Ref: 1) Cengel 23/6/19

Fluid Mechanics (F.M)

Theory → madi sethi
problems: subsamaj

2) Frank M. White

Easy starting: R.K. Bansal, K.L. Kumar etc.., Jagadish babu

→ GATE Marks: 8-10 Marks

Fluid:- Fluid ~~which~~ is a substance which deforms continuously for a small amount of shear force also.

Solid → It not easily shear force are placed (S.M)

{liquid, gas} Fluids → which deforms very small amount of shear force

Introduction properties:-

Density (or) Mass density, specific mass:-

$$\rho = \frac{mass}{volume} = kg/m^3,\ ML^{-3}$$

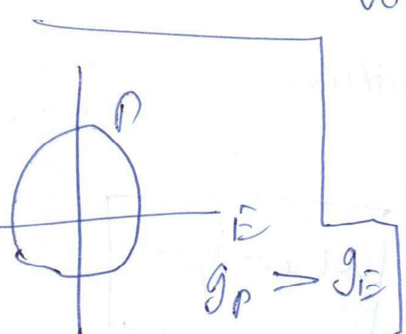
$\rho_{water} = 1000\ kg/m^3$, $\rho_{Air} = 1.208\ kg/m^3$

$\rho_{seawater} = 1025\ kg/m^3$, $\rho_{ice} = 915\ kg/m^3$

Specific weight (or) weight density:

$$\gamma = \frac{weight}{volume} = \frac{N}{m^3}$$

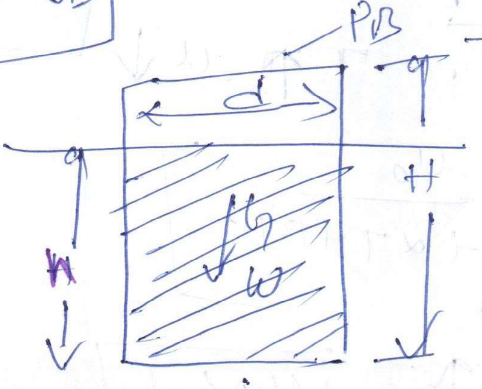
$\gamma_w = \rho_w \times g$
$= 1000 \times 9.81\ m/sec$



$g_{earth} = 9.81\ m/sec^2$

$F_G = M.a$

$W = M.g$



→ weight of the body

$= \gamma_B \times Vol_B$

$W_B = \rho_B \cdot g \times \frac{\pi d^2}{4} \times H$

→ wt. of liquid displaced

$\gamma_{H_2} \times V_{in}$

$= \rho_{liq} \times g \times \pi d^2/4 \times h$

Specific gravity / Relative density (S) :-

$$S_{liq} = \frac{\rho_{Any\ fluid}}{\rho_{reference\ fluid}} = \frac{\rho_{liq}}{\rho_{water}}$$

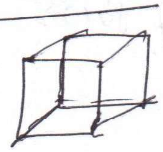
$S_{water} = 1.0$

$S_{Hg} = 13.6$

$S_{CCl_4} = 0.04$

$S_{Kerosene} = 0.8$

Bulk modulus (K) :-



$$K = \frac{\text{Direct stress}}{\text{volumetric strain}} = \frac{-(dP)}{\left(\frac{dV}{V}\right)}$$

-ve sign volume decres pressure ↑

↓ relates

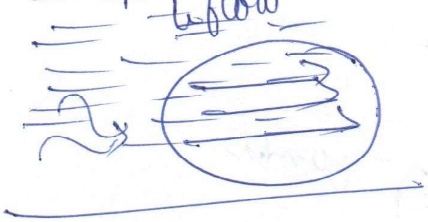
Vanderwall's equation = $\left[\left[P + \left(\frac{a}{V^2}\right)\right](V-b) = RT\right]$

m3 vol m3

$$\boxed{K = \frac{\text{Increase in pressure}}{\text{volumetric strain}}}$$

Viscosity :- (μ)

It is the property of fluid by virtue of which it offers resistance for the movement of one layer over them other layers



Resistance to flow — slip

1) Cohesion → predominent liquids

2) molecular momentum → Exchange

→ Effect of temp :-

$$\boxed{\mu \propto \frac{1}{T}}$$

Liquids : $T \uparrow \ \mu \downarrow$

$$\left[\mu_T = \frac{\mu_0}{1 + \alpha T + \beta T^2}\right]$$

Gas $T \uparrow \ \mu \uparrow$ $\boxed{\mu \propto T}$

$$\boxed{\mu_T = \mu_0 + \alpha T - \beta T^2}$$

In liquid increasing randomness (or) collision molecular agitation on Then viscosity also increased.

