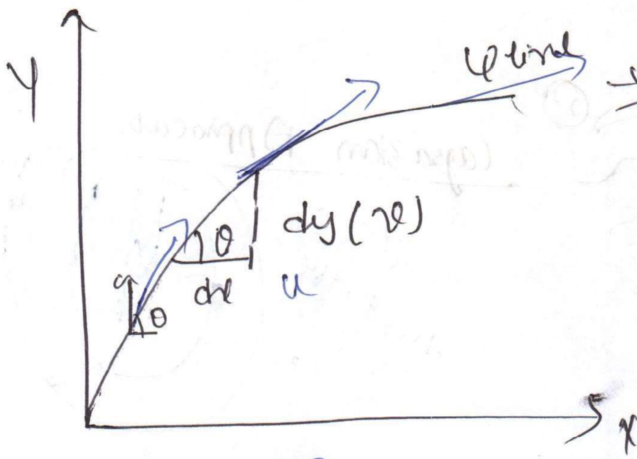
$$\int \partial v = \int -A \cdot e^{x} \cdot \partial y = -A \cdot e^{x} \int 1 \cdot dy$$

$$v = -A \cdot e^{x} (y) + C \rightarrow f(x)$$

Stream line (ψ-line):- In an imaginary curve drawn in flow field such that the tangent to it at any point will give direction of velocity at that point

line → direction

function → magnitude



→ Equation of ψ line

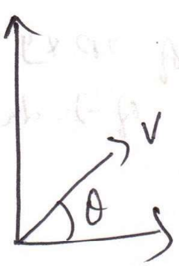
→ $v\,dx - u\,dy = 0$

$$\boxed{\tan\theta = \frac{dy}{dx} = \frac{v}{u}}$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

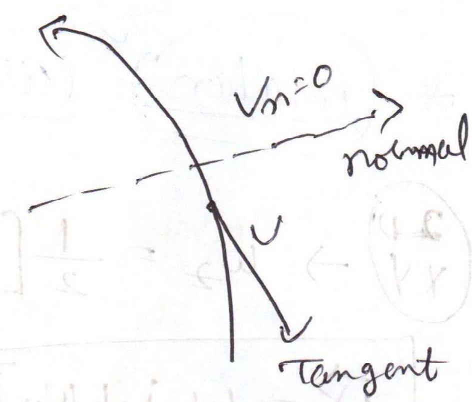
Slope of ψ line m_ψ

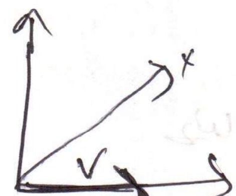
→ **Note**:- 1) The flow across the stream line is '0'



$V_x = V\cos\theta$

$V_y = V\sin\theta$





$V_x = V$

$V_y = 0$

$$Q_n = A_n \cdot \cancel{V_n}^{0}$$

$$= 0$$

NOTE2:-

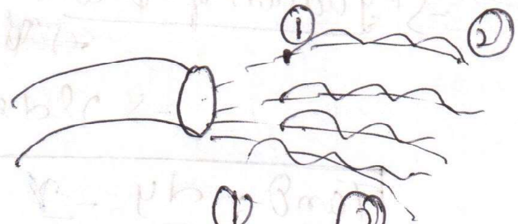
potential line will be ⊥es to stream line

* ϕ line $\frac{\perp}{\text{orthogonal}}$ ψ line

$$\boxed{m_\phi = \frac{-1}{m_\psi} = \frac{-u}{v}}$$

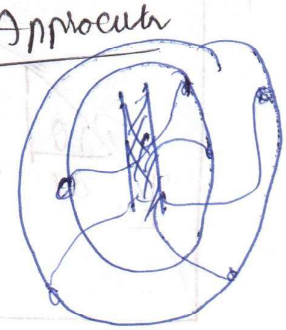
→ m_ϕ = slope of potential line
m_ψ = slope of stream line

