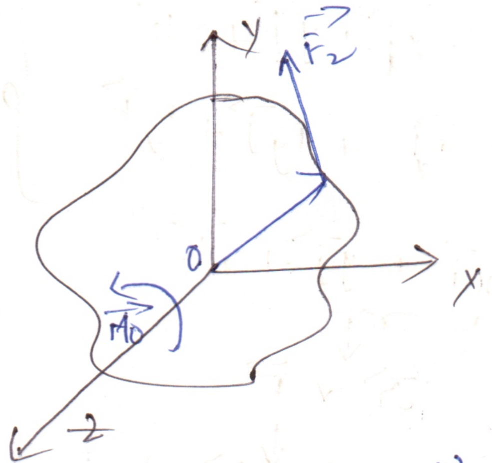
$$\therefore \vec{M_0} = (yF_z - zF_y)\vec{i} - (xF_z - zF_x)\vec{j} + (xF_y - yF_x)\vec{k}$$

$$\vec{M_0} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \Rightarrow \text{in determinant form}$$

$\Rightarrow M_x, M_y$ & M_z are components of $\vec{M_0}$

$$\boxed{M_0 = \sqrt{M_x^2 + M_y^2 + M_z^2}}$$





⇒ clockwise moment
(Negative)

(Anticlockwise moment)
(positive)

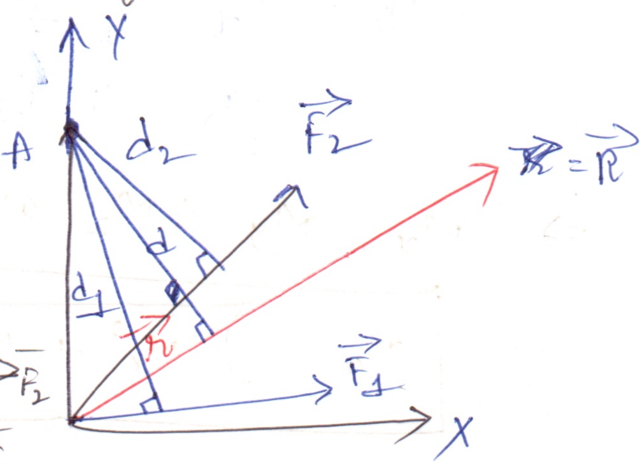
⇒ we can avoid vector approach by considering clockwise moments as Negative & Anticlockwise moment positive

F_x = clockwise & F_y = Anticlockwise

$$\Rightarrow \boxed{M_0 = -yF_x + xF_y}$$

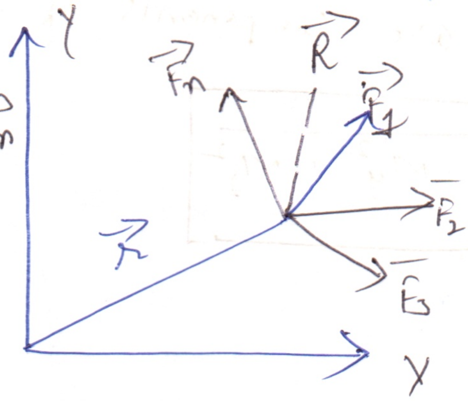
Varignon's theorem (or) Principle of Moments

Def: The varignon theorem states that the moment about a given point 'O' of the Resultant of several concurrent forces is equal to the sum of the moments of individual forces about the same point 'O'



$\vec{R}$ =
$\bar{F}_1 + \bar{F}_2 + \cdots + \vec{F_n}$

$$\boxed{\vec{M_0} = \vec{r} \times \vec{R}}$$



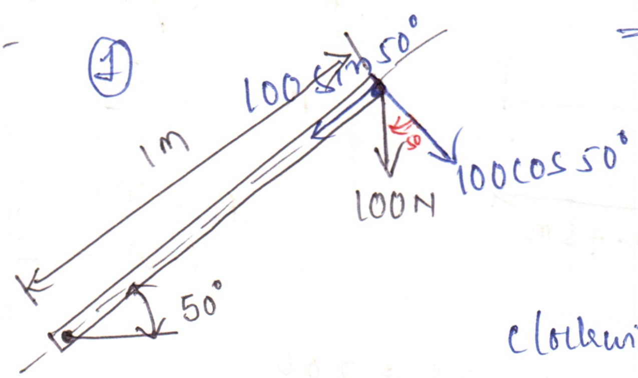
$$\vec{M_0} = (\vec{r} \times \vec{F_1}) + (\vec{r} \times \vec{F_2}) + \cdots (\vec{r} \times \vec{F_n})$$

$$\boxed{\vec{M_0} = \sum (\vec{r} \times \vec{F_i})}$$

1) A 100N force is applied at the end 'A' of a lever "OA" (i) horizontally (ii) vertically and (iii) perpendicular to the lever as shown in figure below. Determine the moment of the force about 'O' in each case. What do you infer from the three moments?

$\boxed{\begin{array}{l} M_O = F \times d \\ M_O = r \times F \end{array}}$

Sol:-

① 

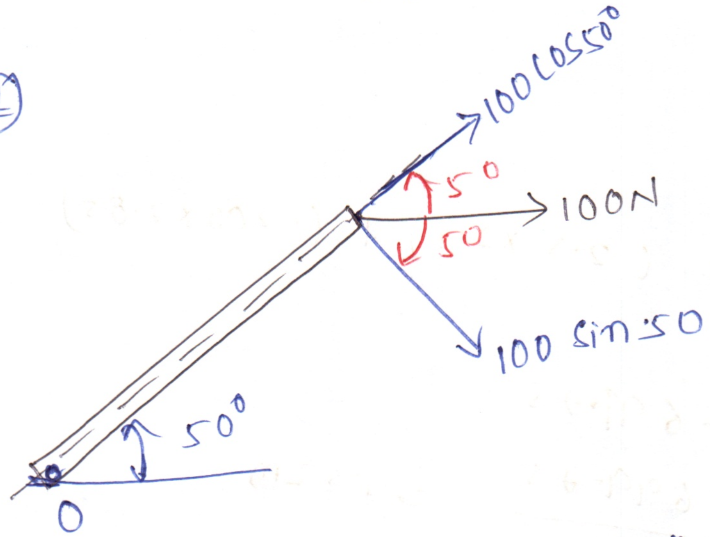
⇒ * when the force is applied vertically

$\Rightarrow M_O = -100\cos 50° \times 1$

$= -62.48\ N\cdot m$

clockwise is ~~+ve~~ "-ve"

⇒ 100 sin 50 pass through the origin does not contribute the moment of a force

② 

when the force is applied vertically

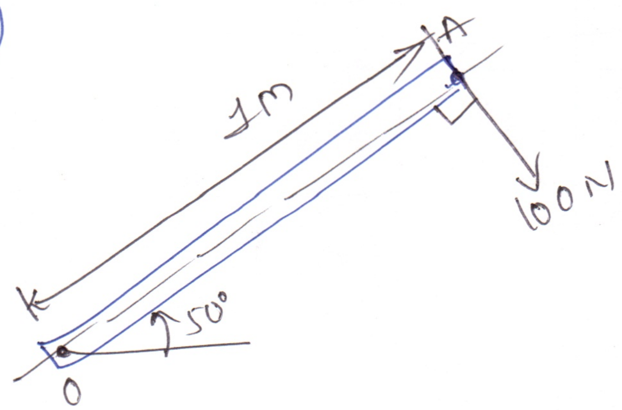
$M_O = -100 \sin 50 \times 1$

$= -76.6\ N\cdot m$

100 cos 50° ~~100cos50°~~ pass through the origin

-ve ⇒ clockwise

⇒ [cos 90° = 0]

