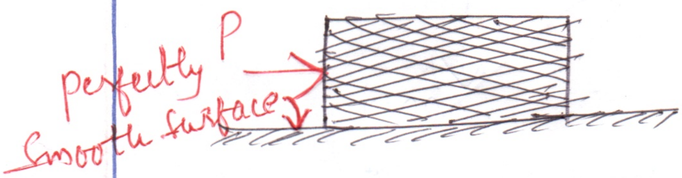
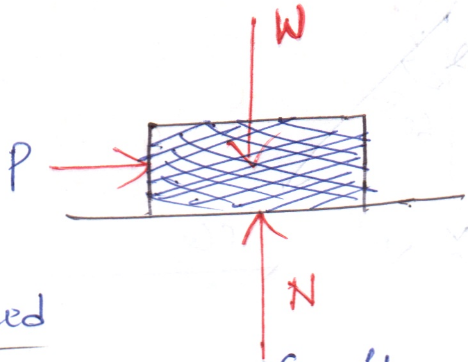
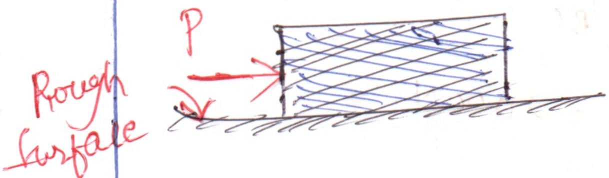
→ FRICTION ←

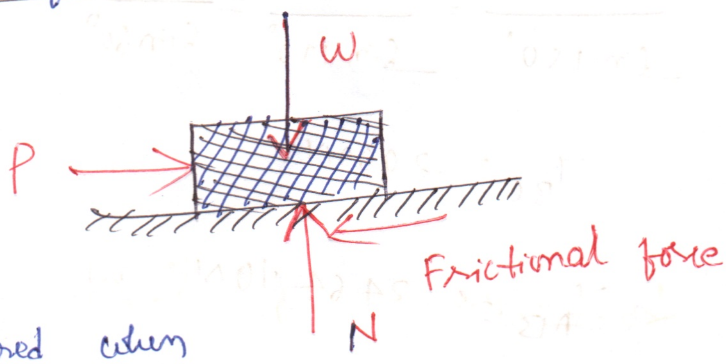
Def:- The force which opposes the relative motion (or) tendency towards such motion of two surfaces in contact is known as friction





⇒ No resistance offered when the surface are smooth.





Resistance is offered when surface are rough

* All frictional force acts opposite to the direction of motion.

⇒ Hence we see that whenever the surface of one body slides (or) tends to slide over another, each body exerts a tangentional frictional force on the apart from the normal reaction. This frictional force tries to prevent the motion of one surface with respect to the other. However these frictional forces are found to be limited in magnitude and will not be able to prevent the relative motion when sufficiently large external forces are applied

Types of friction :-

The friction which generates b/w two surfaces of rigid bodies which are in contact without any fluid between them is called dry friction

This dry friction is further classified in to two types

(1) Static friction

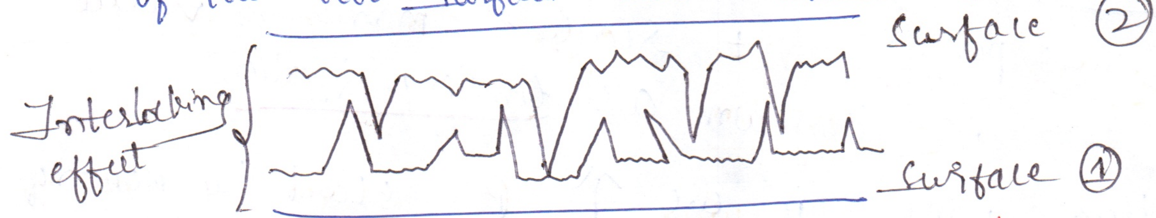
(2) Dynamic (or) kinetic friction.

(1) Static friction :- The friction developed b/w two surfaces which are in contact and are not moving relative to each other is known as static friction.

(2) Dynamic friction :- The friction developed b/w two surfaces which are in contact and are moving relative to each other is known as dynamic (or) kinetic friction

⇒ Development of friction :-

⇒ Due to this interlocking effect, a force will be developed which opposes the relative motion of the two surfaces called "frictional force"