path line:-

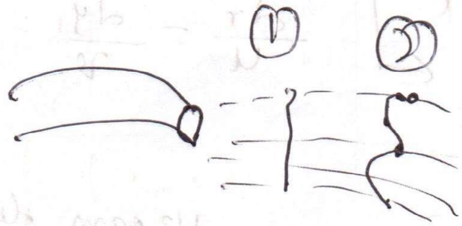


Lagrangian Approach

(ATC) Air traffic line



Streak line :-
(or) Filament line



Eulerian approach

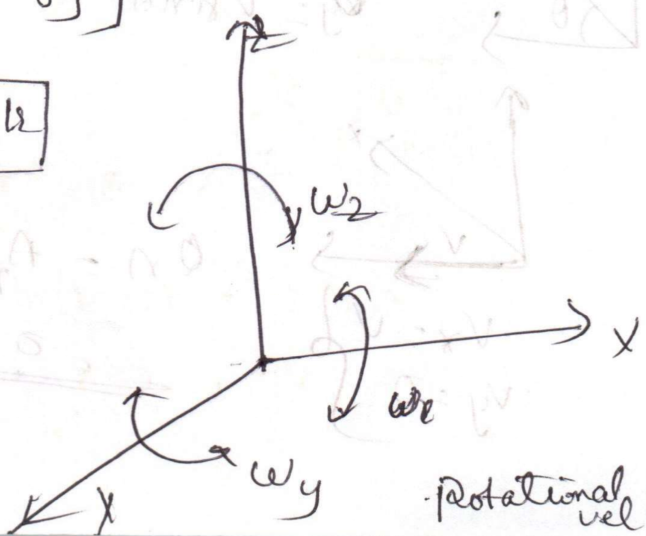
Ex: Rocket propulsion
Cigarette smoke

* Rotation (circulation) → vorticity)

$=0$ IR Rotation of 2D xy
$\neq 0$ R normal of z direction

$$2D\ xy \rightarrow \omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\boxed{\vec{\omega} = \omega_x i + \omega_y j + \omega_z k}$$



Rotational vel / veilos

$\rightarrow \omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$\rightarrow \omega_y = \frac{1}{2}\left[\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right]$

$\rightarrow \omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$

$$\frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Circulation (xy) :- $= \boxed{2 \times \omega_z \times \text{Area}}$

$\rightarrow \text{cir}_{xy} = \cancel{2} \times \frac{1}{\cancel{2}}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \times \text{Area}$

dy
dx

$\rightarrow \boxed{\text{cir}_{xy} = \int_Y \int_X \left[\left(\frac{\partial v}{\partial x}\right) - \left(\frac{\partial u}{\partial y}\right)\right] dx \cdot dy.}$

Vorticity (xy) :- $\frac{\text{Circulation}}{\text{Area}}$

$\rightarrow \boxed{\text{Vorticity}_{xy} = 2 \times \omega_z}$

Additional topics
→ Angular shear deformation
→ source flow
→ sink flow
→ Singulate
→ Doublate

$\boxed{\text{Vorticity vector} = 2\left[\omega_x i + \omega_y j + \omega_z k\right]}$

Gate
velocity field
$\vec{V} = ax\,i + ay\,j$ then equation of stream line passing through (1, 2)

A) $x - 2y = 0$
B) $y - 2x = 0$
C) $x + 2y = 0$
d) $y + 2x = 0$

$\vec{V} = \overset{u}{ax}\,i + \overset{v}{ay}\,j$

$\tan\theta = \frac{dy}{dx} = \frac{v}{u} = \frac{\cancel{a}y}{\cancel{a}x} \Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$

$\log(y) = \log(x) + \log(c)$

$\cancel{\log} y = \cancel{\log} x$

$y = cx \Rightarrow 2 = c \cdot 1 \Rightarrow c = 2$

$y - 2x = 0$

$y - 2x = 0$

Gate
Q.03

IC & IR   I Compressible & I Rotational

P) $u = 2x \quad v = 3y$

Q) $u = 2x + 3y \quad v = 0$

R) $u = 2x, \ v = -2y$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \ (=IC)$ | $\frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \ (IR)$

$2+3 \neq 0$

$2+0 \neq 0$

$2+(-2) = 0$

$1/2\,(0-0) = 0$

IR

(a) P & R

(b) Q & R

(c) Q

(d) R

8) The circulation around circle of radius 2 units for the flow field given by $u = 2x+3y$, $v = -3y$

A) -4π B) -8π

C) -12π D) -24π

$R = 2$ units

Sol.

$\text{Cir}_{(x\text{-}y)} = 2 \times \omega_z \times \text{Area}$

$= \cancel{2} \times \frac{1}{\cancel{2}}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \times \pi R^2$

$= \frac{\partial}{\partial x}\left[\cancel{-3y}\right]^0 - \frac{\partial}{\partial y}\left[2x+3y\right) \times \pi \times 2^2$

$= (0-3) \times 4\pi \rightarrow -12\pi$ units

G-10

9) $\bar{V} = 2xy\,i - x^2 z\,j$

vorticity vector $[1, 1, 1]$

Sol.