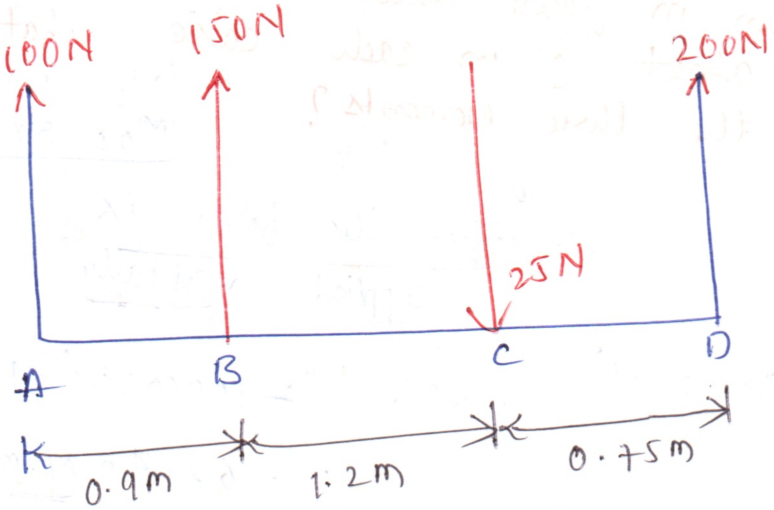
③ 

⇒ when the force is applied ⊥ to the lever

$M_O = -100\,(1)$

$= -100\ N\cdot m$

6



$R = \sqrt{\cancel{\Sigma F_x}^{\,0} + \Sigma F_y}$

$R = \Sigma F_y$

A.C.W = +ve

C.W = −ve

$$R = \Sigma F_y = 100 + 150 - 25 + 200$$
$$= +425\,N$$

$$Rd = \sum_{i=1}^{n} F_i d_i$$

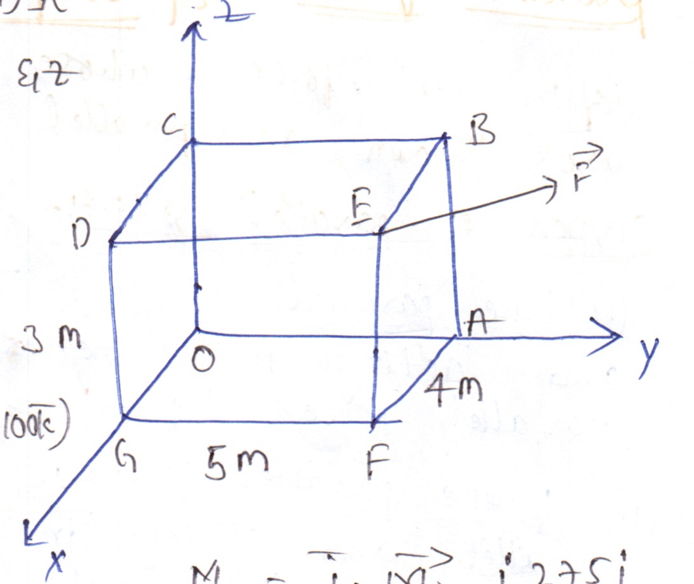
$$= (-150 \times 0.9) + (2.5 \times 2.1) + (-200 \times 2.85)$$

$$Rd = -699.75$$

$$d = -\frac{699.75}{425} = 1.53\,m$$

(3) $\vec{F} = (50N)\vec{i} + (75N)\vec{j} + (100N)\vec{k}$

moment of the force x, y, & z



$\vec{OE} = 4\vec{i} + 5\vec{j} + 3\vec{k}$

$\vec{M_o} = \vec{OE} \times \vec{F}$

$= (4\vec{i} + 5\vec{j} + 3\vec{k}) \times (50\vec{i} + 75\vec{j} + 100\vec{k})$

$= \begin{vmatrix} i & j & k \\ 4 & 5 & 3 \\ 50 & 75 & 100 \end{vmatrix}$

$\vec{M_o} = 275\vec{i} - 250\vec{j} + 50\vec{k}$

$M_x = \vec{i} \cdot \vec{M_o}$ $i \cdot 275i$

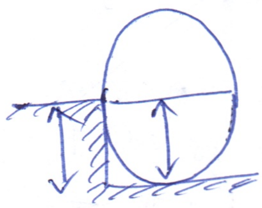
$= 275$ N·m

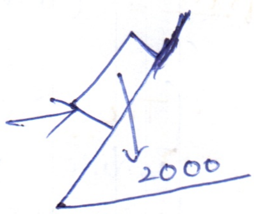
$M_y = \vec{j} \cdot \vec{M_o} = -250$ N·m

$M_z = \vec{k} \cdot \vec{M_o}$

$= 50$ N·m

problems (3)





Parallel forces :- & Couple :-

Def: The force whose lines of action are parallel are known as parallel forces

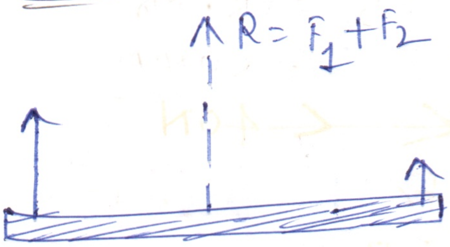
Types of parallel forces :-

(i) **Like parallel forces** :- The forces which are parallel and acting in same direction are called like parallel forces ($\uparrow \uparrow$)

(ii) **unlike parallel forces** :- The forces which are parallel and acting in opposite direction are called unlike parallel parallel forces ($\uparrow \downarrow$)

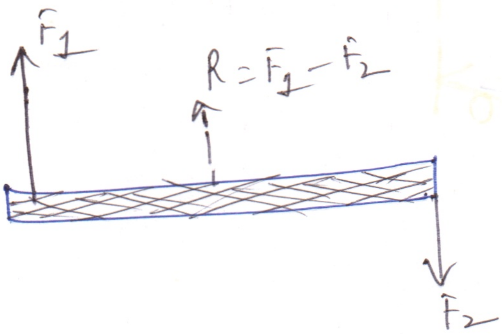
Determine of resultant of parallel forces

(i) **Two like parallel forces** :- (with equal (or) unequal magnitude)



$\therefore$ The resultantant of two like parallel forces F_1 & F_2 is given by $R = F_1 + F_2$

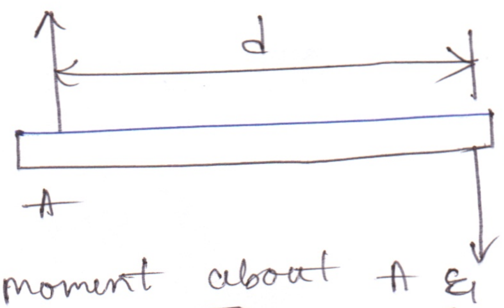
(ii) **Two unlike parallel forces** :- (with unequal magnitude)