Effect of pressure :-

with increase in the pressure the viscosity increases for both liquids and gases But effect on liquids is negligible (∵ it is incompressible)

$P\uparrow$ 1 atm $\longrightarrow$ 1000 atm
$\mu\uparrow$ 1 unit $\longrightarrow$ 2 units

units of viscosity :-

Absolute (or) (μ) dynamic		Kinematic $\boxed{\nu = \frac{\mu}{\rho}}$	
SI	C.G.S	SI	C.G.S
$\frac{N \cdot sec}{m^2}$	$\frac{Dyne \cdot sec}{cm^2}$ (poise)	$\frac{m^2}{sec}$	$\frac{cm^2}{sec}$ (stoke)

$$\boxed{1 \text{ poise} = 10^{-1} \frac{N \cdot sec}{m^2}}$$

$$\boxed{1 \text{ stoke} = 10^{-4} m^2/sec}$$

$\rightarrow \boxed{\frac{N \cdot sec}{m^2} = Pa \cdot sec = kg/m \cdot sec}$

$\rightarrow N = kg \cdot m/sec^2 \times \frac{sec}{m^2}$

$\rightarrow N = kg/m \cdot sec$

ISRO

1) On a given mass fluid if the pressure increases from 3 Megapascal to 3.5 MPas causing the density to increase from 500 kg/m³ to 501 kg/m³. The bulk modulus of fluid (K) = ?

K = ?

Sol → P 3.0 MPa

3.5 MPa ↓

→ ρ → 500 kg/m³

501 kg/m³

$$K = \frac{dP}{\left(\frac{d\rho}{\rho}\right)} = \frac{[3.5 - 3.0]\times10^6}{\left(\frac{501-500}{500}\right)}$$

$$= \underline{250\times10^6} = \underline{250\,MPa}$$

DRDO → The increase in pressure required to decrease unit volume of mercury by 0.1% is ? ($K_{Hg} = 28.5$ MPa)

1) 2.85 kPa

2) 2.85 MPa

3) 28.5 kPa

4) 28.5 MPa

Sol $(+dp) = K \cdot \left[\frac{-dv}{V}\right]$ ← $\boxed{K = \frac{(dP)}{-\left(\frac{dv}{v}\right)}}$

$$= 28.5\times10^6 \left[\frac{0.1}{100}\right] = 28.5\times10^3$$

$$= \underline{28.5\,kPa}$$

Ex

A liquid sp. gravity S = 0.8 & dynamic viscosity = 10 poise from there it to calculate (ν) kinematic viscosity ?

Sol liquid (S) = 0.8

μ = 10 poise

ν = ? [stoke]

$\boxed{\nu = \frac{\mu}{\rho}}$

→ S = 0.8 → ρ = S × ρ_w

= 0.8 × 1000 = 800 kg/m³

Then liquid $S = 0.8 \rightarrow \rho = S \times \rho_w = 0.8 \times 1000$
$= 800\, kg/m^3$

$\mu = 10$ poise $= 10 \times 10^{-1} \frac{N \cdot sec}{m^2}$

$\nu = ?$ [stoke] $\nu = \frac{\mu}{\rho} = \left(\frac{10 \times 10^{-1}}{800}\right) m^2/sec$

$= \left(\frac{10 \times 10^{-1}}{800}\right) \times 10^4$ cm^2/sec

$\Rightarrow \left(\frac{10 \times 10^{-1}}{800}\right) \times 10^4$ $cm^2/sec = \underline{12.5 \text{ Stoke}}$

Applications of viscosity:-

i.e in Newtonian equation:-

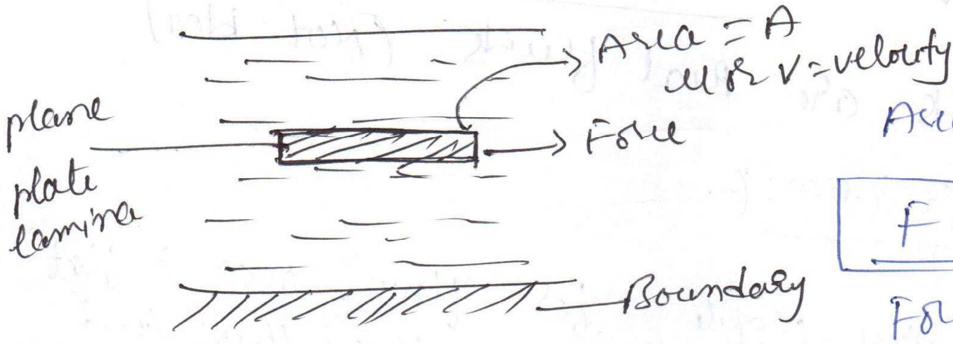
Shear strain $\rightarrow$ $\boxed{\tau = \mu \cdot \left(\frac{du}{dy}\right)}$