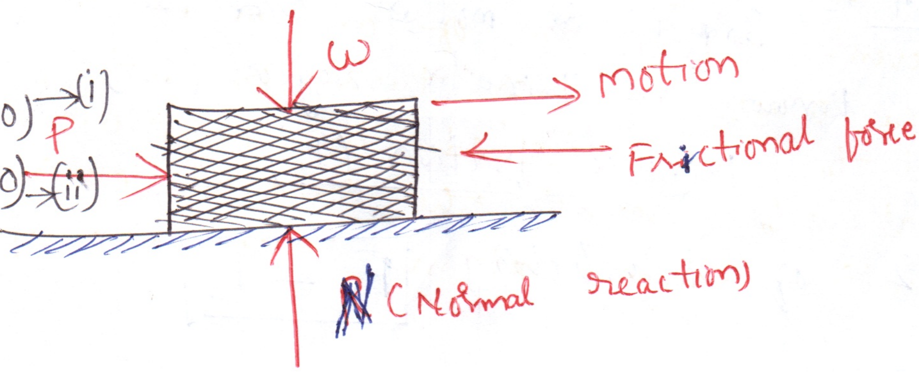


⇒ Limiting friction and impending motion :-

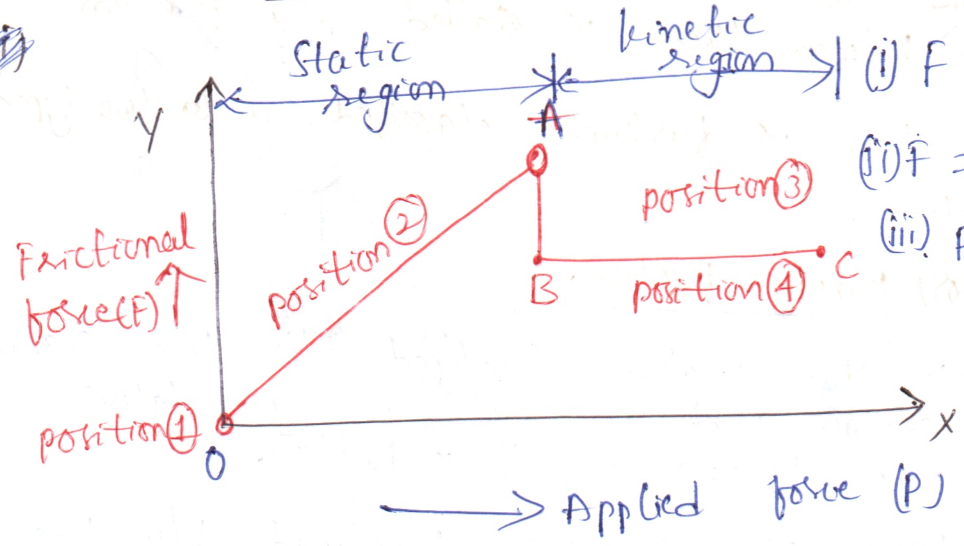
The following experiment is useful to understand the concept of limiting friction and impending motion

$P = F \; (\because \Sigma F_x = 0) \rightarrow (i)$

$W = R \; (\because \Sigma F_y = 0) \rightarrow (ii)$



* variation of frictional force "F" with the applied force "P"



(i) $F < F_{s\,max}$ = No motion occurs

(ii) $F = F_{s\,max}$ Motion impends

(iii) $F > F_{s\,max}$ under motion

(i) position (1) :- $P = 0 \Rightarrow F = 0$ frictional force will be zero
NO Applied force so frictional force

(ii) position (2) :-

P is ↑ ⟹ w.r.to F is ↑ (oppose the motion)

"OA" represents the increment in "F" w.r.to P

(iii) position (3) :- ③

At this position F reaches maximum value is known as "limiting friction"

After this point if we ↑ P start the motion of body known as "impending motion"
At this point is static friction (F_s)

Position 4 :- Body is in motion and the region is known as kinetic region (BC)
kinetic friction (F_K)

Bcz of interlocking $F_K < F_s$

* Coefficient of friction :- (μ) :-

It is the ratio of frictional force "F" and normal reaction "N"

Frictional force "F" is directly proportional to normal reaction "N"

$F \propto N$ where μ = Coefficient of friction

$F = \mu N$

$\mu = \frac{F}{N}$ ⇒
- $\mu_s = \frac{F_s}{N}$ coefficient of static friction
- μ_k = coefficient of kinetic friction $= \frac{F_k}{N}$

~~Colum~~ Coulomb's Law of friction :- (or)

French Coulomb's law of Dry friction :-

(1) Static laws of friction :-

(i) It opposes the direction of motion

(ii) The value of static friction ↑ with ↑ in applied force

(iii) The static frictional force reaches a maximum value (F_s) at the impending motion

(iv) It makes a constant ratio with normal reaction i.e $\frac{F_s}{N} = \mu_s$

(v) It depends only on the interlocking effect but not on the area of contact

(2) Kinetic laws of friction :-

(i) It opposes the direction of motion

(ii) It makes a constant ratio with normal reaction i.e., $\frac{F_k}{N} = \mu_k$

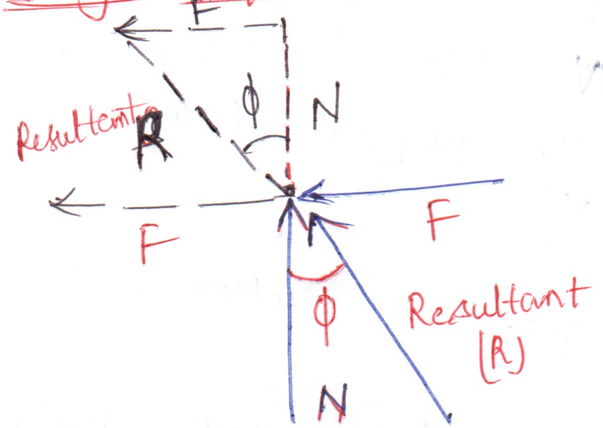
(iii) It depends only on the interlocking effect but not on the area of contact

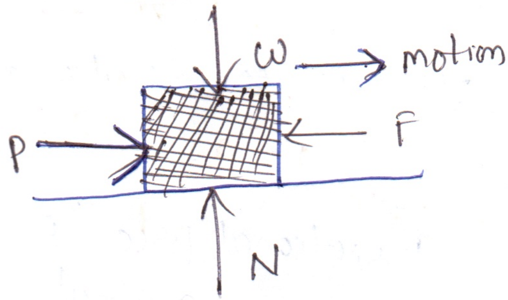
(iv) "μ_k" values are always less than "μ_s"

(v) μ_k remains constant for low & moderate speeds

For very high speeds "μ_k" values ↓ (Interlocking)

Angle of friction :-





The angle ϕ between the resultant (R) (Resultant of F & N)

• The Normal reaction 'N'

$$\tan\phi = \frac{F}{N} \quad \left[\because \mu = \frac{F}{N}\right]$$

(i) $\tan\phi_s = \frac{F_s}{N} \Rightarrow \tan\phi_s = \mu_s$ [Angle b/w static friction]