$\therefore$ [The resultant of two unlike parallel forces F_1 & F_2 is given by $\underline{R = F_1 - F_2}$]

(iii) **Two unlike parallel forces** :- (with equal magnitude)

The resultant of forces $R = F - F = 0$



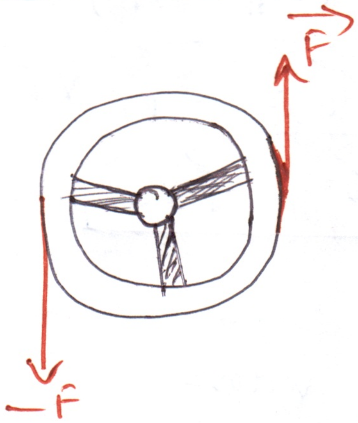
[The resultant of two equal & unlike parallel forces, $R = F - F = 0$,

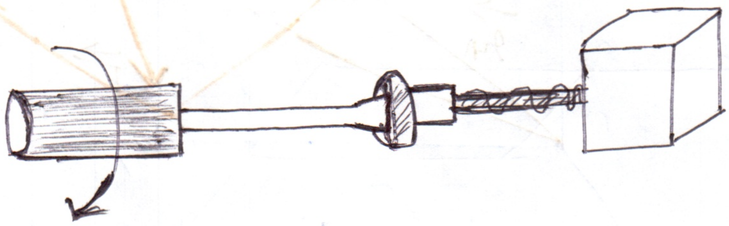
But the moment about A & B is not equal to zero

$$\Rightarrow \boxed{M_A^F = F \times d = M_B^F}$$

Couple :- when two forces $\vec{F}$ & $-\vec{F}$ having the same magnitude, parallel lines of action and opposite sense act on a body then they are said to form a couple

Eg : Steering vehicle & Tightening a screw



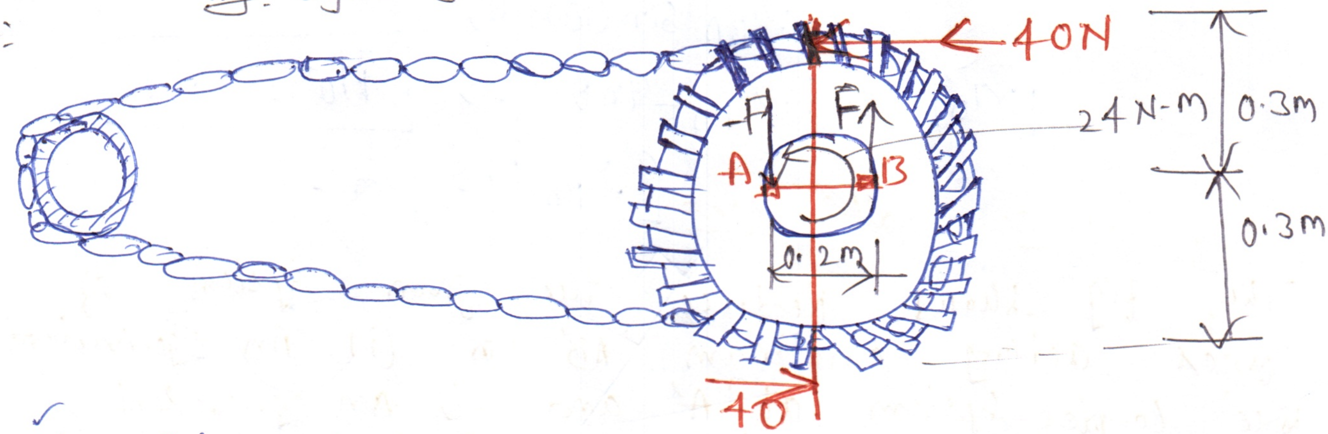


units : N-m

Eg :- A couple acts on the gear teeth, Replace it by an equivalent couple having a pair of force that act through point A & B

Eg: Cycle gear train

Sol:



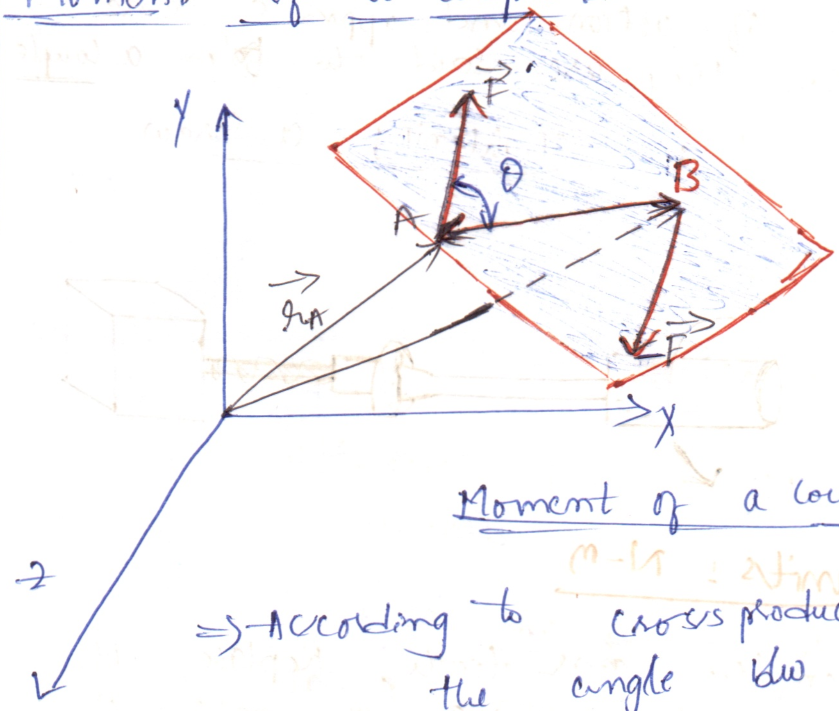
$M = \underline{\underline{Fd}} = 40 \times 0.6 = 24\ N\text{-}m$

Magnitude

$M = Fd \Rightarrow 24\ N\text{-}m = F(0.2m)$

$\underline{\underline{F = 120N}}$

Moment of a Couple :-



$$\vec{M_O} = [\vec{r_A} \times \vec{F}] + [\vec{r_B} \times (\vec{F})]$$

$$= (\vec{r_A} - \vec{r_B}) \times \vec{F}$$

$$\vec{M_O} = \vec{r_{BA}} \times \vec{F}$$

$$\boxed{\vec{M_O} = \vec{F} \times \vec{d}}$$

Moment of a couple

=> According to cross product of vector the angle b/w the forces $\vec{F}$ & $-\vec{F}$ is in positive sense $\vec{F} \rightarrow$ Y-axis (clockwise)

