UNIT-2

Fundamentals of fluid flow

Fluid Kinematics

Introduction :-

It is a branch of fluid mechanics which deals with the motion of fluids such as the displacement, velocity, acceleration, flow rates [Mass flow rate, volume flow rate (or) flow discharge] and other related aspects of space time relations with out the forces and energies causing that fluid motion.

⇒ There are two methods by which the motion of a fluid is described one is legrangian method and other one is Eulerian method.

In the legrangian method the single fluid particle is followed by an observer during its motion and velocity, acceleration density etc. are described is called lagrangian method.

Eulerian method :-

In Eulerian method the velocity, acceleration, pressure, density etc. are described by an observer at a fixed point in the space of a flow fill.

Flow types :-

1. Steady and unsteady flow
2. Laminar and turbulent flow
3. Uniform flow and non-uniform flow
4. compressible and in-compressible flow
5. Rotional and irrotional flow
6. 1, 2, 3 - dimensional flow.

① steady and unsteady flow :-

Steady flow :-

It is at any point of the flowing fluid various characterisation such as velocity, Acceleration, pressure, density, temparature does not change with time.

Mathomatically, it is represented as

$$\left[\frac{\partial v}{\partial t}\right]=0 \quad \text{i.e} \left[\frac{\partial y}{\partial t}\right]=0 \quad \left[\frac{\partial w}{\partial t}\right]=0$$

Where u,v,w are velocity components.

Ex:- Flow of fluid through a pipe at constant rate of discharge

Unsteady flow:-

In that flow parameters at any point change with time.

Mathematically, it is represented as

$$\left[\frac{\partial v}{\partial t}\right] \neq 0, \text{ i.e } \left[\frac{\partial u}{\partial t}\right] \neq 0 \quad \text{i.e } \left[\frac{\partial w}{\partial t}\right] \neq 0$$

u,w are velocity components.

Ex:- Flow in which the quantity of liquid per sec is not constant.

2) Uniform and Non-uniform flow:-

Uniform flow:-

When the velocity [No other variable] of flow of fluid only does not change both in magnitude and direction from point to point in the flowing fluid at any given instead of time.

Ex:- Flow of liquids under pressure through long pipe lines of constant diameter.

Non-uniform flow:-

If the velocity of flow of fluids changes from point to point in the flowing fluid at any instant of time.

3) Laminar and Turbulent flow:-

Laminar flow:-

It is defined as the type of flow in which the fluid particles move along well defined paths (or) stream lines does the particle move in laminar (or) layers by sliding smoothly over the adjacent layer. This type of flow is called as stream line flow (or) laminar flow.

Turbulent flow:-

It is the type of flow in which the fluid particles have move in zig-zag motion (or) way. Then due to the movement of fluid particles in a zig-zag way the eddies formation takes place, which are responsible for the high energy loss. For a pipe flow the type of flow is determined by a non-dimensional number i.e, Renyolds number.

$\Rightarrow$ Renyolds number $= \frac{VD}{\nu}$

where,

D = diameter of pipe

V = means velocity of flow in a pipe

ν = Kinematic viscosity of fluid.

If the Renyold's number is $< 2000 \Rightarrow$ laminar flow

If the Renyold's number is $> 4000 \Rightarrow$ Turbulent flow.

If the Renyold's number lies b/N 2000 - 4000 $\Rightarrow$ transitional flow.

4) compressible and incompressible flow:-

compressible flow:-

compressible flow in that type of flow in which the density of the fluid changes from point to point in other words the density is not constant for the fluid.

Mathematically, compressible flow is,

$$\boxed{\rho \neq \text{constant}}$$

Ex:- Flow of gases through a nossile.

Incompressible flow:-

It is the type of flow in which the density is constant for the fluid flow.

Liquids are generally incompressible while gases are compressible flow. $\boxed{\rho = \text{constant}}$

Ex:- flow of liquid like water and oil.

Rational and irrational flow:-

Rational flow:-

It is the type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis and if the fluid particles while they flowing along stream lines donot rotate about their own axis that type of flow is called irrotational flow.

1, 2, and 3-D flows:-

Type of flow	example.
unsteady and 3D flow	$V = f(x, y, z, t)$ flood flows
steady and 3D flow	$V = f(x, y, z)$
unsteady and 2D flow	$V = f(x, y, t)$
Steady and 2D flow	$V = f(x, y)$ flow b/w two plates
unsteady and 1D flow	$V = f(x, t)$
steady and 1D flow	$V = f(x)$

Rate of flow (or) Discharge (Q) :-

* It is defined as the Quantity of a fluid flowing per second through a section through a pipe (or) a channel.
* For incompressible fluid the rate of flow (or) discharge is expressed as the volume of fluid flowing across the section per second.
* For incompressible flow fluid the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus.

1) For liquids the unit of discharge (Q) is m^3/sec (or) lit/sec.
2) For gases the unit of discharge (Q) is $K \cdot gf/sec$ (or) N/sec.

consider a liquid flowing through a pipe in which

A = cross sectional area of pipe

V = average velocity of the fluid.

$$\therefore \boxed{Q = A \times V}$$

continuity equation:-

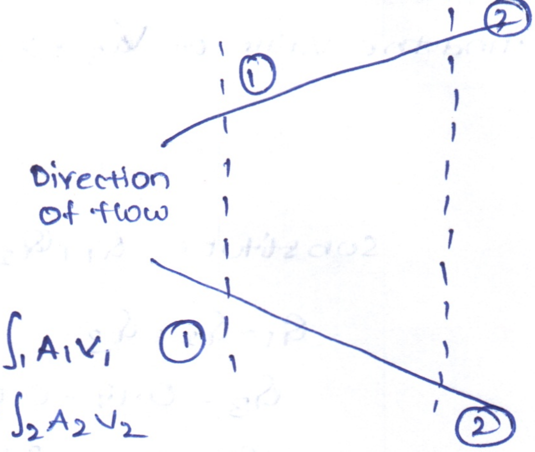
The equation is based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all cross sections the quantity of fluid per second is constant. consider two cross section of a pipe as shown in the figure.

let V_1 = average velocity at cross section 1-1

A_1 = area of pipe at 1-1

ρ_1 = density of fluid at 1-1

And



ρ_2 = density of fluid at 2-2

A_2 = area of pipe at 2-2

V_2 = average velocity at 2-2

Thus the rate of flow at section 1-1 = $\rho_1 A_1 V_1$

rate of flow at section 2-2 = $\rho_2 A_2 V_2$

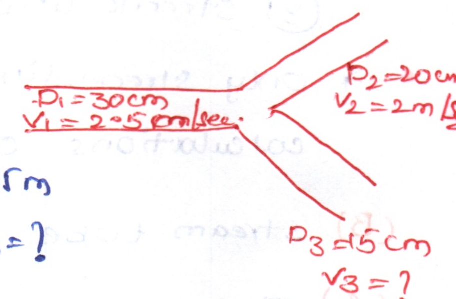
According to law of conservation of mass, the rate of flow at section 1-1 = the rate of flow at section 2-2

$$\therefore \boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$$

The above equation is applicable to the compressive fluids as well as incompressible fluids and is called continuity equation. If density values is equals i.e $\rho_1 = \rho_2$ then $\boxed{A_1 V_1 = A_2 V_2}$

(1) A 30 cm diameter pipe conveying water branches into two pipes of diameter 20 cm and 15 cm respectively. If the average velocity in the 30 cm pipe is 2.5 m/sec. Find the discharge in the pipe also determine the velocity in 15 cm pipe if the average velocity in 20 cm dia pipe is 2 m/sec.