$\left(\frac{du}{dy}\right)$ or $\left(\frac{dv}{dy}\right)$ = velocity gradient (or) (Shear strain Rate)

μ = Dynamic viscosity

Newton Experiment :-

when fluid is passed in certain boundary



$\boxed{F \propto A \text{ (Area)}}$

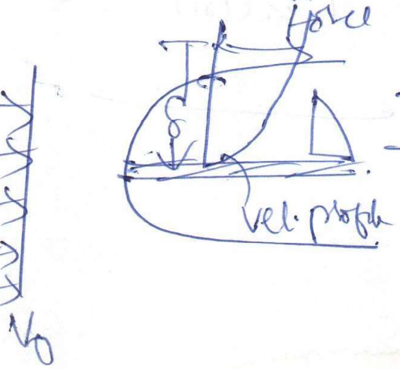
Area is low & Force is low like that

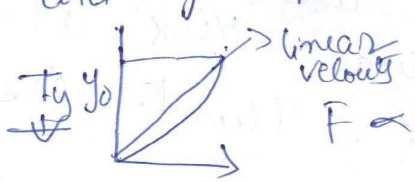
$\boxed{F \propto V}$

Force is high & velocity is higher

Boundary is close to plane

Force will high in plane $\boxed{F \propto \frac{1}{y}}$



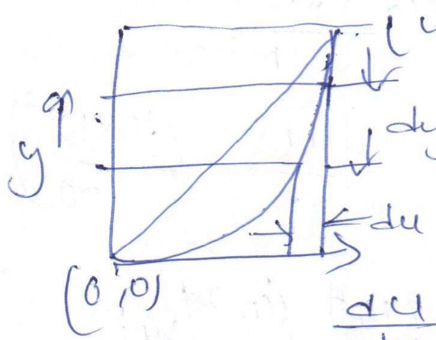


$F \propto \frac{AV}{y} \Rightarrow \boxed{F = \frac{\mu \cdot AV}{y}}$ linear velocity variation

$\boxed{\frac{F}{A} = \tau = \frac{\mu \cdot V}{y}}$

μ = dynamic viscosity

velocity profile Then $\boxed{\tau = \mu \frac{du}{dy}}$



$\tau = \mu \frac{du}{dy}$

→ linear variation

$$\frac{du}{dy} = \left(\frac{u_2 - u_1}{y_2 - y_1}\right) \Rightarrow \frac{du}{dy} = \frac{u}{y}$$

NOTE:- The flow its which follows newtonian equation (8) Newtonian fluids. So which does not follows ~~the~~ the Non newtonian fluids

→ Newtonian fluids:- $\boxed{\tau = \mu \frac{du}{dy}}$

→ Non-newtonian fluids:- $\tau = A \cdot \left(\frac{du}{dy}\right)^n + B$

Real

↓

Rhelogy

In our GATE only Newtonian fluids & Fluid mechanics also

→ Non- Newtonian are advanced

→ Both fluids are Real fluids (Not Ideal)

→ ~~Surface Tension~~:-

Eg:- The velocity profile for flow over a flat plate given by $\frac{3}{4}y - y^2$, If the shear stress ~~at~~ 'K' times at location 0.3m above the plate ~~and 0.2~~ the shear stress at location 0.2m above the plane. then K is ________

Sol: $0.3\,\tau_{y=0.3} = (12)\cdot\tau_{y=0.2}$ ✓? — Then its formula is next

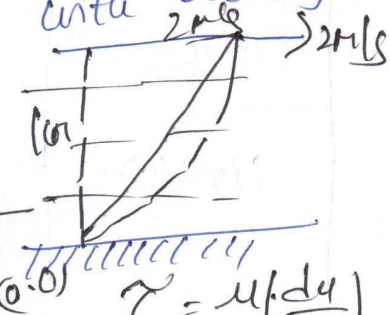
$\Rightarrow \mu\left(\frac{du}{dy}\right)_{y=0.3} = 12\,\mu\left(\frac{du}{dy}\right)_{y=0.2}$

$\Rightarrow \frac{d}{dy}\left[\frac{3}{4}\cdot y - y^2\right] = \frac{3}{4} - 2y$

$\mu\left[\frac{3}{4} - 2\times 0.3\right] = 12\,\mu\left[\frac{3}{4} - 2\times 0.2\right]$