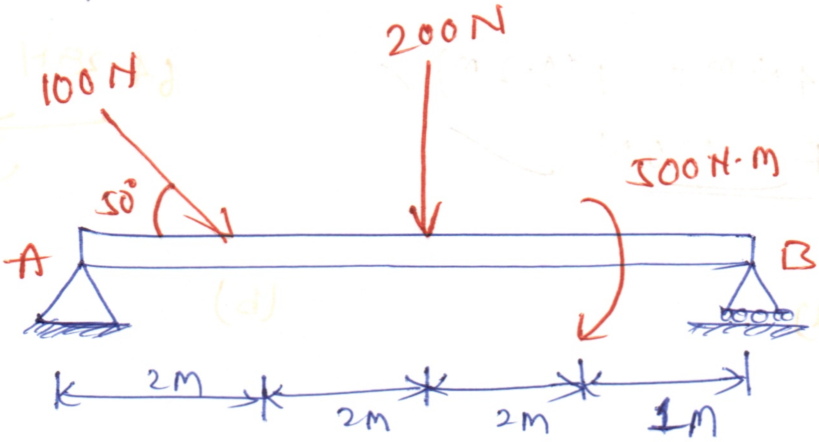
$$|\vec{M_O}| = F r_{BA} \sin(180-\theta)$$

$$F \cdot r_{BA} \sin(180-\theta)$$

$$|\vec{M_O}| = F \cdot r_{BA} \sin\theta \Rightarrow \boxed{\vec{M_O} = Fd}$$

$$[r_{BA} \sin\theta = d]$$

(1) The fig" shown, Reduce the given system of forces acting on a beam 'AB' to (i) An equivalent force couple system at 'A' and (ii) An equivalent force couple system at B



Sol.

Sol: Taking summation of all the forces along x & Y-direction

$$\Sigma F_x = 100\cos 50^\circ$$
$$= \underline{\underline{64.28\ N}}$$
$$\Sigma F_y = -100\sin 50^\circ - 200$$
$$= \underline{\underline{-276.6\ N}}$$

⇒ −ve sign indicates Y-comp of resultant is directed along the negative Y-direction

(i) Equivalent force couple system at 'A'

Taking summation of the moments of all the forces about 'A'

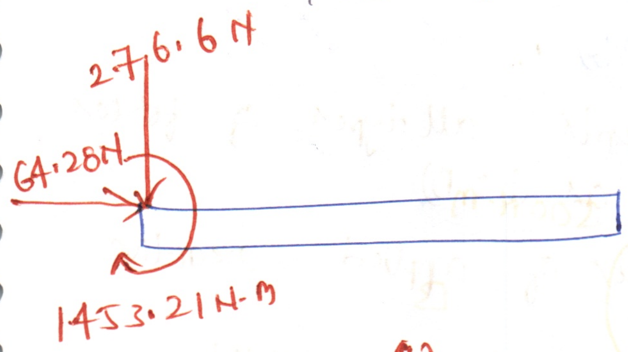
$$\Sigma M_A = -100\sin 50^\circ(2) - 200(4) - 500 = -1453.2\ N\text{-}m$$

−ve sign indicates − clockwise moment

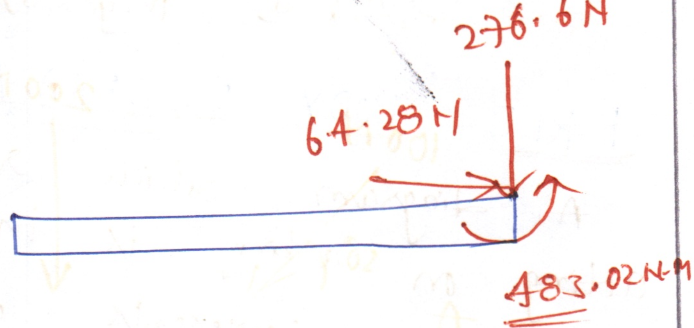
(ii) Equivalent force-couple system at B

$$\Sigma M_B = 100\sin 50^\circ(5) + 200(3) - 500 = 483.02\ N\text{-}m$$

+ve sign indicates − Anticlockwise moment



(a)



(b)

→ Equilibrium of System of forces :-

Equations of Equilibrium :-

A body will remain in equilibrium when

(1). The sum of all the external forces acting on the body is equal to zero ⟹ $\boxed{\Sigma R = 0}$ R = Resultant force

(2). The sum of the moments of the external forces acting on the body is equal to zero ⟹ $\boxed{\Sigma M = 0}$
(M = moment)

(a). Equations of equilibrium in Coplanar system (2D) :-

The eqns of equilibrium of a body subjected to coplanar system are given by

$\Sigma F_x = 0$; $\Sigma F_y = 0$; & $\Sigma M_x = 0$; $\Sigma M_y = 0$

(b). Equations of equilibrium in Non coplanar system (or) Spatial system (3D)

The eqns of equilibrium of body subjected to Non - coplanar (or) spatial forces are given by $\Sigma F_x = 0$, $\Sigma F_y = 0$; $\Sigma F_z = 0$ &
$\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$

FREE BODY DIAGRAM (FBD) :-

A diagram which represents all types of forces acting on a body is known as (FBD)

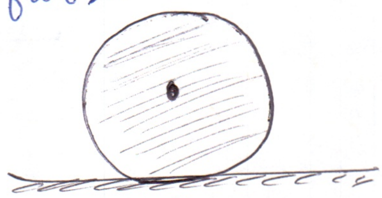
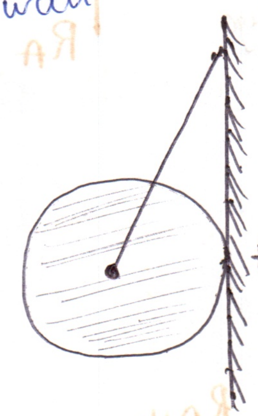
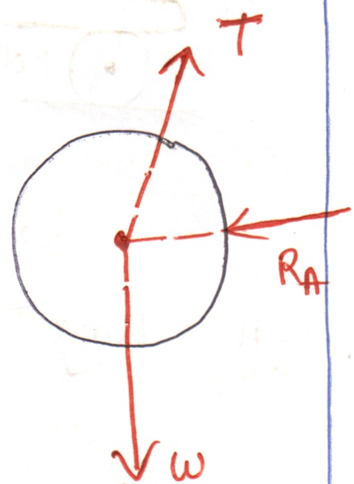
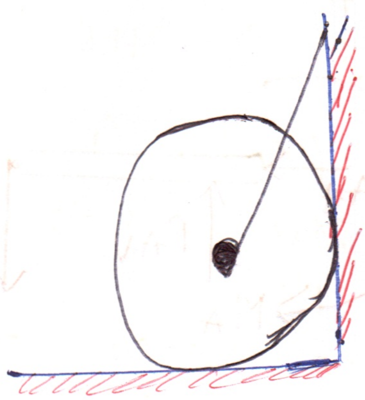
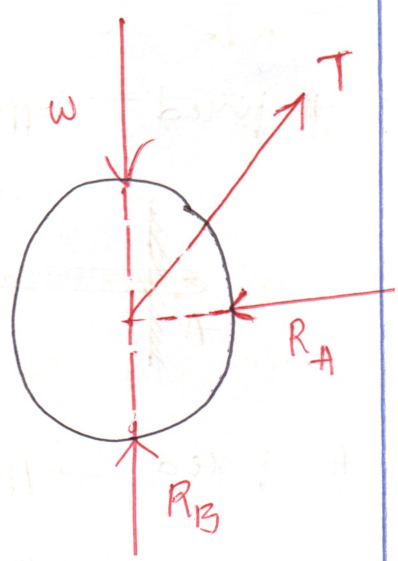
(i) FBD represents all types of applied & reactive forces