Vorticity vector $(1,1,1)$

$= 2\left[\omega_x i + \omega_y j + \omega_z k\right]$

$$\vec{V} = \overset{u}{(2xy)}\,i - \overset{v}{(x^2 z)}\,j + 0\,k$$

$$\frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & \cancel{w}\,0 \end{vmatrix}$$

$2xy \quad x^2 z$

$$\omega_x = \frac{1}{2}\left[0 - \frac{\partial}{\partial z}\left[-x^2 z\right]\right] = \left[\frac{x^2}{2}\right] = \frac{1}{2}$$

$$\omega_y = \frac{1}{2}\left[0 - \frac{\partial}{\partial z}\left[\cancel{2xy}\right]^{0}\right] = 0$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial}{\partial x}\left[-x^2 z\right] - \frac{\partial}{\partial y}(2xy)\right]$$

$$= \frac{1}{2}\left[-2xz - 2x\right] = -2$$

$[1, 1, 1]$

$$\rightarrow 2\left[\omega_x i + \omega_y j + \omega_z k\right]$$

$$= 2\left[\tfrac{1}{2} i + 0\,j + (-2)\cdot k\right]$$

$$= i - 4k$$

Gate 14 :- $V = k\left[y\hat{i} - x\hat{k}\right] \Rightarrow \overset{u}{(ky)}\,\hat{i} + \overset{v}{(0)}\,j + \overset{w}{(-kx)}\cdot k$

vorticity$_{xy}$ = ?

$\rightarrow$ vorticity$_{(xy)}$ $= 2 \times \omega_z$

$$= 2 \times \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \left[0 - \frac{\partial}{\partial y}\left[k \cdot y\right]\right] = -k$$

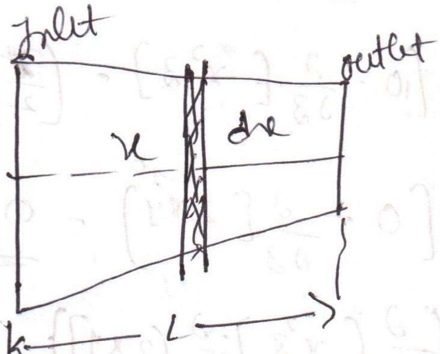
Gate-07 A flow through a nozzle, flow velocity along nozzle axis is given by $V = U_0\left[1 + \frac{3x}{L}\right] i$

where 'L' is the length of the nozzle

'x' is the distance from its inlet plate. Find the time required for a fluid particle on the nozzle axis to travel from its inlet plane to exit plane.

a) $\frac{L}{3U_0}\ln(3)$ (b) $\frac{L}{3U_0}\ln 4$ (c) $\frac{1}{3U_0}\ln(5)$ (d) $\frac{1}{2.5U_0}\ln(3)$

Eg: nozzle $V = u_0\left[1+\frac{3x}{L}\right]i$



$u = \frac{dx}{dt}$

$\Rightarrow \int dt$

$t \Rightarrow \int \frac{dx}{u}$

$t = \int dt$

$t = \int \frac{dx}{u_0\left[1+\frac{3x}{L}\right]}$

$= \frac{1}{u_0}\left[\frac{\log\left[1+\frac{3}{L}x\right]}{3/L}\right]_{x=0}^{L}$

$\frac{L}{3u_0}\left[\log\left(1+\frac{3}{L}\cdot L\right) - \log\left[1+3/L(0)\right]\right]_0^L$

$= \frac{L}{3u_0}\left[\ln(4)-0\right] \Rightarrow \frac{L}{3u_0}\left[\ln(4)\right]$

Potential function (ϕ function) (3D)

Is a function of space & time defined such that the negative derivative w.r.to in x-direction direction will give component of velocity in that

$-\left(\frac{\partial \phi}{\partial x}\right) = u$

$-\left(\frac{\partial \phi}{\partial y}\right) = v$

$-\left(\frac{\partial \phi}{\partial z}\right) = w$

(1) $\rightarrow$ (2)

$\phi_1 > \phi_2$

$\frac{\partial \phi}{\partial x} = \frac{\phi_2 - \phi_1}{x_2 - x_1}$

Notes

1) '$-$' sign indicates flow will always in direction decreasing potential

(2) If ϕ function is continuous with most irrotational

ϕ function $\rightarrow$ Continuous $\rightarrow$ I/R

$w_z = 0 = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$

$$-\frac{\partial}{\partial x}\left(\frac{-\partial\phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial\phi}{\partial x}\right) = 0$$

$$\boxed{\frac{\partial^2\phi}{\partial x\cdot\partial y} = \frac{\partial^2\phi}{\partial y\cdot\partial x}}$$ continuous