(ii) $\tan\phi_k$ = Angle b/w kinetic friction

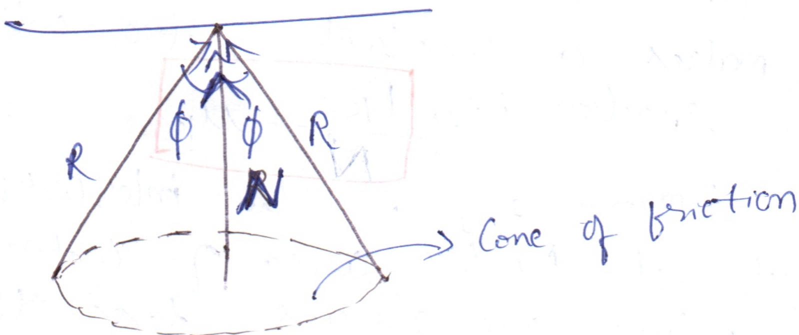
$$\tan\phi_k = \frac{F_k}{N} = \mu_k \Rightarrow \tan\phi_k = \mu_k$$

$$\boxed{\phi_k < \phi_s} \quad [\because \mu_k < \mu_s]$$

Cone of friction :-

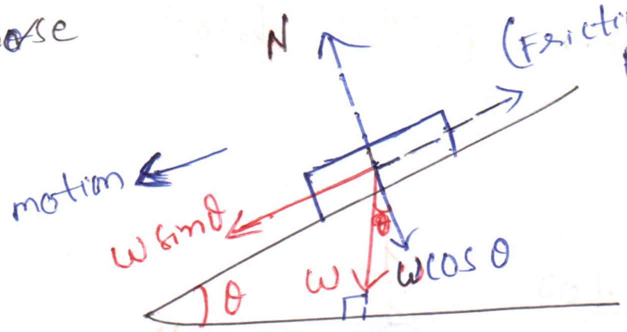
⇒ It is the cone obtained by rotating the resultant "R" (Resultant F & N) about the normal reaction "N"

$[\phi_k < \phi_s]$



Angle of repose :- (θ) :-

The angle at which the motion of the object starts on an inclined plane is known as angle of repose



$F = w\sin\theta$
$N = w\cos\theta$

⇒ From figure $F = w\sin\theta$

$N = w\cos\theta$

$$\mu = \frac{F}{N} = \frac{w\sin\theta}{w\cos\theta}$$

$\boxed{\mu = \tan\theta}$ → ① ⇒ $\boxed{\mu = \tan\phi}$ → ②

⇒ From eq ① & eq ②

⇒ $\tan\theta = \tan\phi$

$\boxed{\theta = \phi}$

(i) θ_s = Angle of ~~friction~~ static repose ⇒

$\tan\theta_s = \tan\phi_s$

$\boxed{\theta_s = \phi_s}$

(ii) θ_k = Angle of kinetic repose

⇒ $\tan\theta_k = \tan\phi_k$

$\theta_k = \phi_k$

Here $\boxed{\theta_k < \theta_s}$ ∴ $\boxed{\phi_k < \phi_s}$

Σ Forces parallel to the plane = 0

$$T + \frac{100}{9.81} a - 100 \sin 60^\circ + F_1 = 0$$

$\therefore$ $\boxed{F_1 = 16.67 N}$

$$T + \frac{100}{9.81} a = 69.93 \quad \text{———} \quad (3)$$

$\Rightarrow$ Now consider 50N block

Σ Forces normal to the plane = 0

$$N_2 = 50 \cos 30^\circ = 43.30 N \longrightarrow (4)$$

$\Rightarrow$ From the law of friction

$$F_2 = \mu N_2 = \frac{1}{3} \times 43.3 = 14.43 N \rightarrow (5)$$

$\Rightarrow$ Σ Forces parallel to 30° plane = 0

$$\frac{50}{9.81} a + F_2 + 50 \sin 30^\circ - T = 0$$

$$\frac{50}{9.81} a - T = -39.43 \quad \text{———} \quad (6)$$

$$F_2 = 14.43$$

Adding eqns (3) & (6)

$$\left[\left(\frac{100}{9.81}\right) + \left(\frac{50}{9.81}\right)\right] a = 69.93 - 39.43$$

$$a = 1.9947 \ m/sec^2$$

③ A body of weight 100 N is placed on a rough horizontal plane. Determine coefficient of friction if a horizontal force of 50 N just causes the body to slide over the horizontal plane

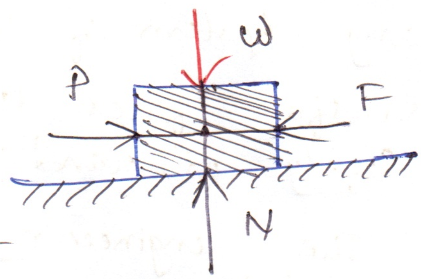
Sol: $\boxed{W = N}$ & $\boxed{F = P}$

Given, $W = 100\,N = N$ ✓

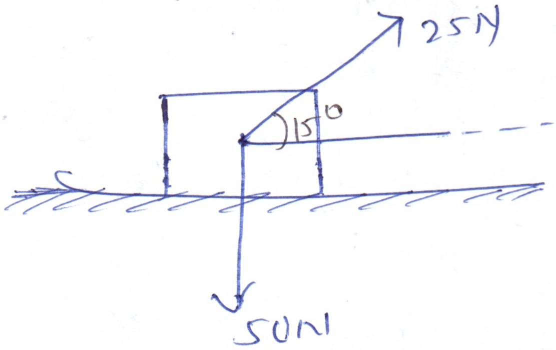
$P = 50\,N = F$ ✓

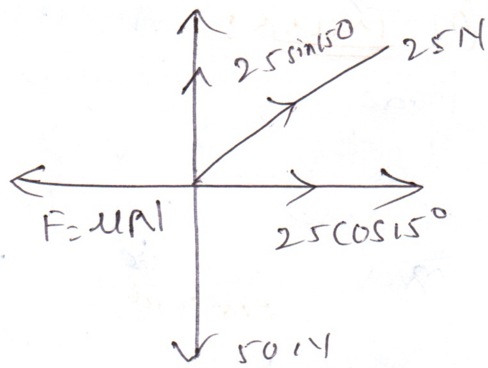
$$\mu = \frac{F}{N} \Rightarrow \mu = \frac{50}{100} = \frac{1}{2} = 0.5$$

$\mu = 0.5$



④ Determine "μ" from the figure





Sol:

$25\cos 15° = \mu N$ $\quad \boxed{\Sigma F_x = 0}$

$$\boxed{\mu = \frac{25\cos 15°}{N}}$$ ✓

$\Rightarrow N + 25\sin 15° = 50$ $\quad \boxed{\Sigma F_y = 0}$

$\Rightarrow N = 50 - 25\sin 15°$

Substitute N value in $\mu = \dfrac{25\cos 15°}{50 - 25\sin 15°}$

$$\mu = \frac{24.14}{43.53} = \mu = 0.554$$

Analysis of Structures:-

Introduction:- Structures may be defined as any system of connected members built to support (or) transfer forces acting on them and to ~~Satisfy~~ Safely withstand these forces.