(ii) FBD shows all the angles b/w the forces

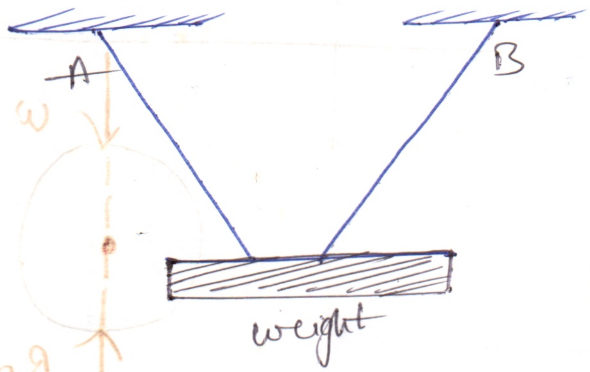
(iii) FBD Converts a physical situation into a mathematical problem

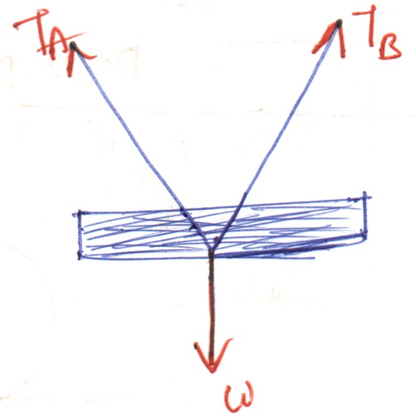
Hence after completing the FBD, we can write the eqn of equilibrium to solve the unknown quantities Hence after completing the FBD, we can write eqns of equilibrium to solve the unknown quantities.

Examples for Free Body Diagram (FBD)

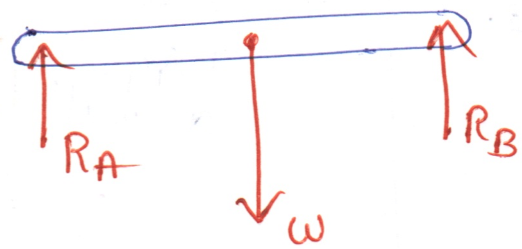
S.No	Description	FBD
1.	⇒ A ball resting on floor 	
2.	⇒ A ball suspended with a rope resting against a wall 	
3.	⇒ A ball suspended with a rope resting against a wall & floor 	

4. A weight supported by two string:

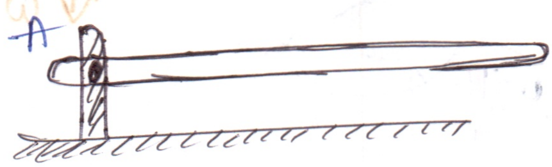


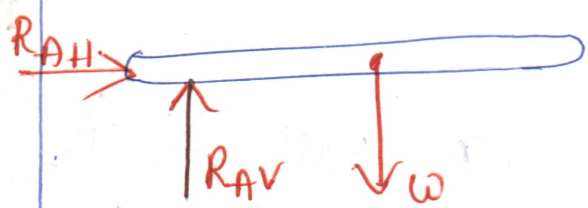


5). A weight supported by a roller and knife edge:

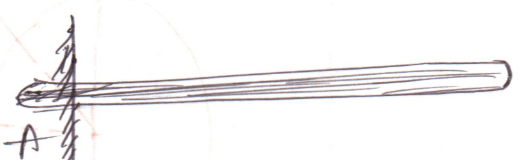


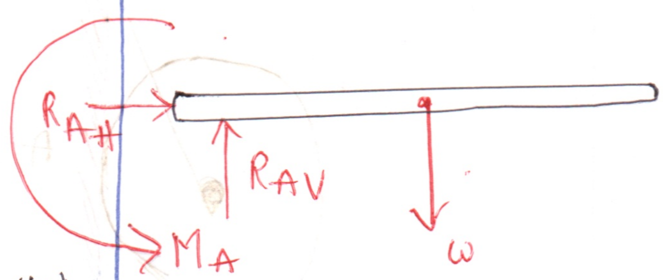
6. A hinge support



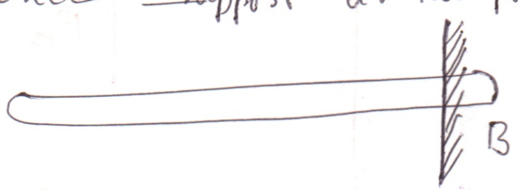


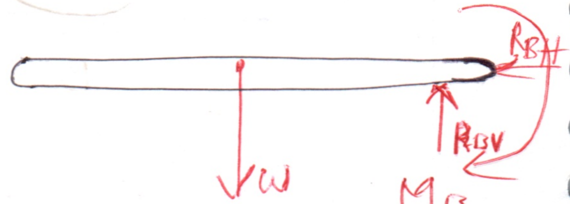
7. A fixed support at the point 'A'



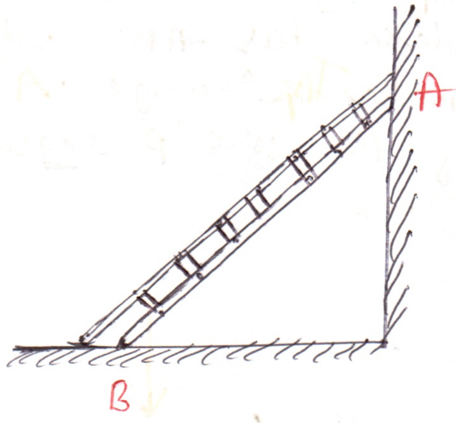


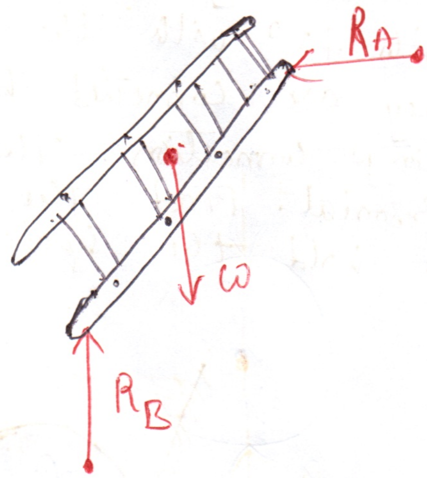
A fixed support at the point 'B'



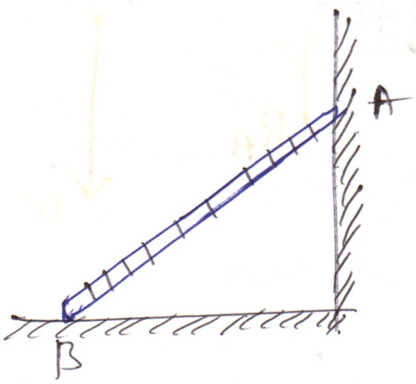


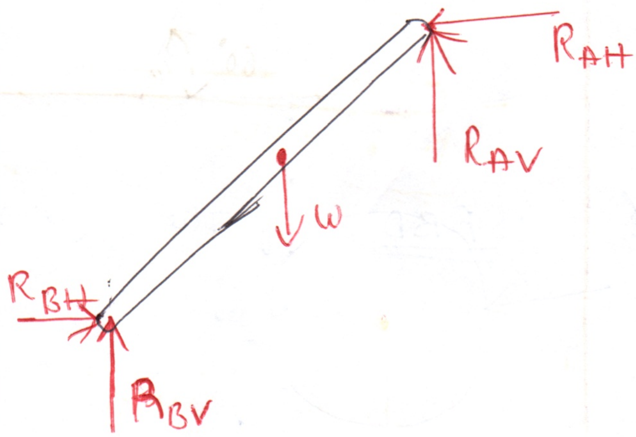
8). A ladder resting against a wall and floor