Sol) Given,



Diameter, $D_1 = 30cm = 30 \times 10^{-1} = 0.3m$

$D_2 = 20cm = 0.2m$, $D_3 = 15cm = 0.15m$

$V_1 = 2.5$ m/sec, $V_2 = 2$ m/sec, $V_3 = ?$

W K T,

$$Q = A \times V \Rightarrow Q = \frac{\pi}{4}(d^2) \times V \Rightarrow \frac{\pi}{4}(0.3)^2 \times 2.5 = 0.07m^2$$

$$A_2 = \frac{\pi}{4}(D_2)^2 = \frac{\pi}{4}(0.2)^2 = 0.03m^2$$

$$A_3 = \frac{\pi}{4}(D_3)^2 = \frac{\pi}{4}(0.15)^2 = 0.017m^2$$

To find discharge in pipe 1 (or) Q_1

Velocity in pipe of diameter 15 cm i.e V_3

Let Q_1, Q_2, Q_3 are discharges in pipe 1, 2 & 3 then according to continuty equation $Q_1 = Q_2 + Q_3 \rightarrow (1)$

The discharge Q_1 in pipe-1 is given by $Q_1 = A_1 \times V_1$

$$= 0.07 \times 2.5$$

$$Q_1 = 0.17 \ m^3/sec.$$

To find the value of V_3 => $Q_2 = A_2 \times V_2$

$$= 0.03 \times 2$$

$$Q_2 = 0.06\ m^3/sec.$$

Substitute Q_1, Q_2 in ① We get

$$Q_1 - Q_2 = Q_3$$

$$Q_3 = 0.17 - 0.06$$

$$Q_3 = 0.11\ m^3/sec.$$

The value of V_3 => $Q_3 = A_3 V_3$

$$0.11 = 0.017 \times V_3$$

$$\boxed{V_3 = 6.47\ m/sec}$$ //.

Flow Pattern :-

To visualize the flow of fluids, the following pattern generated by the lines are used.

(A) Types of Lines :-

① Path line

② Stream line

③ Streak line

* Only stream lines are convenient for mathamatical analysis and calculations and other lines are used only for visualization.

(B) Stream tube

(C) Flow Net

(A) Types of lines :-

① Path line :-

A path line is actual path tracked by a single fluid particle.

①

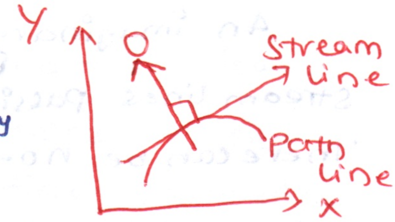
②

② Stream line :-

A stream line as a line tangent to the velocity vector of every point in a given instant. An imaginary curve has drawn through a following fluid. In such a way that the tangent to it at any point gives the direction of the velocity of flow at that point also called as flow line since flow is along the stream lines.

* stream line is tracking of motion of bulk modulus of fluid.

Different equation of stream line:



Two dimensional i.e $\frac{dx}{du} = \frac{dy}{V} = \frac{dz}{W}$ Where dx, dy and dz are direction and u, v are velocity components.

* Stream lines can't intersect itself.
* No two stream lines intersect each other.
* stream lines spacing inversly as the velocity.
* converging of the stream lines indicates acceleration flow in that direction.
* Stream lines indicate tracing of motion of a group of particles.
* There can be no fluid flows across a stream line.
* for steady flow stream line pattern remains same at different times for unsteady flow it varies from time to time and hence instantaneous.

① The equation of stream line passing through the origin in a flow fluid $u = \cos\alpha$, $v = \sin\alpha$ a constant α is determind as ___?

Sol) The stream line equation is, $\frac{dx}{u} = \frac{dy}{v}$

$$\Rightarrow \frac{dx}{\cos\alpha} = \frac{dy}{\sin\alpha}$$

integrating on both sides

$$\Rightarrow \int \frac{dx}{\cos\alpha} = \int \frac{dy}{\sin\alpha}$$

$$\sin\alpha \cdot x = \cos\alpha \cdot y + c$$

Where $x=0, y=0, c=0$

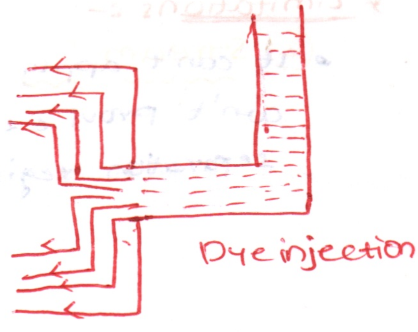
$$\sin\alpha \cdot x = \cos\alpha \cdot y$$

$$\Rightarrow y = \frac{\sin\alpha \cdot x}{\cos\alpha}$$

$$\boxed{y = \tan\alpha \cdot x}$$

③ Streak line:-

A streak line is a locus of particle which has earlier passed through a choosen point (or) a streak line traced by a single fluid particle passing through a fixed point in a flow field.



Ex:- The trial of a colour die injected at a point, path taken by a smoke coming out of exhaust.

Here acceleration in x, y, z directions become

$$\therefore a_x = \frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{dw}{dt} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

* local acceleration $A = a_x\bar{i} + a_y\bar{j} + a_z\bar{k} = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Local acceleration and convective acceleration:-

Local acceleration is defined as the rate of increase of velocity w.r.to time at a given point in a flow field in equation given by (1) the expression $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ is known as local acceleration.

Convective acceleration is defined as the rate of change of velocity due to the change of position of fluid particle in a fluid flow.

Continuity equation in 3-D:-

Consider a fluid element of length dx, dy, dz in the direction of x, y, z.