2D

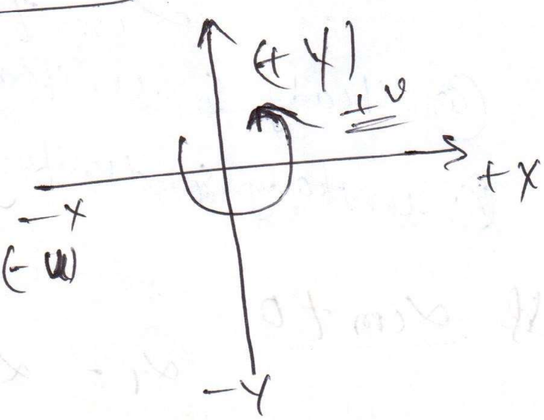
Stream function → (ψ function)

It is function of space & time defined such that the derivative w.r.to any direction will give the component of velocity at right angle in counter clockwise direction

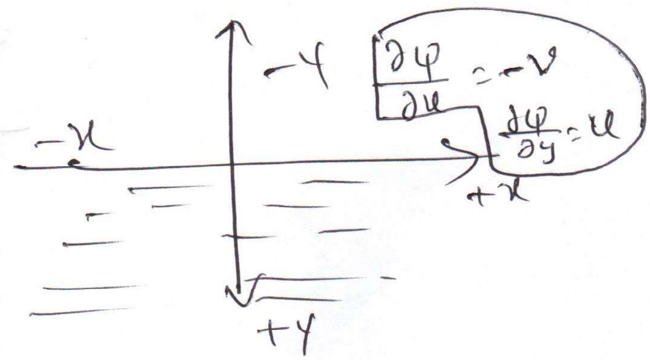
$$\psi = f[\text{space}, \text{time})$$

$$\frac{\partial\psi}{\partial x} = v$$

$$\frac{\partial\psi}{\partial y} = -u$$



It is 2-Dimensional function



Note:-1. The stream function satisfies the laplace equation it must be irrotational

$$IR \rightarrow \omega_z = 0$$

$$\frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0 \Rightarrow \frac{\partial}{\partial x}\left[\frac{\partial\psi}{\partial x}\right] - \frac{\partial}{\partial y}\left(\frac{-\partial\psi}{\partial y}\right] = 0$$

$$\rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \quad \boxed{\nabla^2\psi = 0}$$ laplace equation

Cauchy Remains equations :-

$\rightarrow u = -\left(\frac{\partial \phi}{\partial x}\right) = -\left(\frac{\partial \psi}{\partial y}\right) \Rightarrow \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$ ✓

$\rightarrow v = \left(\frac{-\partial \phi}{\partial y}\right) = \frac{\partial \psi}{\partial x} \Rightarrow \boxed{\frac{\partial \phi}{\partial y} = \frac{-\partial \psi}{\partial x}}$ ✓

Gate-14 :-

fluid flow having convective acceleration

$\alpha_{con} \neq 0$

(a) steady & uniform (c) unsteady Non-uniform

(b) unsteady & uniform ✓ (d) steady, Non-uniform

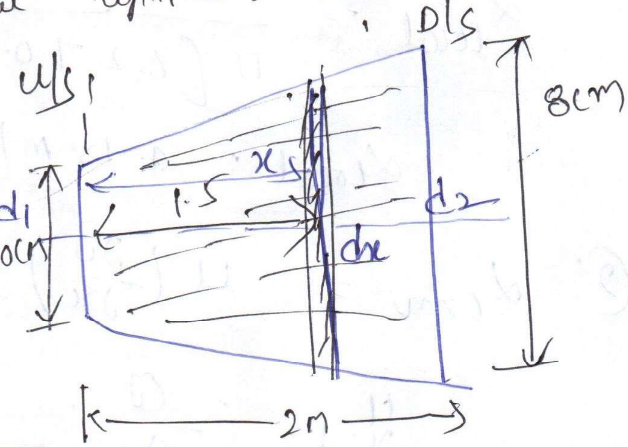
If $\alpha_{con} \neq 0$

$\alpha_T = \alpha_{conv} + \alpha_{local} = 0$

$\neq 0$ Non uniform, $= 0$ steady ✓

6/7/17

A 2m long conical diffuser with 20 cm dia at the upstream end, 80 cm dia in downstream end at instant the flow rate as estimated as 200 ~~…~~ liters/sec. It was found to increase at a rate of 50 litre/sec. Find local convective and total acceleration at a distance of 1.5 metres from the upstream end.