The engineering structures may be broadly divided into

(a) Trusses

(b) Frames

(c) Machines

(a) **Truss**:- A truss is a structure that is made of straight, slender bars that are jointed together to form a pattern of triangles. Trusses are usually designed to transmit over ~~realitively~~ realitively long spans:

Eg: Bridge trusses & Roof trusses

The members of a truss are straight and the loads are applied only at the joints. Every member of a truss is a two force member

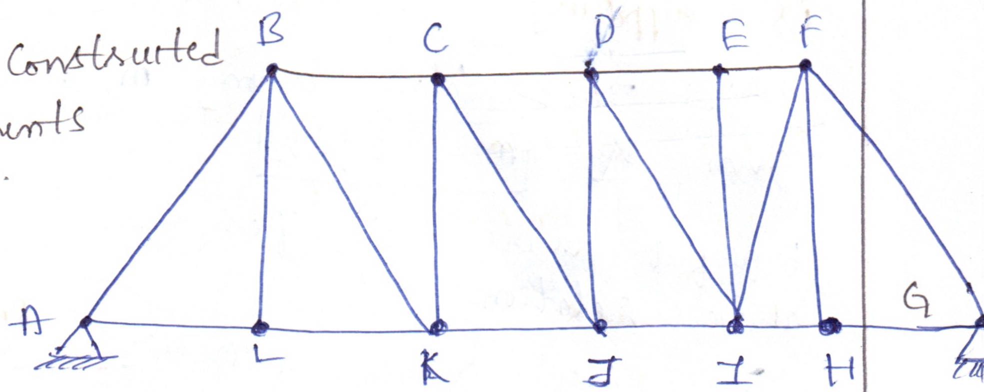
(b) **Frame**:- It is a structure consisting of several bars (or) members pinned together and in which one (or) more than one of its members are subjected to more than two forces. They are designed to support loads and these frames are stationary structures

(c) **Machine**:- Machines are structures designed to transmit and modify forces and contain some moving members

In this chapter, we focus on the types and analysis of trusses.

Elements of a truss :-

A simple truss is constsurted from different elements as shown in figure.



(a) Chords :- There are members which form the outline of the truss. The members like CD, LK, & EF are chords.

(b) verticals : These are the vertical members of the truss like CK, DJ, FH, etc--,

(c). Diagonals : These are the inclined members inside the truss like BK, DI, FI etc--,

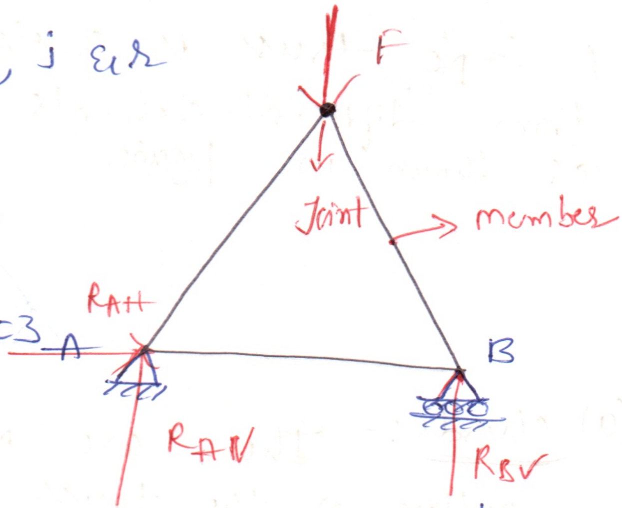
(d). End points : These are the members at the ends of the truss, may be vertical (or) inclined, like AB & FG

Types of truss :-

Trusses can be classified into two types. based on the relation among the number of joints (J) & No. of members (m), number of support reaction (r).

Perfect truss :- The truss consisting just sufficient members to keep it in equilibrium, when it is supporting an external load is called perfect truss

Eq 1: The relation among m, j & r given by $m = 2j - r$



A perfect truss satisfies the above relation

In fig, number of members = 3

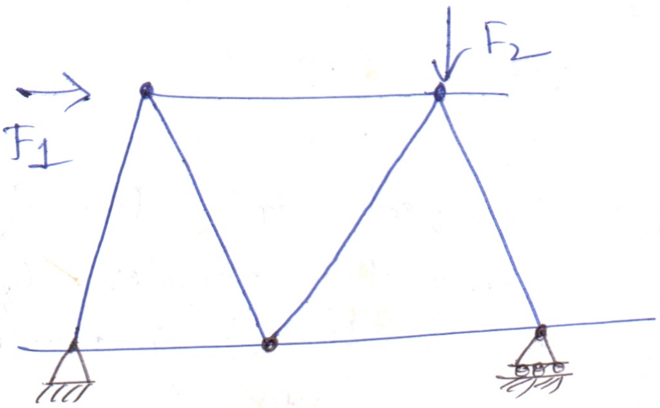
No. of joints ⇒ J = 3

No. of reactions ⇒ r = 3

(Generally for a truss, one end is hinged and other end is placed on a roller support. So at 'A' the reactions are R_{AV}, R_{AH} and at "B", the reaction is R_{BV}, The total reaction are 3)