$12 = \frac{(3/4 - 0.6)}{[3/4 - 0.4]} = 6/14 = 3/7$ ✓

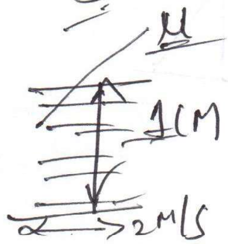
Ex 12
The max. shear stress develop in lubricating oil having viscosity 0.981 poise filled b/w two parallel plate 1CM apart and moving with velocity of 2 m/sec



Sol:

$\mu = 0.981$ poise $\times 10^{-1}$ N. sec/m²

$y = 1\,CM = 10^{-2}\,m \quad V = 2\,m/s \qquad \tau = \mu\left(\frac{du}{dy}\right)$



$\tau = \frac{\mu\cdot v}{y} = \frac{0.98\times 10^{-1}\times 2}{10^{-2}} = 19.62\,N/m^2$

is above plat is moving — Shear stress →

Gate-04: A lubricating oil with a specific gravity $S = 0.88$ at kinematic viscosity $\nu = 7.4\times 10^{-6}\,m^2/sec$ filled two parallel plates. If the top plate is moving with velocity of 0.5 m/sec while the bottom is stationary assuming linear velocity variation over a gap of 0.5 MM b/w these plates [the max. shear stress develop at the fixed plate]

moving plate
Same

Sol

$S = 0.88$, $\nu = 7.4 \times 10^{-7}\ M^2/sec$

~~y = 0.5 m/sec~~ $V = 0.5\ M/sec$, $y = 0.5\ MM = 0.5 \times 10^{-3}\ m$

$$\tau = \frac{\mu . V}{y} = \frac{S . \rho_w \times \nu \times V}{y}$$

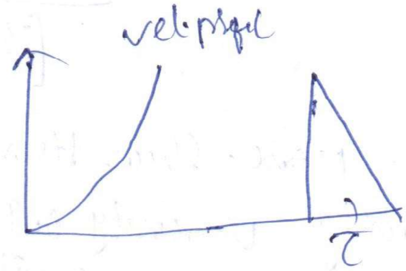
$$\tau = \frac{0.88 \times 1000 \times 7.4 \times 10^{-7} \times 0.5}{0.5 \times 10^{-3}}$$

$$\tau = 0.651\ N/M^2$$

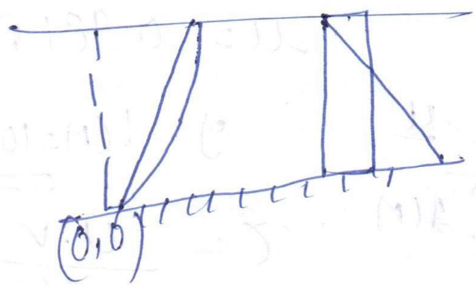
Velocity	Shear stress (τ)
parabolic	linear
cubic	parabolic
linear	constant

$$\tau = \mu \left(\frac{du}{dy} \right)$$

$$\mu \left(\frac{u - 0}{y - 0} \right)$$



$$\tau = \mu \left(\frac{du}{dy} \right)$$



Q-16 A circular shaft of 500 mm dia was rotating inside a sleeve (or) bearing of 502 mm dia is running at 200 R.P.M the ~~angu~~ Annular space was filled with lubricant of viscosity 5 poise calculate power loss due to friction for sleeve length of 100 MM

Sol

Q. Shaft (radius) → 500 mm dia

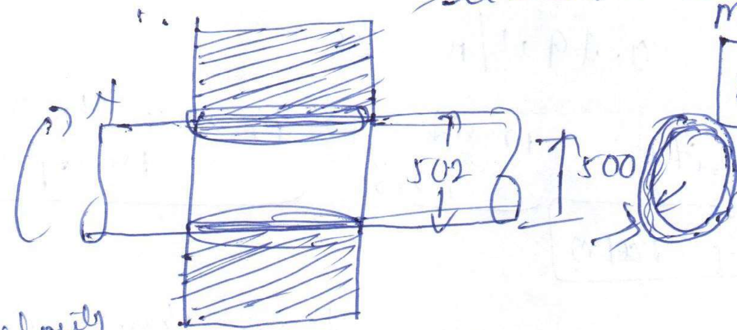
Sleeve → 502 mm dia $P_{loss(friction)} = ?$

$N = 200$ RPM $L = 100$ mm

$T = F \times R$ (radius)

Annular space $\mu = 5$ poise $P = \frac{2\pi N T}{60}$ ($T = F \times R$)

$\rightarrow \mu = 5 \times 10^{-1} \frac{N \cdot sec}{m^2}$



$$\boxed{P_{loss} = F \times Velocity = \left(\frac{\mu A V}{y}\right) \times V}$$