* Let u, v, w are the velocity components in x, y, z respectively.
* Mass of the fluid entering of face ABCD per second

$= \rho \times$ Velocity in x-direction $\times$ Area of ABCD

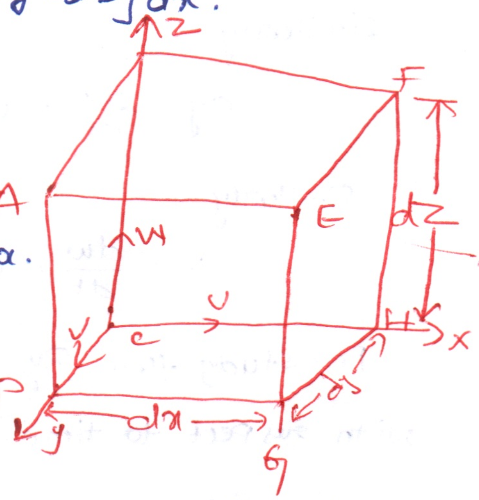
$= \rho \times u \times [dy \times dz]$.

* The mass of fluid leaving the face EFGH per second

$$= \rho \times u \times dy \times dz + \frac{\partial}{\partial x}[\rho \cdot u \cdot dy \cdot dz]dx.$$

* Gain of mass in x-direction is equal to,

Gain = mass through ABCD − Mass through EFGH per second



$$\text{Gain} = \rho \times u \times [dy \times dz] - \rho \times u \times dy \times dz - \frac{\partial}{\partial x}[\rho \times u \times dy \times dz]dx.$$

$$\boxed{\text{Gain} = -\frac{\partial}{\partial x}[\rho \times u \times dy \times dz]\,dx}$$

* Gain of mass in x-direction $= -\frac{\partial}{\partial x} \times \rho \times u \times [dx \cdot dy \cdot dz]$

Ily y-direction $= -\frac{\partial}{\partial y}(\rho \cdot v)[dx \cdot dy \cdot dz]$

Ily z-direction $= -\frac{\partial}{\partial z}(\rho \cdot w)[dx \cdot dy \cdot dz]$

$\Rightarrow$ Net gain of masses $= -\left[\frac{\partial}{\partial x}(\rho \cdot u) + \frac{\partial}{\partial y}(\rho \cdot v) + \frac{\partial}{\partial z}(\rho \cdot w)\right](dx \cdot dy \cdot dz) \rightarrow (1)$

Since the mass is neither created nor destroyed in the fluid element the net increase of mass per unit time in the fluid element must be equal to the rate of ~~flow~~ increase of mass of the fluid in the element.

But mass of fluid in the element is $= \rho \cdot [dx \cdot dy \cdot dz]$ and its rate of increase with time is $= \frac{\partial \rho}{\partial t}[dx \cdot dy \cdot dz] \rightarrow (2)$

Equating the above expressions we get.

$$\Rightarrow -\left[\frac{\partial}{\partial x}[\rho u] + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho \cdot w)\right](dx \cdot \cancel{dy} \cdot \cancel{dz}) = \frac{\partial \rho}{\partial t}(dx \cdot \cancel{dy} \cdot \cancel{dz})$$

$$\Rightarrow -\left[\frac{\partial}{\partial x}[\rho u] + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho \cdot w)\right] = \frac{\partial \rho}{\partial t}$$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho \cdot w) = 0 \rightarrow (3)$$

The above equation is continuity equation in cartesian co-ordinate system. This equation is applicable to (1) steady and unsteady flow

(2) uniform and non-uniform flow

(3) compressible and incompressible flow.

* The above eq (3) in steady flow $\frac{\partial \rho}{\partial t} = 0$ Hence eq (3) becomes

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho \cdot w) = 0 \rightarrow (4)$$

* If the fluid is incompressible then ρ is constant the above eq^n becomes $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ in 3-D.

(1) A fluid flow field is given by resultant velocity $V = x^2 y \bar{i} + y^2 z \bar{j} - [2xyz + yz^2]\bar{k}$. prove that it is a case of possible steady incompressible fluid flow. calculate the velocity and acceleration at the point (2, 1, 3).

Sol) For the given fluid flow, is the velocity component

$u = x^2 y$

$v = y^2 z$

$w = -[2xyz + yz^2]$

$$\frac{\partial u}{\partial x} = 2xy, \quad \frac{\partial v}{\partial y} = 2yz$$

$$\frac{\partial w}{\partial z} = -[2xy + 2yz]$$

For a case of possible steady incompressible fluid flow the continuity equation should be satisfy.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\cancel{2xy} + \cancel{2yz} - \cancel{2xy} - \cancel{2yz} = 0$$

Hence the velocity field at (2,1,3) , the velocity at (2,1,3) .
$x \; y \; z$
Substitute the valus in given equation of fluid flow field is,

$$V = (2)^2(1)\bar{i} + (1)^2(3)\bar{j} - [2(2)(1)(3) + (1)(3)^2]\bar{k}$$

$$V = 4\bar{i} + 3\bar{j} - 21\bar{k}$$

Resoltant Velocity $V = \sqrt{(4)^2 + (3)^2 + (-21)^2}$

$$V = \sqrt{16 + 9 + 441}$$

$$V = 21.58 \text{ m/sec.}$$

The acceleration at (2,1,3),

The acceleration components a_x, a_y, a_z for steady flow are

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\frac{\partial u}{\partial x} = 2xy, \quad \frac{\partial u}{\partial y} = x^2, \quad \frac{\partial u}{\partial z} = 0$$

$$a_x = x^2y(2xy) + y^2z(x^2) + (-2xyz - yz^2)(0)$$

$$a_x = 2x^3y^2 + x^2y^2z$$

$$a_x = 2(2)^3(1)^2 + (2)^2(1)^2(3)$$

$$a_x = 28 \text{ units.}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$\frac{\partial v}{\partial x} = 0, \quad \frac{\partial v}{\partial y} = 2yz, \quad \frac{\partial v}{\partial z} = y^2$$

$$a_y = x^2y(0) + y^2z(2yz) + [-2xyz - yz^2](y^2)$$

$$a_y = 2y^3z^2 - 2xy^3z - y^3z^2$$

$$a_y = 2(1)^3(3)^2 - 2(2)(1)^3(3) - (1)^3(3)^2$$

$$a_y = 18 - 12 - 9$$

$$a_y = -3 \text{ units.}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

$$\frac{\partial w}{\partial x} = -2yz, \quad \frac{\partial w}{\partial y} = -2xz - z^2, \quad \frac{\partial w}{\partial z} = -2xy - 2yz$$

$$a_z = x^2y(-2yz) + y^2z(-2xz - z^2) + [-2xyz - y^2](-2xy - 2yz)]$$

$$a_z = -2x^2y^2z - 2xy^2z^2 - y^2z^3 + 4x^2y^2z + 4xy^2z^2 + 2xy^2z^2 + 2y^2z^3$$

$$a_z = -2(2)^2(1)^2 3 - (1)^2(3)^3 + 4(2)^2(1)^2 3 + 4(2)(1)^2(3) + 2(1)^2(3)^3$$

$$a_z = -24 - 27 + 48 + 54 + 72 = 123 \text{ units}$$

Acceleration $= a_x \bar{i} + a_y \bar{j} + a_z \bar{k}$

$= 28\bar{i} + 3\bar{j} + 123\bar{k}$

Resultant acceleration $= \sqrt{(28)^2 + (-3)^2 + (123)^2}$

$= 126.11$ units//.