⇒ So it is perfect truss

Eg (ii):



m = 7

J = 5

R = 3

$m = 2j - r$

7 = 2(5) − 3

7 = 7

Hence given truss is perfect

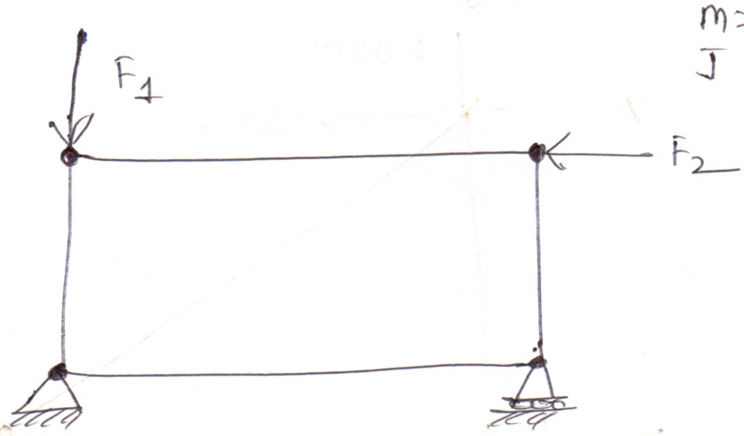
Imperfect truss :-

A truss which does not satisfies the relation $m = 2j - r$ is called imperfect truss.

These are two types of imperfect truss

(1) Imperfect Deficient truss :-

A truss which satisfies the relation $m < 2j - r$ is called imperfect deficient truss



$m = 4 \quad r = 3$
$J = 4$

$m = 2j - r$

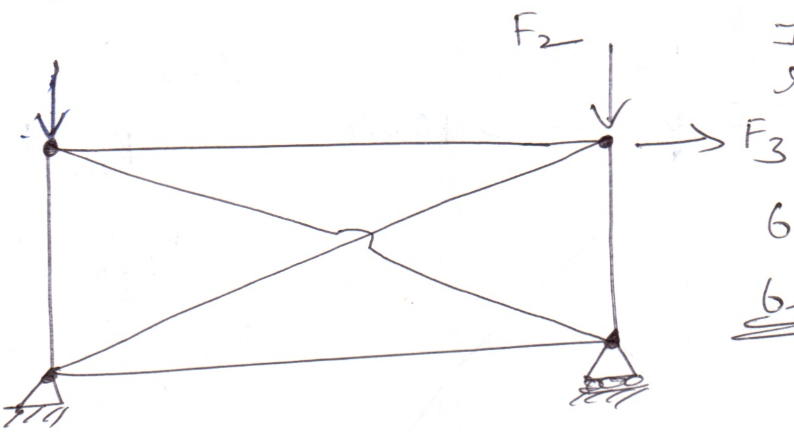
$4 = 2(4) - 3$

$4 = 8 - 3$

$\boxed{4 < 5}$

(2) Imperfect Redundant truss :

A truss which satisfies the relation $m > 2j - r$



$m = 6$
$J = 4$
$r = 3$

$6 > 2(4) - 3$

$6 > 5$

Basic Assumption for a Perfect truss

(1) All the members of a truss are straight & connected to each other at their ends by frictionless pins. Therefore, the joints cannot resist moments.

(2) All loads on the truss are applied at the pins only.

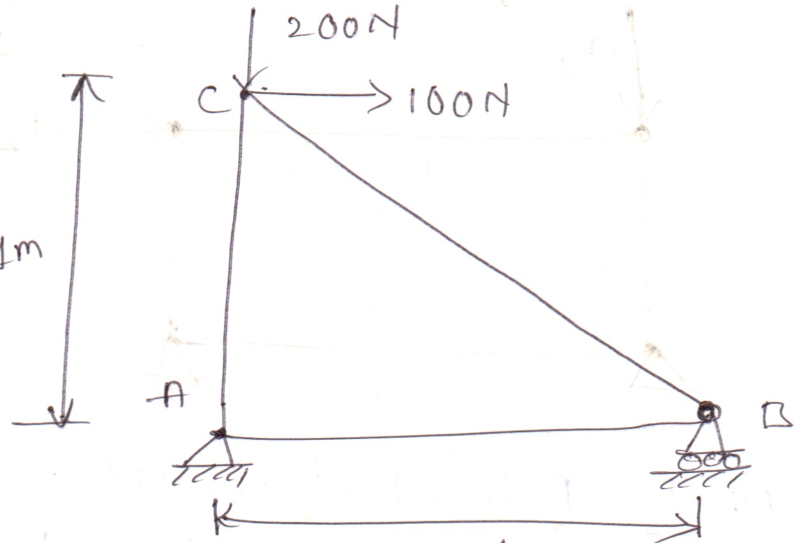
(3). All the members of the truss are assumed to be weightless

4). All the members of truss and external forces acting at pins lie in same plane

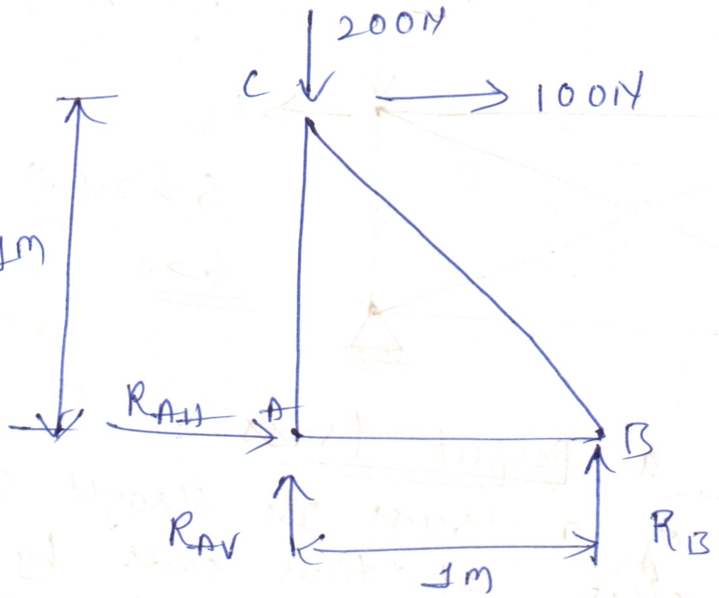
(5). For the analysis of perfect trusses, static equilibrium eqⁿs are used i.e., $\Sigma F_x = 0$, $\Sigma F_y = 0$ & $\Sigma M = 0$

Eg: Find the forces in the members of the given truss.