$V = \frac{\pi d N}{60}$ (velocity) Area under shear $= \pi d L$

$\rightarrow y = \frac{D_o - D_i}{2} = \frac{502 - 500}{2} = 1\,mm$ ✓ $\boxed{P_{loss} \propto \mu}$

$$P = \frac{5 \times 10^{-1} \times [\pi \times 0.5 \times 0.1] \times \left(\frac{\pi \times 0.5 \times 200}{60}\right)^2}{1 \times 10^{-3}}$$

$P = 2.153\,kN$ ✓

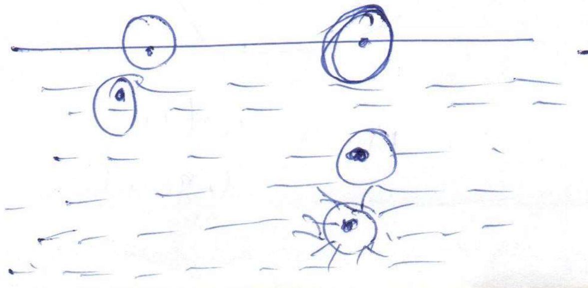
Surface tension (σ):-

	P
water droplet	$\frac{4\sigma}{d}$
Soap ~~bubble~~ bubble	$\frac{8\sigma}{d}$
liquid jet	$\frac{2\sigma}{d}$

Def:

→ It is the property of liquid surface film to exert tension. It is the force required to maintain a unit length in equilibrium

$\sigma = N/m$



→ It is because of cohesion

$$\boxed{\sigma \propto \frac{1}{T}}$$

$T\uparrow \rightarrow \sigma \downarrow \qquad \sigma \propto \frac{1}{T}$

Note:- $\sigma_{water} = 0.073\ N/m$ 30°C

$= 0.589\ N/m$ 100°C

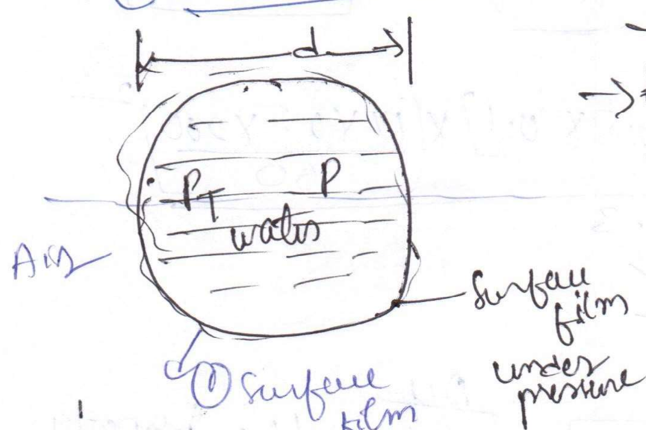
$\sigma_{Hg} = 0.49\ N/m$ 30°C

→ P = pressure intensity inside 1,2,3 in excess to outside Atmo pressure

$\boxed{P_I > P_{atm}}$

→ In water droplet is more than atm. press its stage is Burst excess pressure

① water droplet:



For equation equilibrium No burst

→ $F_{Bursting} = F_{Resisting}$

$F_{press} = F_{\sigma}$

$P \times A = \sigma \times L$

Surface tension = F/A

= Force per unit length | Area

$\boxed{\sigma = F/L \Rightarrow F_{\sigma} = \sigma \times L}$ $\boxed{P = F/A \Rightarrow F_P = P \times A}$

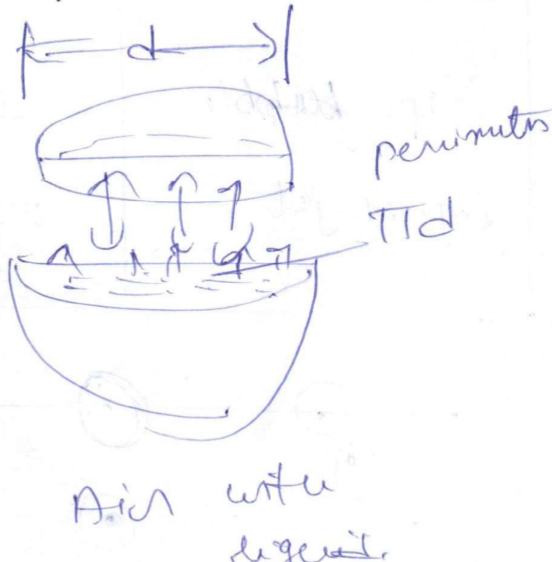
$F_{Burst} = F_{resist}$

$F_P = F_{\sigma}$

$P \times A = \sigma \times L \rightarrow \pi d$

$\frac{P \times \pi d^2}{4} = \sigma \times \pi d$

$\boxed{P = \frac{4\sigma}{d}}$



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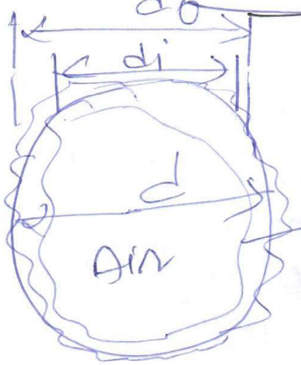
(7) Soap bubble :-