Rotational and irrotational motion:-

Rotational flow:-

The fluid particles while moving in the direction of flow rotate about their mass centres.

Ex:- Liquid in rotating tanks where the velocity of each particle varis directly as the distance from the centre of rotation.

Irrotational flow:-

If the liquid particles while moving in the direction of flow do not rotate about their mass centres.

Mathematically in irrotational flow,

curl of vector i.e, $\boxed{\nabla \times V = 0}$

⇒ Rotational angular velocity of fluid in 3-D.

We write the equation in matrix form.

i.e,
$$\omega = \frac{1}{2}\begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{bmatrix}$$

Angular velocity in x-axis

$$\omega_x = \frac{1}{2}\begin{bmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v & w \end{bmatrix}$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

Angular velocity in y-axis direction,

$$\omega_y = \frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right]$$

Angular velocity in z-direction

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

For irrotational flow $\omega_x = 0$ then

$$\frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] = 0 \Rightarrow \frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

lly $\omega_y = 0$ then

$$\frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right] = 0 \Rightarrow \frac{\partial w}{\partial x} = \frac{\partial u}{\partial z}$$

lly $\omega_z = 0 \Rightarrow \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0 \Rightarrow \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$.

* For fluids are flows of large viscosity flow is in variable in rotational motion then
for fluids such as air and water having small viscosity, the flow in the region away from the boundary may be treated as irrational.

* In the case of rapidly converging / accelerating flows flow may be treated as irrational.

Circulation and verticity:-

circulation (Γ):-

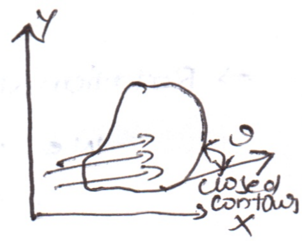
It deals with flow along a closed curve.

* It is a line integral of the tangential component of the velocity taken round a closed contour.

* It is the circulation of fluid particle in a closed curve, it is denoted by (Γ) gamma.

$$\Gamma = \oint V\cos\theta \, ds$$

where $V.\cos\theta = V_t$.



proof:-

consider a stream line, that forms a closed loop. The velocity of the stream line at any point is tangential to the radius of curvature i.e, 'P' is rotating at angular velocity i.e ω.

* Now consider a small length of the stream line i.e "ds".

* The circulation is defined as $\boxed{\Gamma = \oint V_t \cdot ds}$

* Integrate around the entire closed loop and substituting $V_t = \omega . R$, $ds = R . d\theta$.

$$\Gamma = \oint \omega R^2 d\theta$$

* The closed loop boundaries are 0 and 2π then

$$\Gamma = \oint_0^{2\pi} \omega R^2 d\theta$$

$$\Gamma = 2\pi \omega R^2$$

* In terms of V_t

$$\boxed{\Gamma = 2\pi V_t \cdot R}$$

Verticity :-

The limiting value of circulation divided by the area of closed contour as the area tends to '0'. It is denoted by (Ω)

$$\boxed{\Omega = \frac{\text{circulation}}{\text{Area}}}$$

Velocity Potential :- (ϕ)

Velocity potential is defined as a scalar function of space and time such that it's negative derivative w.r.to any direction gives the fluid velocity in that direction.

* velocity potential is denoted by "ϕ", for unsteady flow

$$\phi = f(x, y, z, t)$$

* for steady flow, $\phi = f(x, y, z)$
* Then velocity components $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$, $w = \frac{\partial \phi}{\partial z}$
* -ve sign indicates that 'ϕ' decreases with an increase in x, y, z directions.
* Incompressible steady flow, the continuity equation is,

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ then substitute the above values we get

$$\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial z}\right) = 0$$

$$\Rightarrow \boxed{\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0}$$

This is laplace's equation.

Rotational Motion / Components :-

Rotational components are average of shear strain in two mutually ⊥er directions.

* Angular velocity in rotational components in x-direction.

$$\omega_x = \frac{1}{2}\left[-\frac{\partial w}{\partial y}, \frac{\partial v}{\partial z}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial}{\partial y}\left[-\frac{\partial \phi}{\partial z}\right] - \frac{\partial}{\partial z}\left(-\frac{\partial \phi}{\partial y}\right)\right]$$

$$\omega_x = \frac{1}{2}\left(\frac{-\partial^2 \phi}{\partial z\, \cancel{\partial y}} + \frac{\partial^2 \phi}{\partial y\, \partial z}\right)$$

$||^{ly}$ $\omega_y = \frac{1}{2}\left(\frac{-\partial^2\phi}{\partial z \partial x} + \frac{\partial^2\phi}{\partial z \partial x}\right)$

$||^{ly}$ $\omega_z = \frac{1}{2}\left[\frac{-\partial^2\phi}{\partial x \partial y} + \frac{\partial^2\phi}{\partial x \partial y}\right]$

Rotational Components in x, y, z directions.

* If ϕ is continous function then in irrotational components $\omega_x = \omega_y = \omega_z = 0$

then $\omega_x \Rightarrow \frac{\partial^2\phi}{\partial y \partial z} = \frac{\partial^2\phi}{\partial y \partial z}$

$\omega_y \Rightarrow \frac{\partial^2\phi}{\partial x \partial z} = \frac{\partial^2\phi}{\partial x \partial z}$

$\omega_z \Rightarrow \frac{\partial^2\phi}{\partial x \partial y} = \frac{\partial^2\phi}{\partial x \partial y}$

Irrotational flow components

* Thus if velocity potential satisfies laplace equation. It represents the possible steady, incompressible irrotational flow. often an irrotational flow is known as potential flow

1) The velocity components in a two-dimensional flow are $u = \frac{y^3}{3} + 2x - x^2 y$ and $v = xy^2 - 2y - x^3/3$ show that these components represent a possible case of an irrotational flow.

Sol: Given $\Rightarrow u = \frac{y^3}{3} + 2x - x^2 y$

$\frac{\partial u}{\partial x} = 2 - 2xy$

$\Rightarrow v = xy^2 - 2y - x^3/3$

$\frac{\partial v}{\partial y} = 2xy - 2$

(i) For 2-D flow continuity eqn is $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ substituting the values we get

$\frac{\partial u}{\partial x}$ & $\frac{\partial v}{\partial y}$ $\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - 2xy + 2xy - 2 = 0$

$\Rightarrow \therefore$ It is a possible case of fluid flow

Then given velocity components is u & v

Then Rotation ω_z is given by $\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$= \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$$v = xy^2 - 2y - x^3/3 \qquad u = y^3/3 + 2x - x^2y$$

$$\frac{\partial v}{\partial x} = y^2 - \frac{\cancel{3}x^2}{3} = y^2 - x^2 \quad \& \quad \frac{\partial u}{\partial y} = y^2 - x^2$$

$$\Rightarrow \omega_z = \frac{1}{2}\left[(y^2 - x^2) - (y^2 - x^2)\right] = 0$$

Rotation is zero, which means it is case of irrotational flow.