Sol

A 2m long

20cm upstream

80cm down stream

$Q = 200\ L.P.S \times 10^{-3}\ m^3/sec$

$\left(\frac{dQ}{dt}\right) = 50\ L.P.S \times 10^{-3}\ m/sec$

α_{local}, α_{conv} & α_{total}

$$\boxed{\underset{\text{Total}}{\alpha_x} = \underset{\text{Convective}}{u\left(\frac{\partial U}{\partial x}\right)} + \underset{\text{Local}}{\frac{\partial U}{\partial t}}} \Rightarrow \boxed{U_x = \frac{Q}{(A_x)} = \frac{Q}{\frac{\pi}{4}(d_x)^2}}$$

Diameter distance of x

$$d_x = d_1 + \left(\frac{d_2 - d_1}{2}\right)x = \frac{0.2 + (0.8 - 0.2)}{2}x$$

$$= \boxed{d_x = 0.2 + 0.3x}$$

(1) $\alpha_{local} = \frac{\partial U}{\partial t} = \frac{\partial}{\partial t}\left[\frac{Q}{A_x}\right] = \frac{1}{A_x}\left[\frac{\partial Q}{\partial t}\right] = \frac{1}{A_x}\left[\frac{\partial Q}{\partial t}\right]$

$$= \frac{1}{\frac{\pi}{4}(0.2 + 0.3x)^2}\left(\frac{\partial Q}{\partial t}\right)$$

$\alpha_{local} = \frac{1}{\ }$

(1) $\alpha_{local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{A_x}\right) = \frac{1}{A_x}\left[\frac{\partial Q}{\partial t}\right]$

$x = 1.5$

$= \frac{1}{\pi/4}\left[0.2 + 0.3x\right]^2\left[\frac{\partial Q}{\partial t}\right]$

$\alpha_{local} = \frac{4}{\pi\left[0.2 + 0.3 \times 1.5\right)^2} \times 50 \times 10^{-3}$

$\alpha_{local} = 0.15\ m/sec^2$

(2) $\alpha_{conv} = u\left(\frac{\partial u}{\partial x}\right)$

$u_{x=1.5} = \frac{Q}{A_x} = \frac{Q}{\pi/4\left[0.2 + 0.3x\right)^2}$

$u = \frac{200 \times 10^{-3}}{\frac{\pi}{4}\left(0.2 + 0.3(1.5)\right)^2} = 0.6$

$u = 0.6\ m/sec$

(3) $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[\frac{Q}{\pi/4\left[0.2 + 0.3x\right]^2}\right]$

$= \frac{4Q}{\pi}\frac{\partial}{\partial x}\left[\frac{1}{(0.2 + 0.3 \times 1.5)^2}\right]$

$= \frac{4Q}{\pi}\frac{(-2)}{(0.2 + 0.3 \times 1.5)^3} \times 0.3$

$= \frac{4(0.2)(-2) \times 0.3}{\pi \times (0.2 + 0.3 \times 1.5)^3}$

$\frac{\partial u}{\partial x} = -0.55\ m/sec$

$$\alpha_{conv} = u\left(\frac{\partial u}{\partial x}\right) = 0.6 \times (-0.55)$$

$$\alpha_{conv} = -0.33\, m/sec^2$$

$$\alpha_{total} = \alpha_{conv} + \alpha_{local}$$

$$\alpha_{total} = \alpha_{conv} + \alpha_{local}$$

$$= (0.15) + (-0.33)$$

$$\alpha_{total} = -0.18\, m/sec^2$$

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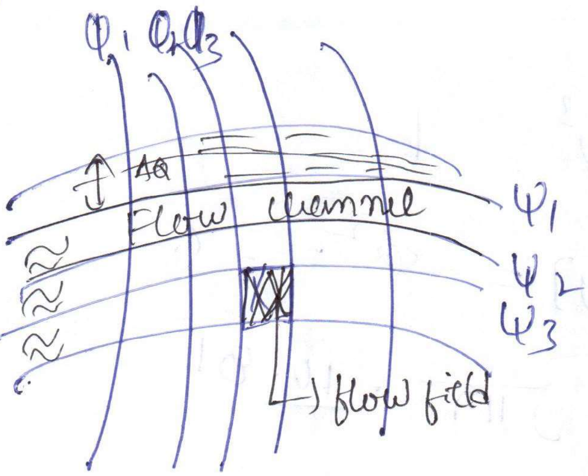
* Flow net Analysis *

→ **Flow net:-** → It is a graphical representation of solution of laplace equation



φ (Potential)
ψ (Stream)

ΔQ = Flow Rate

→ It consist of 'φ' lines & 'ψ' lines there are ⊥ each other

→ The intersected areas are approximated square square

→ Flow rate measurement

* Let ΔQ is the flow rate through which each flow channel remains constant and is equal for all channels