Air with soap bubble



2 surface

water & air internally

$$\boxed{d_o = d_i = d}$$

$\Rightarrow F_p = F_\sigma$

$P \times A = \sigma \times L$

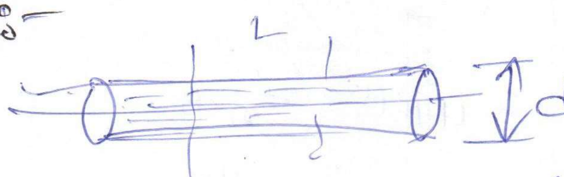
$P \times \frac{\pi d^2}{4} = \sigma [\pi d_o + \pi d_i]$

$= \sigma \cdot 2\pi d$

$P = \left(\frac{4\sigma}{d}\right) \times 2 = \frac{8\sigma}{d}$

→ Soap bubble 2 interface

(8) Liquid jet :-



$F_p = F_L$

$P \times A = \sigma \times L$

$P \times L \times d = \sigma [2L] + \cancel{2d}$

So we can consider only length

Liquid is contact with in the surface not in liquid so only length will be considered

$$\boxed{P = \frac{2\sigma}{d}}$$

NOTE :- Surface tension can also expressed as

$(\sigma)' = \frac{\text{surface energy}}{\text{surface Area}} = \frac{\text{Joule}}{\text{Area}} = \frac{N \cdot m}{m^2} = \frac{N}{m} = \frac{F}{L}$

Eg :-
→ work required to burst a water droplet

$= \sigma \times$ surface Area → (surface tension × surface area)

$$\boxed{W_{droplet} = \sigma \times 4\pi R^2}$$

In soap bubble

$\Rightarrow 2\sigma \times 4\pi R^2$

→ If n: no. of water droplet then the solution is

$$E_1 + W = E_2$$

$$\Rightarrow W = E_2 - E_1$$

64 No of waterdroples

$$W = n\sigma 4\pi r^2 - \sigma 4\pi R^2$$

σ will be same in both cases because in our fluid measurement units

$$E_1 = \sigma \times 4\pi R^2, \quad E_2 = n \times \sigma \cdot 4\pi r^2$$

Volume 1 & Volume 2 — V_1 volume

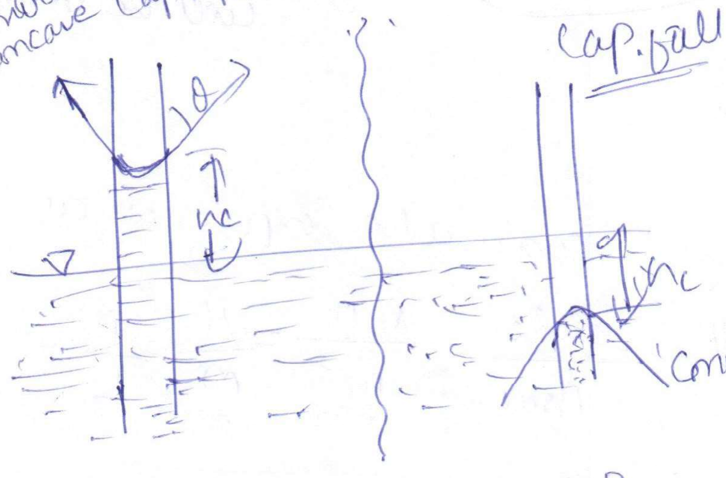
$$V_1 = V_2 = \tfrac{4}{3}\pi R^3 = n \cdot \tfrac{4}{3}\pi r^3$$ → volume of the droplet

$$W = \sigma\, 4\pi R^2 \left(\frac{n}{n^{2/3}} - 1\right)$$

$$\Rightarrow W = \sigma \cdot 4\pi R^2 \left[n^{1/3} - 1\right]$$

→ **Capilarity** :-

Concave Cap. Rise



d < 6mm — Capillary dia

d > 10MM — Manomethic dia

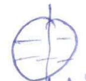
Higher adhesion More height

Larger cohesion low height

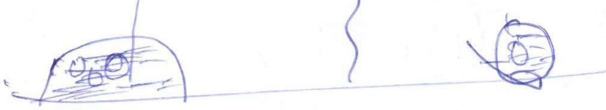
Cap. Rise	Cap. fall
Adh > Coh	Coh > Adh
Concave	Convex
$\theta < 90^\circ$ (Accute)	$\theta > 90^\circ$ [Obtuse]