Equi-potential line :- An equi-potential line is one along which velocity potential (ϕ) is constant.

$d\psi = 0$

$\phi = f(x, y)$ for steady flow

$$\Rightarrow d\phi = \frac{\partial \phi}{\partial x} \cdot dx + \frac{\partial \phi}{\partial y} dy \qquad \left[\frac{\partial \phi}{\partial x} = -u \quad \& \quad \frac{\partial \phi}{\partial y} = -v\right]$$

$$\Rightarrow d\phi = -(u dx + v dy)$$

For equipotential line $d\phi = 0$

$$\Rightarrow u dx + v dy = 0$$

$$\boxed{\frac{dy}{dx} = -\frac{u}{v}} \qquad \frac{dy}{dx} = \text{slope of equipotential line}$$

Stream function :- (ψ) :- It is a function of space & time such that its partial derivative w.r.to any direction gives the velocity component at right angles to this direction.

3 - Dim flow

$\psi = f(x, y, z, t) \rightarrow$ For unsteady flow

$\psi = f(x, y, z) \rightarrow$ For steady flow

$$\frac{\partial \psi}{\partial y} = u \quad \& \quad v = -\frac{\partial \psi}{\partial x}$$

For 2-D flow Continuity eqn

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial x}\left[\frac{\partial \psi}{\partial y}\right] + \frac{\partial}{\partial y}\left[-\frac{\partial \psi}{\partial x}\right] = 0$$

$$\Rightarrow \frac{\partial^2 \psi}{\partial x \cdot \partial y} + \left[-\frac{\partial^2 \psi}{\partial y \cdot \partial x}\right] = 0 \Rightarrow \boxed{\frac{\partial^2 \psi}{\partial x \cdot \partial y} - \frac{\partial^2 \psi}{\partial y \cdot \partial x} = 0}$$

Fluid may be rotational (or) irrotational

$\Rightarrow$ Rotational component $\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

Substitute u & v value get

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left[-\frac{\partial \psi}{\partial x}\right] - \frac{\partial}{\partial y}\left[+\frac{\partial \psi}{\partial y}\right]\right]$$

$$\omega_z = \frac{1}{2}\left[-\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}\right]$$

$$\omega_z = -\frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2}\right]$$ Eqn is know as poissions eqn. For an irrotational flow

since $\omega_z = 0$

Eqn becomes $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$ i.e., $\nabla^2 \psi = 0$

The stream function can also be defined as the flux (or) flow rate b/w two stream lines

Properties of stream function :-

1). On any stream line, ψ is constant everywhere [ψ = constant, Represents the family of streamline]

2). If the flow is continuous the flow around any path in the fluid zero

3). The rate of change of "ψ" with distance in arbitrary directional is propostional to the component of velocity normal to the direction.

4). The algebraic sum of stream function for two incompressible flow patterns is the stream function for the flow resulting from the superimposition of these patterns

i.e, $$\boxed{\frac{\partial \psi}{\partial s} + \frac{\partial \psi}{\partial s} = \frac{\partial(\psi_1 + \psi_2)}{\partial s}} \quad \text{—— (1)}$$

Cauchy Riemann eq^{n}s :-

From the above discussion of streamline function and velocity potential function and we arrive at the following conclusions

1) potential function (ϕ) exists only for irrotational flow

2). stream function (ψ) applies to both the rotational and irrotational flows [which steady & incompressible]

3). Incase of irrotational flow both the stream function and velocity function satisfy laplace eq^n and as such they are interchangble for irrotational and incompressible flow, the following relationship b/w ϕ & ψ hold's good

$$\left.\begin{aligned} u &= -\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \\ v &= -\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} \end{aligned}\right\} \rightarrow (2)$$

These eqn in hydrodynamics, & are sometimes called cauchy riemann eqn

Relation b/w stream function & velocity potential :-

FLOW THROUGH PIPES

* we have seen that when the "Re" is less than 2000 for pipe flow, the flow is laminar flow.
* where as the "Re" is more than 4000 the flow is known as turbulent flow.
* In this chapter the turbulent flow of fluids through pipes running full will be considered
* Here we will consider flow of fluids through pipes under pressure only.

⇒ **Loss of energy in pipes**

when a fluid is flowing through a pipe the fluid experiencles some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as:

Energy losses

⇒ 1. **Major Energy losses**

This is due to friction & it is calulated by the following formulae:

(a) Darcy – weisbach formula

(b) chezy's formula

⇒ 2. **Minor energy losses**

This is due to

(a) Sudden expansion of pipe

(b) Sudden contraction of pipe

(c) Bend in pipe

(d) Pipe fittings etc..

(e) An obstruction in pipe

⇒ (a) **Darcy – weisbach formula** :-

The loss of head (or energy) in pipes due to friction is calulated from Darcy – weisbach eqn

$$\boxed{h_f = \frac{4fLV^2}{2gd}}$$

where h_f = loss of head due to friction

f = Co-efficient of friction which is a function of Reynolds number

$$f = \frac{16}{R_e} \text{ for } R_e < 2000 \text{ (viscous flow)}$$

Laminar flow

$$f = \frac{0.079}{R_e^{1/4}} \text{ for } R_e \text{ varying from } 4000 \text{ to } 10^6$$

Turbulent flow.

L = Length of pipe, V = mean velocity of flow

d = diameter of pipe.

(b) Chezy's formula for loss of head due to friction in pipes :- f' = Frictional resistance per unit wetted area per unit velocity.

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \text{——— (1)}$$

where h_f = loss of head due to friction,

A = Area of c/s of pipe

V = mean velocity of flow

P = wetted perimeter of pipe

L = Length of pipe

Now the ratio of $\frac{A}{P}\left[= \frac{\text{Area of flow}}{\text{perimeter (wetted)}}\right]$ is called hydraulic mean depth (or) hydraulic radius and is denoted by "m"

$$\therefore \text{ Hydraulic mean depth, } m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$$

Substituting $\frac{A}{P} = m$ (or) $\frac{P}{A} = \frac{1}{m}$ in eqn (1)

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m} \text{ (or) } V^2 = \frac{h_f \times \rho g}{f'} \times m \times \frac{1}{L}$$

$$= \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}}\sqrt{m\frac{h_f}{L}} \quad \text{--- (2)}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where "C" is constant known as chezy's constant and $\frac{h_f}{L} = i$, where "i" is loss of head per unit length of pipe

⇒ Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ & $\sqrt{\frac{h_f}{L}}$ in eq (2)

$$\boxed{V = C\sqrt{mi}} \longrightarrow$$ it is known as chezy's formula

Minor energy (Head) losses :-

The loss of energy due to change of velocity of the following fluid in magnitude (or) direction is called minor loss of energy (or head) includes the following cases

1. Loss of head due to sudden enlargement
2. Loss of head due to sudden contraction.
3. Loss of head at the entrance of a pipe
4. Loss of head at the exit of a pipe
5. Loss of head due to an obstruction in a pipe
6. Loss of head due to bend in the pipe
7. Loss of head in various pipe fittings.