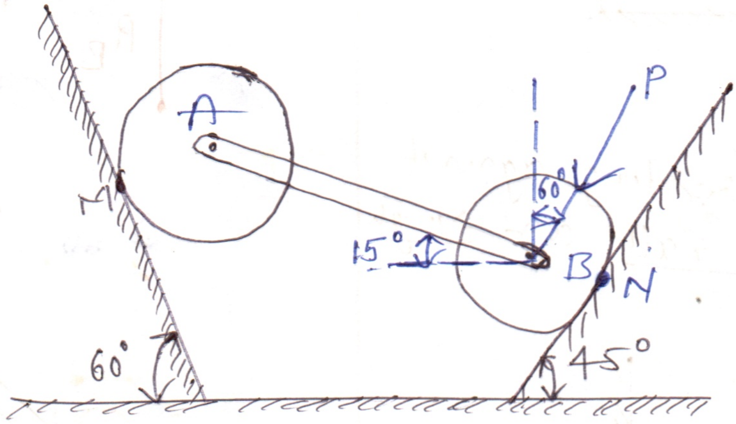


9). A ladder resting against a rough wall and rough floor

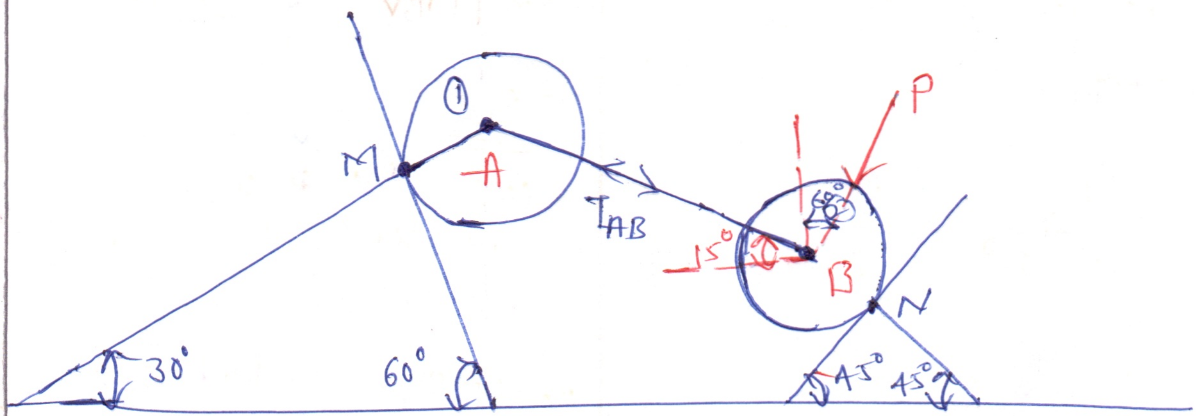


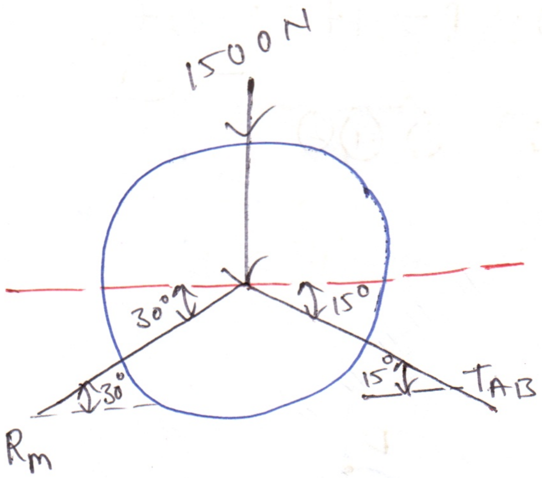


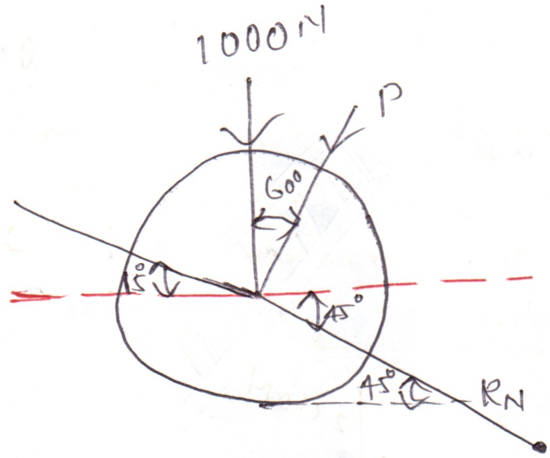
(1) Two spheres having weights $W_A = 1500\,N$, $W_B = 1000\,N$ are resting on smooth inclined planes having inclinations $60°$ to $45°$ with the horizontal respectively as shown in figure. They are connected by a weightless bar "AB" with hinge connection. The bar 'AB' makes $15°$ angle with the horizontal. Find the magnitudes of the force 'P' required to hold the system in equilibrium.

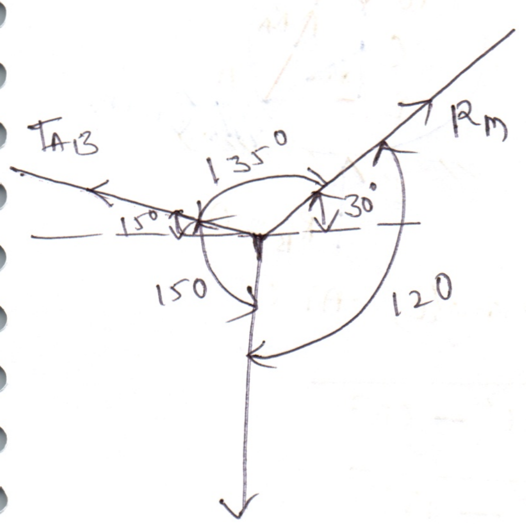


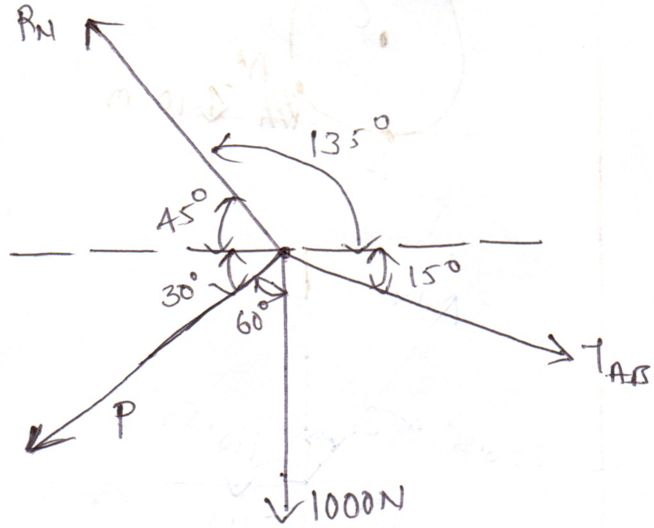
Sol. FBD of the given system is shown fig