Sol:



⇒ FBD of the given truss



$\Sigma F_x \Rightarrow$

$R_{AH} + 100 = 0$

$R_{AH} = -100\,N$

R_{AH} is equal to 100 N in oppⁿ dir

$\Sigma F_y = 0 \Rightarrow R_{AV} + R_B - 200 = 0$ $R_{AV} + R_B = 200\,N$

$\Sigma M_A = 0 \Rightarrow -(100 \times 1) + (R_B \times 1)$

$R_B = 100\,N$

$R_{AV} = 100\,N$ $(\because R_{AV} + R_B = 200\,N)$

⇒ Analysis of trusses

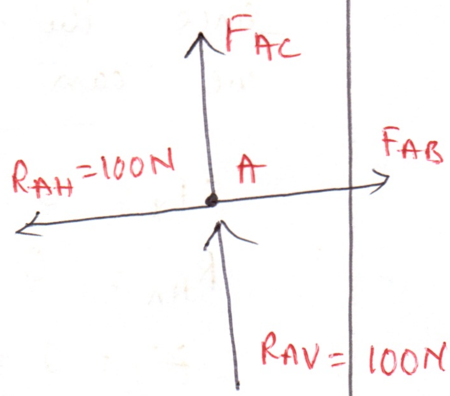
(i) The reaction at the supports
(ii) The force in the supports.

(a) method of joints (b) method of sections
(c) method of tension coefficients
(d) Graphical method.

⇒ Now consider all the forces in members as tension at all joints

At joint 'A':-

The FBD of joint 'A' is shown in figure



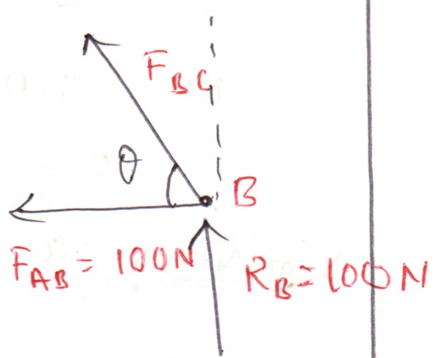
$\Sigma F_x = 0 \Rightarrow -100 + F_{AB} = 0$

$F_{AB} = 100\,N$

$\Sigma F_y = 0 \Rightarrow F_{AC} + 100 = 0$

$F_{AC} = -100\,N$

At joint 'B': The FBD of joint 'B' is shown in figure



$\Sigma F_x = 0 \Rightarrow -100 - F_{BC}\cos\theta = 0$

$\Rightarrow F_{BC}\cos\theta = -100$

From the figure

$\tan\theta = \frac{1}{1} \Rightarrow \theta = 45^\circ$

$F_{BC}\cos 45^\circ = -100$

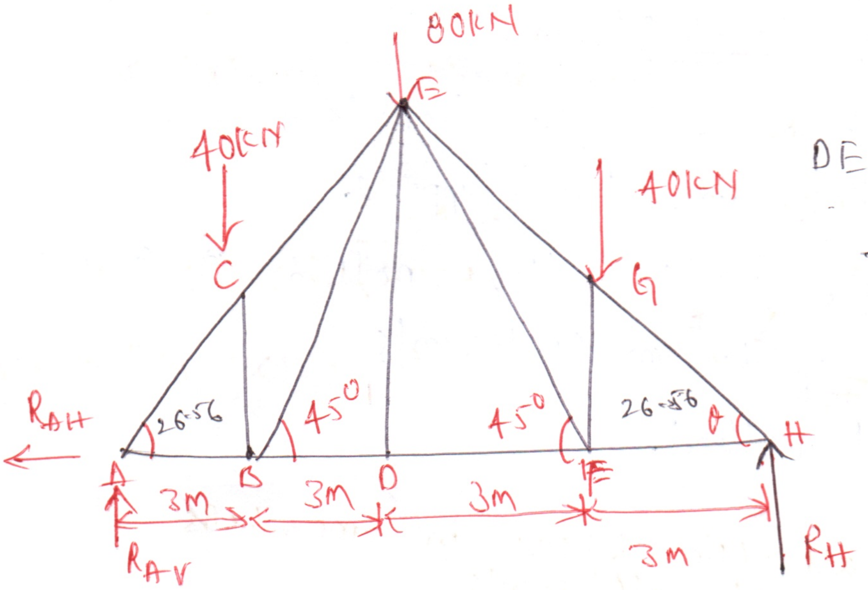
$F_{BC} = \frac{-100}{\cos 45} = -\underline{141.42}$

Hence the magnitude & nature of the forces in the members are

$F_{AB} = 100\,N$ (Tension)
$F_{BC} = -141.42$ (comp)
$F_{AC} = -100\,N$

(2)

Q:



DE = BD tan 45 = 3m

$\tan\theta = \frac{DE}{AD} = \frac{3}{6} = 0.5$

θ = 26.56

Ans: Consider the FBD of the given truss to find the supportive reactions

Since the member DE is a zero - force member we can remove DE in the FBD

$\Sigma F_x = 0$,

$R_{Ah} = 0$,

$\Sigma F_y = 0 \Rightarrow R_{AV} + R_H = 40 + 80 + 40$

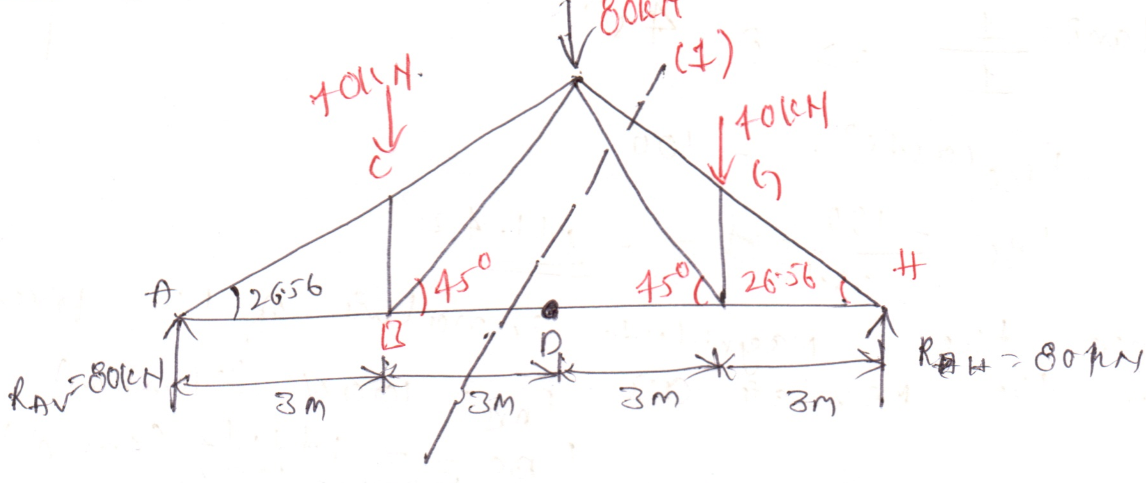
$R_{AV} + R_H = 160 kN$

⇒ loading are symmetric ∴ $R_{AV} = R_H = 80 kN$

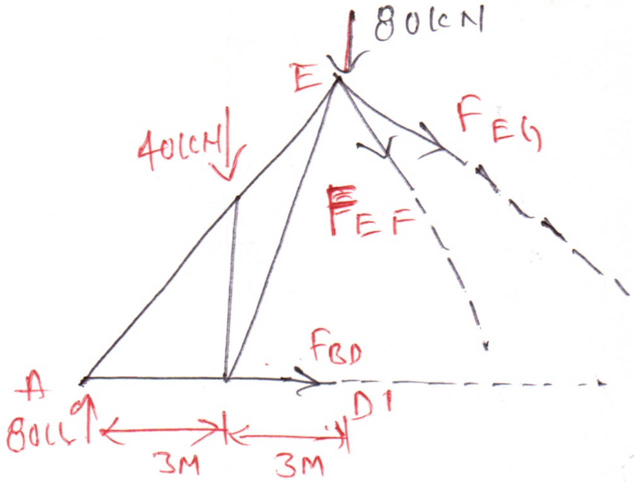
$DE = BD \tan 45 = 3m \Rightarrow \tan\theta = \frac{DE}{AD} = \frac{3}{6} = 0.5$

θ = 26.56

⇒ Consider a section (1-1) :-



Draw the FBD of the left portion (1)-(1)
Assume all the forces in the members as tension

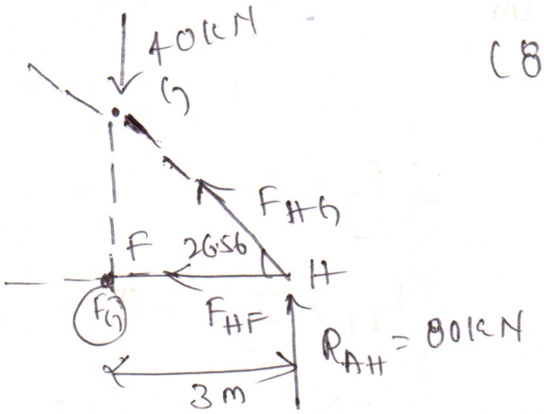


$\Sigma M_E = 0$

$-(80\times6)+(40\times3)+(F_{BD}\times3)=0$

$F_{BD} = 120KN$ (Tension)

⇒ Section 2 - 2



$\Sigma M_G = 0$

$(80\times3) - (F_{HF}\times F_G) = 0$

$F_G = Tan26.56\times3$

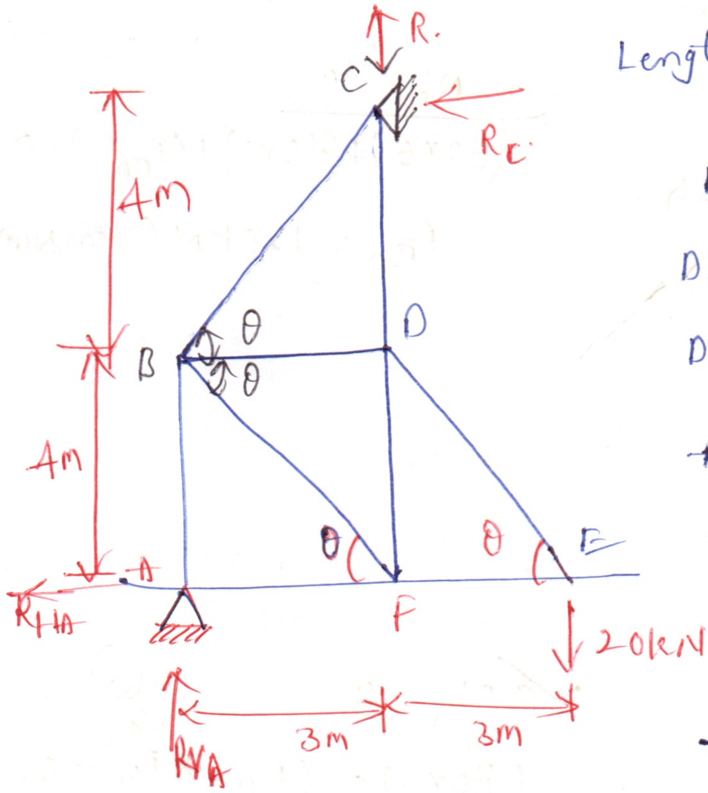
$F_G = 1.5M$

$\left[Tan\theta = \frac{F_G}{3}\right]$

$F_{HF} = 160kN$ (Tension)

(2) Find the forces in all members of the following truss

Q:



Length of the inclined Numbers

DEF

$DE^2 = EF^2 + DF^2$

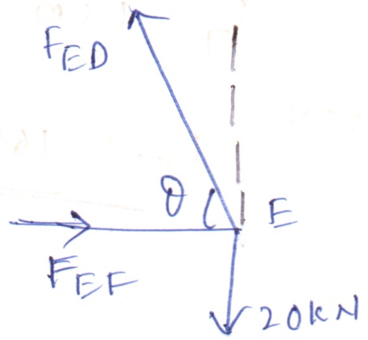
$DE = \sqrt{3^2+4^2} = 5m$

And $\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

⇒ At joint 'A', At joint 'C'
At joint 'B', At joint 'F'

At joint E :-



$\Sigma F_y = 0$

$F_{ED} \times \frac{4}{5} - 20 = 0$

$\Rightarrow F_{ED} = 25\,kN$ (Tension)

$\Sigma F_x = 0$

$F_{EF} + F_{ED}\cos\theta = 0$

$F_{EF} = 25 \times \frac{3}{5} = -15\,kN$ (comp)

At this stage as no other joint is having only two unknowns, no further progress is possible

Let us find the reactions at the supports Considering the whole structure

$\Sigma M_A = 0$

$R_C \times 8 - 20 \times 6 = 0$

$R_C = 15\,kN$

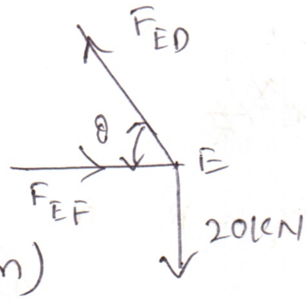
$\Sigma V = 0$

$V_A = 20\,kN$

$\Sigma H = 0$

$H_A = R_C = 15\,kN$

At joint E :-



⇒ $\Sigma F_y = 0$

$F_{ED} \times \frac{4}{5} - 20 = 0$

$F_{ED} = 25 kN$ (Tension)

⇒ $\Sigma F_x = 0$ Gives

$F_{EF} + F_{ED} \cos\theta = 0$

$\therefore F_{EF} = (-25 \times \frac{3}{5}) = -15 kN$ (compression)

⇒ At this stage as no other joint is having only two unknowns, no further progress is possible

let us find the reactions at the supports considering the whole structure.

$\Sigma M_A = 0$

$R_C \times 8 - 20 \times 6 = 0$

$R_C = 15 kN$

⇒ $\Sigma F_y = 0$

$V_A = 20 kN$ ✓

⇒ $\Sigma F_x = 0$

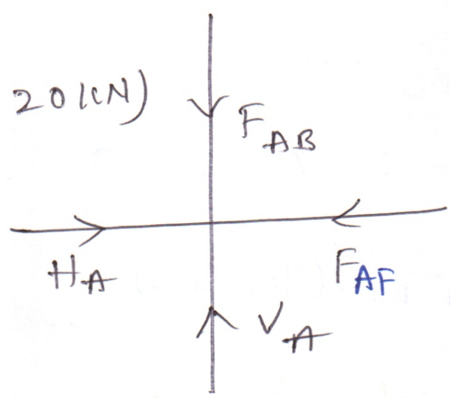
$H_A = R_C = 15 kN$ ✓

⇒ At joint A :-

$\Sigma F_y = 0$

$-F_{AB} + V_A = 0$ [$\because V_A = 20 kN$]

$F_{AB} = -20 kN$ (comp)



$\Sigma F_x = 0$

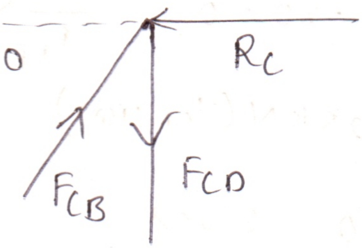
⇒ $-F_{AF} + H_A = 0$ ✓

⇒ $F_{AF} = 15 kN$ (comp) ✓

At joint "C" :-

$$\Rightarrow \Sigma F_x = 0$$

$$-F_{CB} \times \frac{3}{5} + R_C = 0 \text{ (or) } F_{CB} \times \frac{3}{5} - R_C = 0$$



$$F_{CB} = 15 \times \frac{5}{3} = 25\,kN \text{ (comp)}$$

$$F_{CB} = 25\,kN \text{ (comp)}$$

$$\Sigma F_y = 0$$

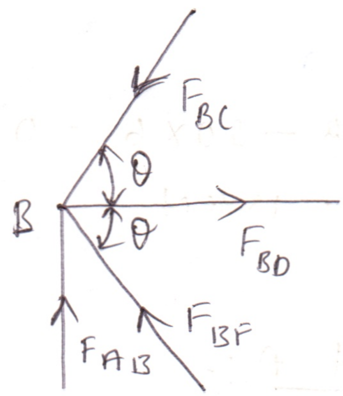
$$F_{CD} = F_{CB} \sin\theta$$

$$F_{CD} = 25 \times \frac{4}{5} = 20\,kN \text{ (Tension)}$$

At joint B :-

$$\Sigma F_y = 0$$

$$F_{BF} \times \frac{4}{5} - F_{BC} \times \frac{4}{5} + F_{AB} = 0$$



$$F_{BF} \times \frac{4}{5} = 25 \times \frac{4}{5} - 20$$

$$\therefore F_{BF} = 0$$

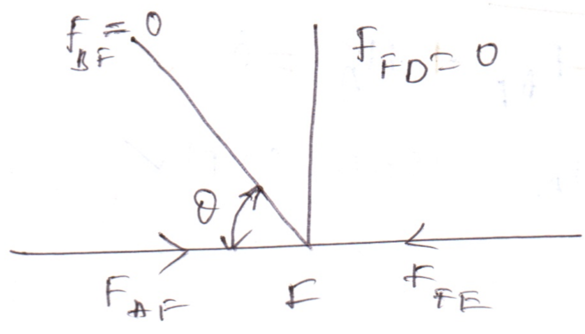
$$\Sigma F_x = 0$$

$$F_{BD} - 25 \times \frac{3}{5} = 0$$

$$F_{BD} = 15\,kN \text{ (Tension)}$$

At joint F

$$\Sigma F_y = 0 \Rightarrow$$



$$F_{FD} = 0$$

$$F_{BF} = 0$$