⇒ In case of long pipe the above losses are small as compared with loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe. these losses are comparable with the loss of head due to friction.

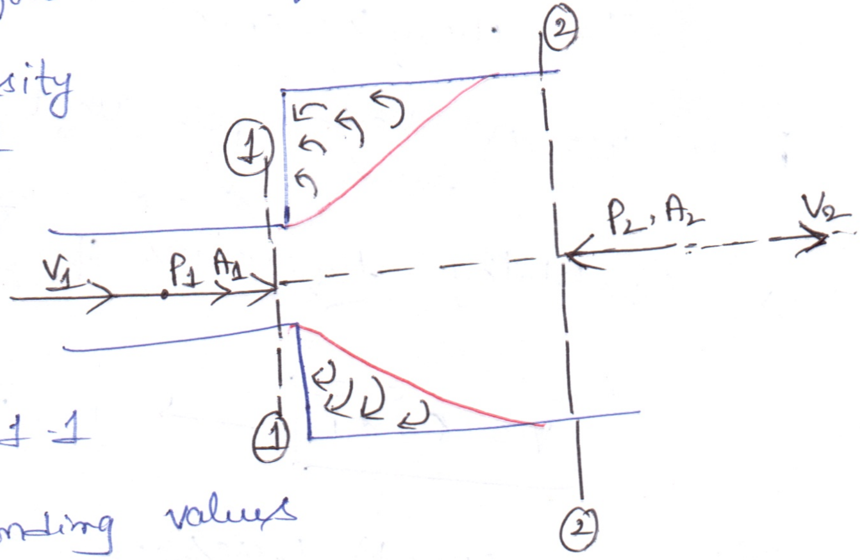
Loss of head due to Sudden enlargement :-

Consider a liquid flowing through a pipe which has Sudden enlargement as shown in fig. Consider two Sections (1)-(1) & (2)-(2) before and after the enlargement

Let P_1 = pressure intensity at section 1-1

V_1 = Velocity of flow at section 1-1

A_1 = Area of pipe at section 1-1



$P_2\ V_2$ & A_2 = corresponding values at Section 2-2

⇒ Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt changes of the boundary. Thus the flow seperates from the boundary and turbulent eddies are formed as shown in fig.

⇒ The loss of head (or energy) takes places due to the formation of these eddies

⇒ Let p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

h_e = loss of head due to sudden enlargement

Applying Bernoulli's eqn at Section 1-1 & 2-2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2} +$$

loss of head due to sudden enlargement

$z_1 = z_2$ as pipe is horizontal

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

$$h_e = \left[\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right] + \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] \quad \text{---} (1)$$

Consider the control volume of liquid b/w section 1-1 & 2-2 Then the force acting on the liquid in the direction of flow is the control volume in the direction of flow is given by

$$F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2$$

But experimentally it is found that $\boxed{p' = p_1}$

$$\therefore \; F_x = p_1 A_1 + p_1 (A_2 - A_1) - p_2 A_2$$

$$= p_1 A_2 - p_2 A_2$$

$$\boxed{F_x = (p_1 - p_2) A_2} \quad \text{---} (ii)$$

Momentum of liquid / sec at section 1-1 = mass × velocity

$$= \rho_1 A_1 V_1 \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid / sec at section 2-2 =

$$\rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$$

$$\therefore \text{ change of momentum / sec} = \rho A_2 V_2^2 - \rho_1 A_1 V_1^2$$

But from continuity eqn we have

$$A_1 V_1 = A_2 V_2 \text{ (or) } A_1 = \frac{A_2 V_2}{V_1}$$

$$\therefore \text{ change of momentum / sec} = \cancel{\rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2 \; \rho A_2 V_1 V_2}$$

$$= \cancel{\rho A_2}$$

$$= \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2 = \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 [V_2^2 - V_1 V_2] \quad \text{——— (3)}$$

Now Net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum (or) change of momentum per second

Hence equating (ii) & (iii)

$$(P_1 - P_2) A_2 = \rho A_2 [V_2^2 - V_1 V_2]$$

(or)

$$\frac{P_1 - P_2}{\rho} = V_2^2 - V_1 V_2$$

Dividing by "g" on both sides, we have

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g} \quad \text{(or)} \quad \frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2 - V_1 V_2}{g}$$

Substituting the value of $\left[\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right]$ in eqn (1)

we get

$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1 V_2 + V_1^2 - V_2^2}{2g}$$

$$h_e = \frac{V_2^2 + V_1^2 - 2V_1 V_2}{2g} = \left[\frac{[V_1 - V_2]}{2g}\right]^2$$

$$\boxed{h_e = \frac{(V_1 - V_2)^2}{2g}} \quad \text{——— (4)}$$

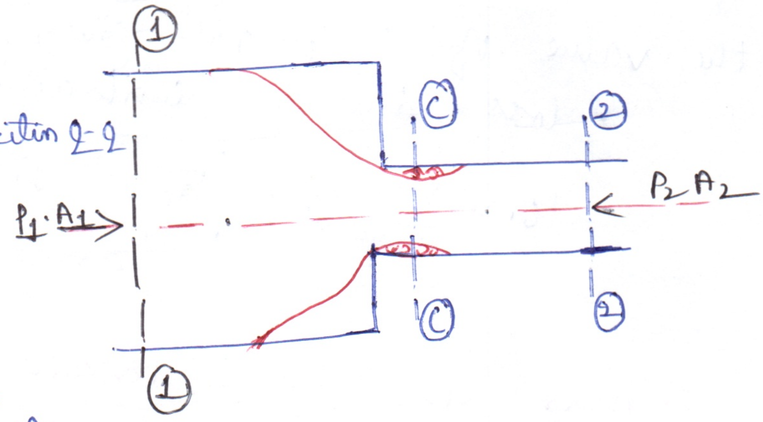
Loss of head due to Sudden Contraction :-

Let A_c = Area of flow at section c-c

V_c = velocity of flow at section c-c

A_2 = Area of flow at section 2-2

V_2 = velocity of flow at section 2-2



h_c = Loss of head due to sudden contraction

Now h_c = Actual loss of head due to enlargement from Section c-c to section (2)-(2) & is given by eqn. $h_e = \frac{(V_1 - V_2)^2}{2g}$

$$\frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right] \text{ ——— (i)}$$