using equation of Lamis theorem

$$\frac{T_{AB}}{\sin 120^\circ} = \frac{R_M}{\sin 105^\circ} = \frac{1500}{\sin 135^\circ}$$

$$T_{AB} = \frac{1500 \times \sin 120^\circ}{\sin 135^\circ}$$

$$\boxed{T_{AB} = 1837.11\,N}$$

$$R_M = \frac{1500 \times \sin 105^\circ}{\sin 135^\circ}$$

$$R_M = 2049.03\,N$$

using eqn of equilibrium

(i) $\Sigma F_x = 0$

$$R_N \cos 135^\circ + P\cos(180+30) + 1000\cos 270^\circ + T_{AB}\cos(360^\circ-15^\circ) = 0$$

$$-0.707R_N - 0.866P + 1837.11(0.965) = 0$$

$$= 0.707R_N + 0.866P = 1772.811 \longrightarrow (1)$$

(ii) $\Sigma F_y = 0$

$$R_N \sin 135^\circ + P\sin(180+30) + 1000\sin 270^\circ + T_{AB}\sin(360-15) = 0$$

$$\Rightarrow 0.707R_N - 0.5P - 1000 + 1837.11(-0.258) = 0$$

$\Rightarrow$ 0.

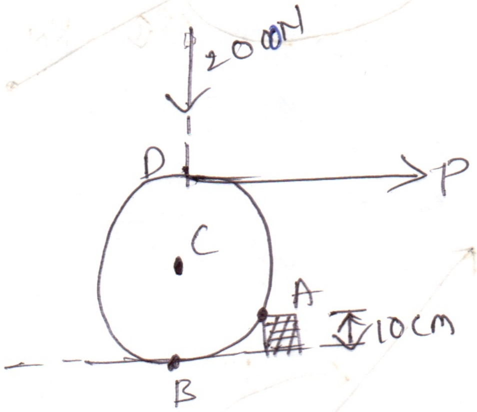
$0.707 R_N - 0.5P = 1473.974$ —— (2)

solving eqn (1) & (2)

$P = 218.767 N$

$R_N = 2239.54 N$

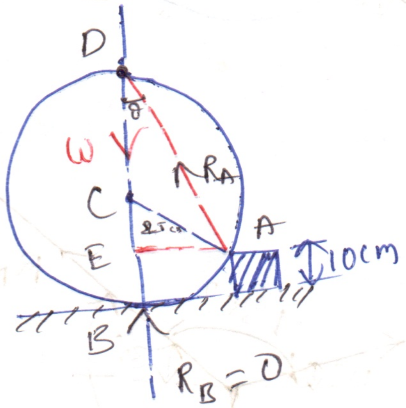
(2)

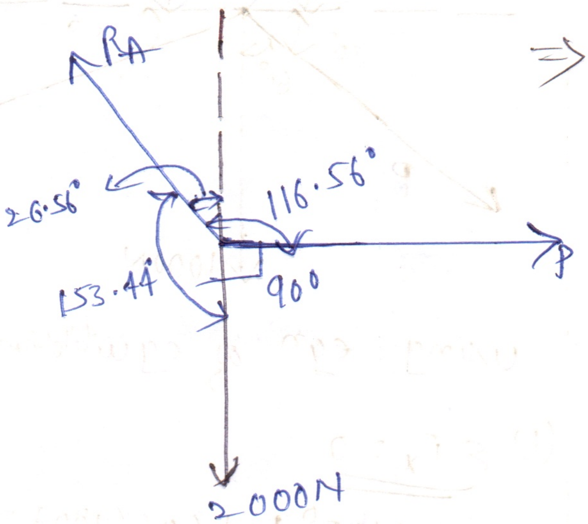


$D = 50 cm$

$R = 25 cm$

$W = 2000 N$





⇒ From triangle, AEC

$AE = \sqrt{AC^2 - CE^2}$

$= \sqrt{25^2 - 15^2}$

$= \sqrt{625 - 225}$

$= 20 cm$

$\tan\theta = \frac{AE}{DE} \Rightarrow \tan\theta = \frac{20}{40} \Rightarrow \theta = 26.56$

Lami's theorem

$$\frac{2000}{\sin(116.56)} = \frac{P}{\sin(153.44^\circ)} = \frac{R_A}{\sin 90^\circ}$$