→ water & kerosene | Hg

→ Adh > Coh | → Coh > Adh



wetting fluid | Non wetting

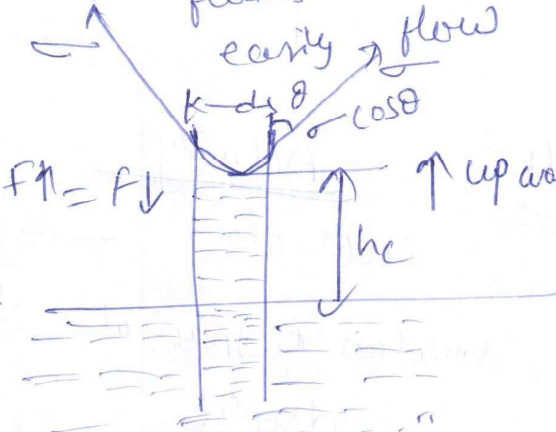


* Capillary action water never over flow
In high cohesion for Mercury

→ But mercury is only thermometers for measuring temp
not related to this

* when Adhesion is high capillary rise

* wetting fluids is easily flow, But non wetting fluids is not easily flow



$$\rightarrow \boxed{h_c = \frac{4\sigma\cos\theta}{\gamma \cdot d}}$$

weight of balance

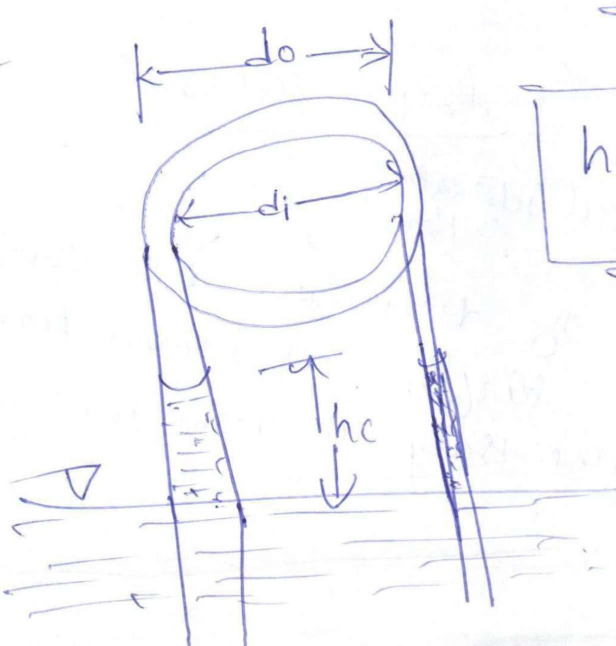
$$F\uparrow = F_G \downarrow = W \downarrow$$

$$\sigma\cos\theta \times L = \gamma \times vol = \gamma \times \frac{\pi d^2}{4} \cdot h_c$$

$$\sigma\cos\theta \times \pi d = \gamma \times vol = \gamma \times \frac{\pi d^2}{4} \cdot h_c \Rightarrow \boxed{h_c = \frac{4\sigma\cos\theta}{\gamma \cdot d}}$$

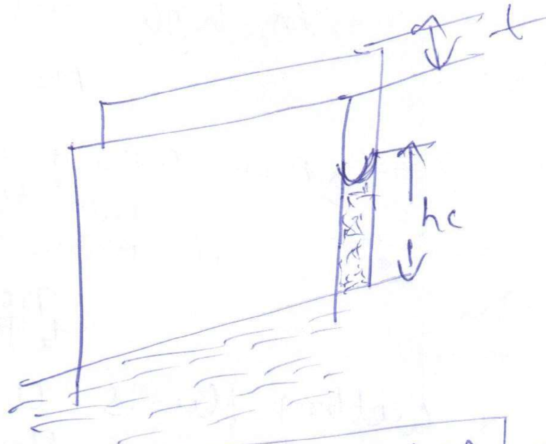
→ $\theta = 0^\circ, 25^\circ, 128^\circ$ for pure water, glass contaminated, Hg - glass