From continuity eqn we have

$$A_c V_c = A_2 V_2 \quad (\text{or}) \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c/A_2)} = \frac{1}{C_c}$$

$$\left[\because C_c = \frac{A_c}{A_2}\right]$$

Substituting the value of $\frac{V_c}{V_2}$ in (i) we get

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

$$= \frac{k V_2^2}{2g} \text{ where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62 then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

Thus h_c becomes $h_c = \frac{k V_2^2}{2g} = 0.375 \frac{V_2^2}{2g}$

If the value of "C_c" is not given then the head loss due to contraction is taken as

$= 0.5 \frac{V_2^2}{2g}$ (or) $\boxed{h_c = 0.5 \frac{V_2^2}{2g}}$

Loss of head at the entrance of a pipe :-

This is the loss of energy which occurs when liquid enters a pipe which is connected to a large tank (or) reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. This loss is slightly more for a sharp edge entrance, than a rounded or bell mouthed entrance.

In practice the value of loss of head at the entrance (or inlet) of a pipe with sharp concerned entrance is taken $= 0.5 \frac{V^2}{2g}$, where V = velocity of liquid in pipe

This loss is denoted by h_i

$\therefore$ This loss $\boxed{h_i = 0.5 \frac{V^2}{2g}}$ ——— (1)

Loss of head at the exit of pipe :-

This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet [If outlet of the pipe is free] or it is lost in the tank (or) reservoir [If the outlet of the pipe is connected to the tank (or) reservoir]. This loss is equal to $\frac{V^2}{2g}$, where "V" is the velocity of of liquid at the outlet of pipe. This loss is denoted h_o

$$h_o = \frac{V^2}{2g}$$ where V = velocity at outlet of pipe

Loss of head due to an obstruction in a pipe

whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the c/section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow

beyond the obstruction due to which loss of head takes place as shown in fig.

Consider a pipe of area of c/s "A" having an obstruction as shown in the fig

Let a = maximum area of contraction
A = Area of pipe
V = velocity of liquid in pipe

Then (A − a) = Area of flow of liquid at section 1-1,

As the liquid flows and passes through section, a vena-contracta is formed beyond section 1-1 after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform & equal to velocity 'V' in the pipe

This situation is similar to the flow of liquid through sudden enlargement.

Let V_c = velocity of liquid at vena-contracta

Then loss of head due to obstruction = loss of head due to enlargement from vena-contracta to section 2-2

$$= \frac{(V_c - V)^2}{2g} \quad \text{——— (i)}$$

From continuity we have $\underline{a_c \times V_c = A \times V}$ ——— (2)

where a_c = area of c/s at vena-contracta

If C_c = Co-efficient of contraction

Then $C_c = \dfrac{\text{area at vena-contracta}}{(A-a)} = \dfrac{a_c}{(A-a)}$

$\therefore a_c = C_c \times (A-a)$

$\therefore$ Substituting this value in (2) we get

$$C_c \times (A-a) \times V_c = A \times V \quad \therefore \boxed{V_c = \frac{A \times V}{C_c(A-a)}}$$

$\therefore$ Substituting this value of 'V_c' in eqn (i) we get

Head loss due to obstruction

$$= \frac{(V_c - V)^2}{2g} = \frac{\left[\dfrac{A \times V}{C_c(A-a)} - V\right]^2}{2g}$$

$$\boxed{h_{ob} = \frac{V^2}{2g}\left[\frac{A}{C_c[A-a)} - 1\right]^2}$$

Loss of head due to Bend in pipe :-

When there is any bend in a pipe, the velocity of flow changes, due to which the seperation of the flow from the boundary and also formation of eddies takes place. Thus the energy lost. Loss of head in the pipe due to bend is expressed as

$$\boxed{h_b = \frac{kV^2}{2g}}$$

h_b = loss of head due to bend
V = velocity of flow
k = coefficient of bend

The value of 'k' depends on

(i) Angle of bend (ii) Radius of curvature of bend
(iii) Diameter of pipe.

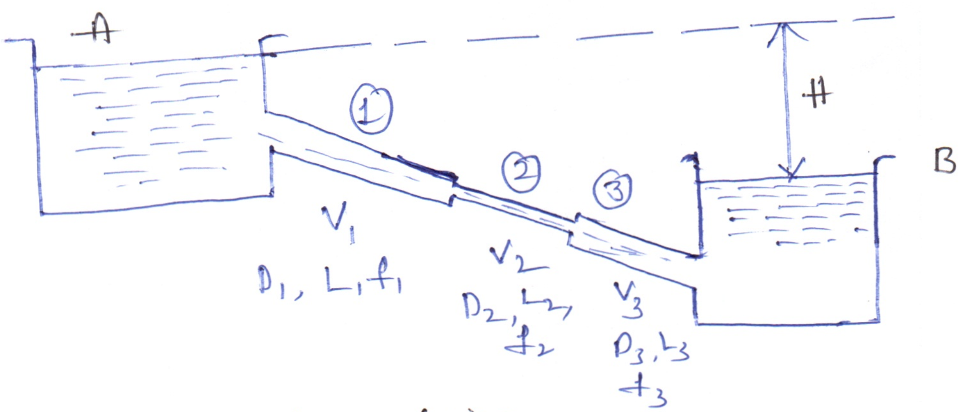
Loss of head in various pipe fittings :

The loss of head in the various pipe fittings such as valves, couplings etc.,

It is expressed as $= \dfrac{kV^2}{2g}$

v = velocity of flow, k = coefficient of pipe fitting.

⇒ Pipes in Series (or) Compound pipes :-



Pipes in Series

The above figure shows a system of pipes in series

Let V_1, V_2, V_3 = velocities of flow through 1, 2 & 3 respectively
D_1, D_2, D_3 = Diameter of pipes 1, 2, 3 respectively