$P = ?$

$R_A = ?$

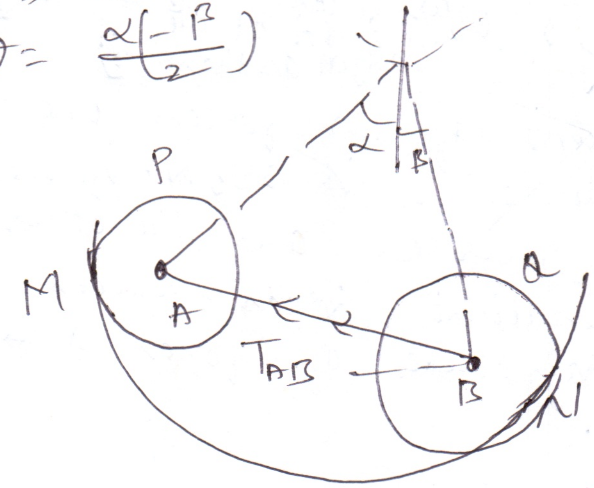
(3) $P = 250N$

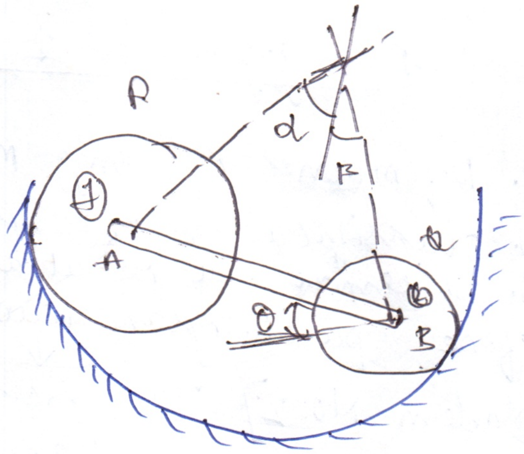
$Q = 500N$

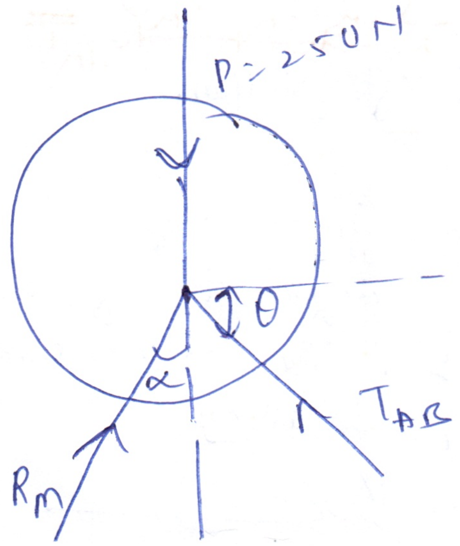
Radius & Angle $\alpha + \beta = 90$

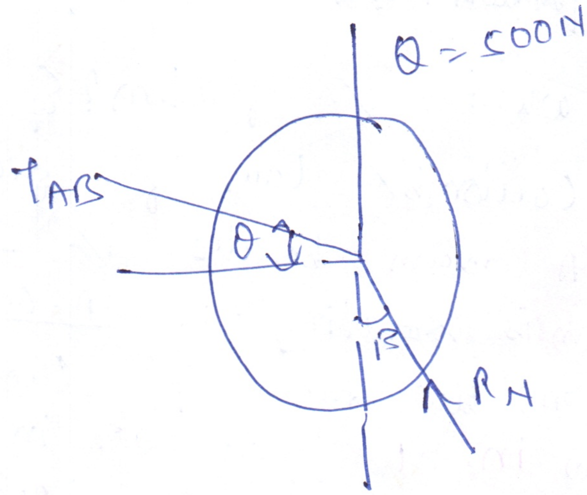
$\Rightarrow AB$

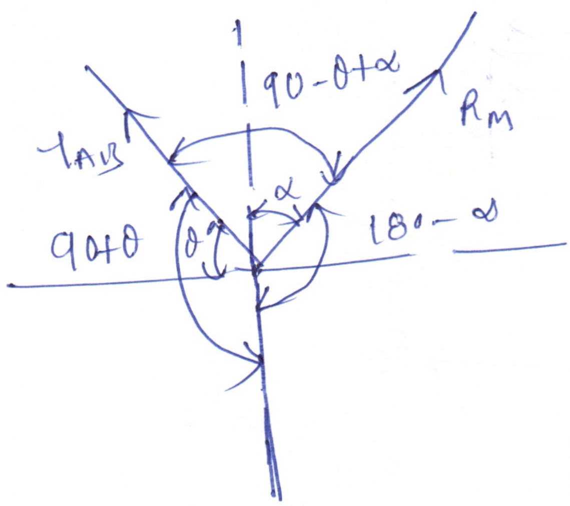
$\theta = \dfrac{\alpha - \beta}{2}$

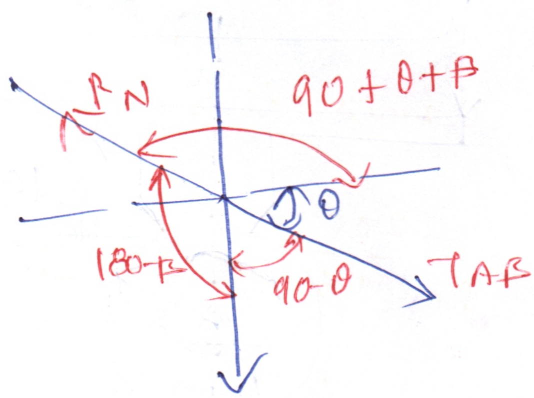








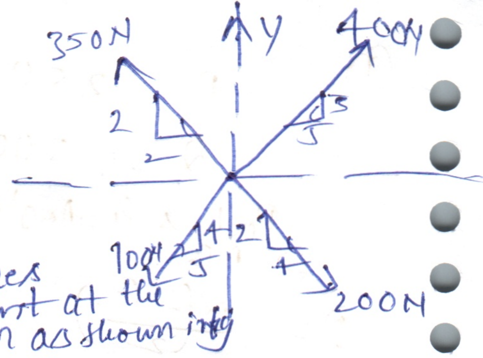




$$\frac{250}{\sin(90+(\alpha-\theta))} = \frac{T_{AB}}{\sin(180-\alpha)} = \frac{R_M}{\sin(90+\theta)}$$

$$\frac{R_N}{\sin(\cancel{90+\theta})\ 90-\theta} = \frac{T_{AB}}{\sin(180-\beta)} = \frac{500}{\sin(90+\theta+\beta)}$$

Assignment - 1   UNIT-1



(1) 1). (a). Definition of engg mechanics

(b). classification of system of forces

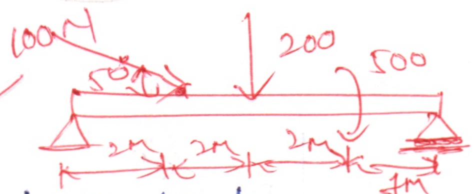
4th problem ← (c). Determine the resultant of four forces concurrent at the origin as shown in fig

(2) 2)(a) Define parallelogram law? X

(b). [problem No: 2] The resultant of two forces one of which is double than the other is 235 N. If the direction of large force is reversed and the other remains unaltered the resultant forces reduces to 155N. Determine the magnitude of forces and the angle b/w the forces.

(c) Explain moment of a force & moment of couple

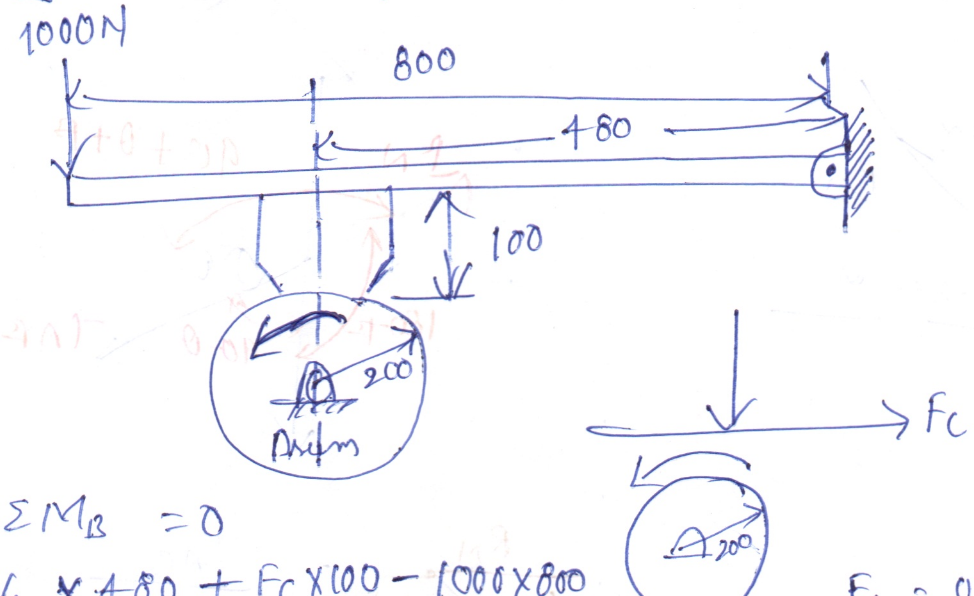
problem No: 1



(3) 3)(a) what is friction? & Coefficient of friction.

(b) Coulomb's Law of friction.

(c) A drum brake is shown in fig. The drum is rotating in A.C.W. The coefficient of friction b/w drum & shoe is 0.2. The dimensions shown in the figure are in mm. The braking torque (in N-M) for the brake shoe is ________



Sol:

$\Rightarrow \Sigma M_B = 0$

$V_c \times 480 + F_c \times 100 - 1000 \times 800 = 0$

$\Rightarrow F_c = \mu V_c = 0.2 V_c$

$480 V_c + 0.2 V_c \times 100 = 800000$

$500 V_c = 80,000 \Rightarrow V_c = 1600N$

$F_c = 0.2 V_c = \underline{320 N}$

$M = 0.2 \times F_c = 0.2 \times 320$

$= \underline{64 N\text{-}m}$