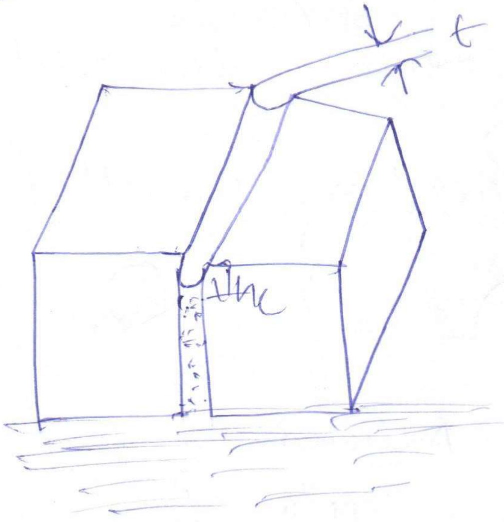
Sp. case 1



$$\boxed{h_c = \frac{4\sigma \cdot \cos\theta}{\gamma(d_o - d_i)}}$$

Sp case-II





$$\boxed{h_c = \frac{2\sigma\cos\theta}{\gamma \cdot t}}$$

Gate: Activity → property

lubrication → viscosity

Formation of spherical droplet → surface tension $\boxed{P_{inside} > P_{vapour}}$

Raise of sap in tree → capilary

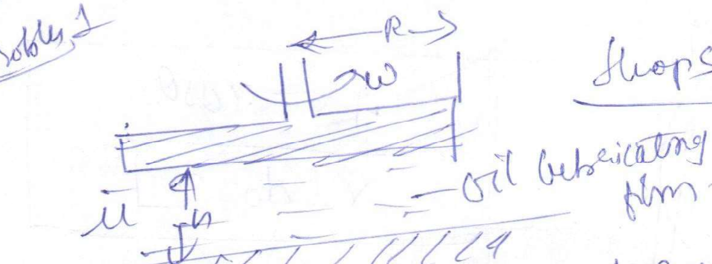
Cavitation → vapour pressure

Hammering effect → valve closure (surge tank)

Gate:

problem 1

Shops shuttles



Circular disc of diameter 'd' rotating above a fixed surface as shown find the torque necessary to maintain angular velocity 'ω'

Sol:

$\mu = \mu$

$A = \pi R^2$

$V = R \cdot \omega$

$y = h$

Torque $T = F \times \text{dist } R$

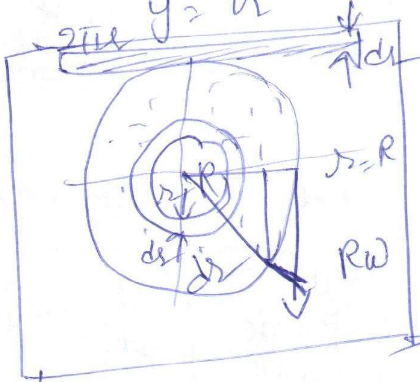
$$F = \frac{\mu A V}{y}$$

$$T = \left[\frac{\mu A V}{y}\right] \times R$$

$$\frac{\mu(\pi R^2)(R\omega) \cdot R}{h}$$

velocity is not constant

$$\boxed{\cancel{T = \frac{\pi \mu \omega R^4}{h}}}$$



→ other method it is correct

→ $\mu = \mu$, $A = dA = 2\pi r\, dr$, $V = r\omega$, $y = h$

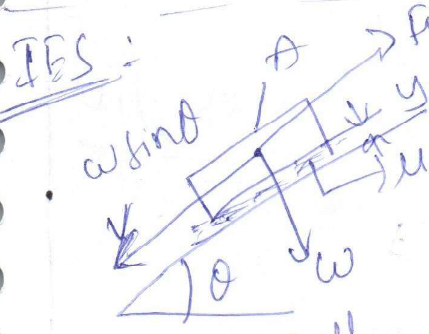
$$\rightarrow T = \int dT = \int dF \cdot r \Rightarrow \int_{r=0}^{r=R} \frac{\mu(2\pi r \cdot dr)(r\omega) \cdot r}{h}$$

$$\Rightarrow \frac{2\pi \mu \cdot \omega}{h} \cdot \int_0^R r^3 dr = \frac{2\pi \mu \omega}{h} \left[\frac{r^4}{4}\right]_0^R$$

$$\boxed{T = \frac{\pi \mu \omega R^4}{2h}}$$

→ Integration is used for these problems because of change of rotation of angular velocity with radius

IES:

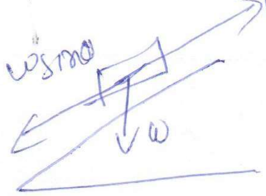


Component of angle θ

Frictional force is occurred in rectangular block

$$W \sin\theta = F_{frict}$$

$$W \sin\theta = \frac{\mu \cdot A \cdot V}{y}$$



$$\boxed{F_{pull} = W \cdot \sin\theta + \frac{\mu A \cdot V}{y}}$$ — to pull the rectangular block