L_1, L_2, L_3 = length of pipes 1, 2 & 3 respecti

BOUNDARY LAYER FLOW (B.L) Theory

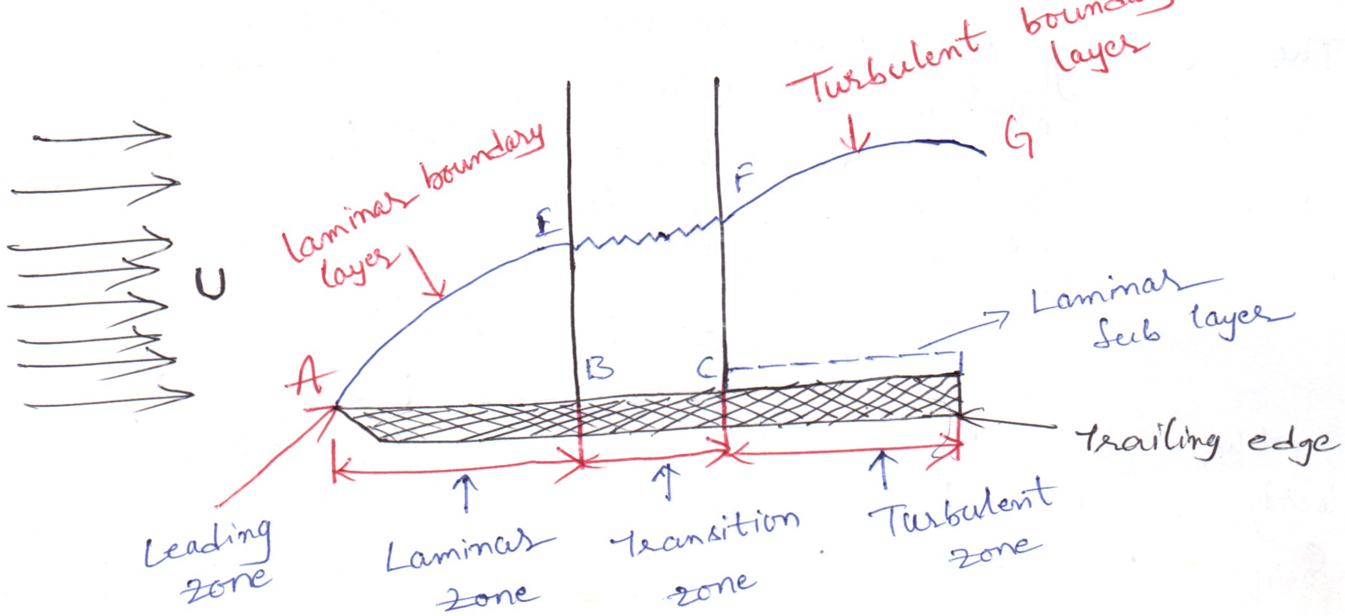
Introduction:-

Concept of Boundary layer (B.L) theory was introduced by L. Prandtl. It is an external flow over objects like air-foils, flow past a blunt body, & circular cylinder, experiences boundary layer formation. Two regions exist in an external flow over a body:

a) Flow outside the boundary layer, where viscosity influence is negligible and free stream velocity is uniform. Flow is irrotational. Hence ideal flow theories may be used.

b) Flow immediately adjacent to the object surface where ~~various~~ viscous and inertia forces causing flow rotational and velocity gradient (du/dy) exist normal to the boundary surface corresponding shear stress appreciable.

Figure below shows a long thin plate held stationary in the direction parallel to the flow in a uniform stream of velocity U_∞ effected by B.L



Flow over a plate

Terminology :-

(i) Free stream velocity :- It is the velocity of external flow with zero incidence angle and parallel to surface. It is also called ambient (or) potential velocity

(ii) Boundary layer :- when a solid body is effected by fluid flow, there is a narrow region of the fluid in the neighborhood of the solid body, where the velocity of fluid varies from zero to free stream velocity. Such narrow region of fluid, is called Boundary layer (B.L).

⇒ The fluid exerts a shear stress on the wall in the direction of motion i.e., $\tau = \mu\, du/dy$.

⇒ Zero (or) uniform pressure gradient is observed $(dp/dx = 0)$ over flat plate. The flow is rotational and shear stress decreases as flows downstream in the B.L region.

Laminar Boundary layer :-

The velocity of fluid on the surface of the plate should be equal to the velocity of the plate. But plate is stationary and hence velocity of fluid on the surface of the plate is zero.

But a distance away from the plate, the fluid is having certain velocity. Thus a velocity gradient is set up in the fluid near the surface of the plate.

⇒ The velocity gradient develops shear resistance, which retards the fluid. Thus the fluid with a uniform free stream velocity (U) is retarded in the vicinity of the solid surface of the plate and boundary layer region begins at the sharp ~~edge~~ leading edge

⇒ At subsequent points ~~downstream~~ the leading edge, the boundary layer region increases because the retarded fluid is further retarded. This is also referred as the growth of boundary layer.

⇒ Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar through the main flow is turbulent. This layer of the fluid is said to be laminar boundary layer. This is shown by "AE" in figure

⇒ The length of the plate from the leading edge, upto which laminar boundary layer exists, is called laminar zone This is shown by the distance AB. The distance "B" from the leading edge is obtained from "Re" equal to 5×10^5 for a plate. Because upto this 'Re' the boundary layer is laminar. The "Re" is given by $(Re)_x = \frac{U x}{\nu}$ where x = Distance from leading edge

U = Free-stream velocity of fluid

ν = Kinematic viscosity of fluid.

Hence for laminar boundary layer

$$5 \times 10^5 = \frac{U x}{\nu}$$

Transition Boundary layer:- The value of Re_x at which the boundary layer may change from higher laminar to initial turbulent varies from 3×10^5 to 6×10^5

Turbulent Boundary layer:

Further downstream the transition zone, the boundary layer is turbulent and continues to grow in thickness. This layer of boundary is called turbulent boundary. ~~B~~ layer which is shown by the portion FG in figure.

Laminar sub layer:- (δ'):- If the plate is very smooth, even in the region of tubulent boundary layer, there is very thin layer adjacent to the boundary in which flow is laminar. This layer is known as "laminar sub layer". It is observed in all turbulent boundary layers. The velocity distribution in this region is linear. throughout turbulent boundary δ' which is constant region.

∴ Thus the shear stress in the laminar sub layer $\tau_0 = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \mu \frac{u}{y}$

$\left[\text{For linear variation, } \frac{\partial u}{\partial y} = \frac{u}{y}\right]$

Boundary layer thickness (δ):- It is defined as the distance (vertically) from the boundary surface in which the velocity reaches 99% of the velocity of the main (or) free stream velocity.

It is also called nominal thickness (or) disturbance thickness

δ_{lam} = Thickness of laminar boundary layer

δ_{tur} = Thickness of turbulent boundary layer.

Factors affecting boundary layer thickness along a smooth plate

(a) It increases as the distance from leading edge increases

(b) It decreases with the increase in the velocity of flow approaching stream of fluid.

(c) Greater is the kinematic viscosity of fluid greater is the Boundary layer thickness.

Laminar Boundary layer thickness (δ_{lam}):

The velocity distribution is parabolic. As per Blasius,

$$\frac{\delta}{x} = \frac{k}{\sqrt{Re_x}}$$

$$\Rightarrow \delta = k\sqrt{x\nu / U_\infty}$$

"k" is Blasius constant varies from 4.64 to 5

"x" = distance from leading edge of plate

ν = kinematic viscosity of fluid

$$\therefore \boxed{\delta_{lam} \propto x^{1/2}}$$

Turbulent boundary layer thickness (δ_{tur})