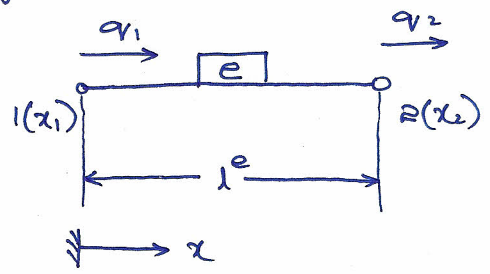
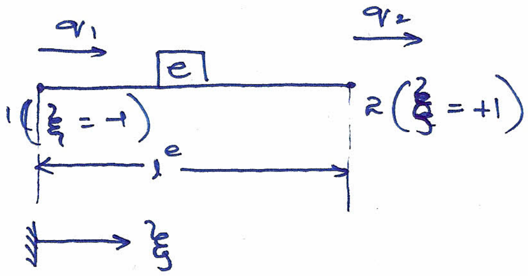
Intrinsic (or) Natural co-ordinate system

Cartesian co-ordinate system (x, y, z) is converting into intrinsic co-ordinate system (ξ, η, ψ) by normalizing or limiting the values of elements





As node (1) $x = x_1, \xi = -1$

As node (2) $x = x_2 = \xi = +1$

$$\xi = mx + c \quad \text{—— (1)}$$

$$-1 = mx_1 + c \quad \text{—— (2)}$$

$$+1 = mx_2 + c$$

$$-2 = m(x_1 - x_2) \Rightarrow m = \frac{2}{x_2 - x_1}$$

m value in eq (2)

$$-1 = \left(\frac{2}{x_2 - x_1}\right) x_1 + c$$

$$c = -1 - \frac{2x_1}{x_2 - x_1}$$

Substitute c & m values into eq (1)

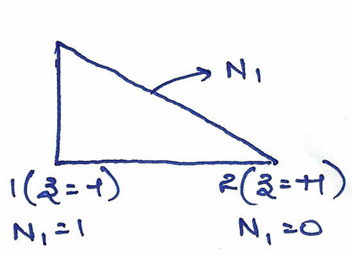
$$\xi = \left[\frac{2}{x_2 - x_1}\right] x - 1 - \frac{2x_1}{x_2 - x_1}$$

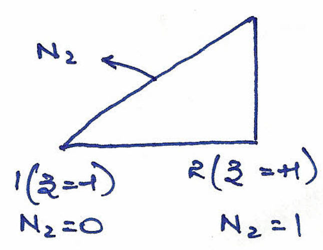
$$\xi = \frac{2(x - x_1)}{x_2 - x_1} - 1$$

{Sub $x = x_1$ & $x = x_2$ check the values of -1 & $+1$}

Interpolation function can be written as $q(\xi)$

$$q(\xi) = N_1 q_1 + N_2 q_2 \quad (N_1 \,\&\, N_2 \text{ in terms of } \xi)$$





$N_1 = C_1(1-\xi)$ $\quad$ $N_2 = C_2(1+\xi)$

at Node (1), $N_1 = 1$, $\xi = -1$ $\quad$ at Node 2, $N_2 = 1$, where $\xi = +1$

$1 = C_1(1-(-1))$ $\quad$ $1 = C_2(1+1)$

$1 = C_1 \times 2 \Rightarrow C_1 = 1/2$ $\quad$ $C_2 = \frac{1}{2}$

$N_1 = \frac{1}{2}(1-\xi)$ $\quad$ $N_2 = \frac{1}{2}(1+\xi)$

$$N_1 + N_2 = \frac{1}{2}(1-\xi) + \frac{1}{2}(1+\xi) = 1$$

$$q(\xi) = \frac{1}{2}(1-\xi)q_1 + \frac{1}{2}(1+\xi)q_2$$

Strain $\epsilon = \dfrac{\partial q}{\partial x} = \dfrac{\partial q}{\partial \xi} \cdot \dfrac{\partial \xi}{\partial x}$ $\quad$ $\xi = \dfrac{2(x-x_1)}{x_2-x_1} - 1$

$$\frac{\partial q}{\partial \xi} = -\frac{1}{2}q_1 + \frac{q_2}{2}, \quad \frac{\partial \xi}{\partial x} = \frac{2}{x_2-x_1} = \frac{2}{l^e}$$

$$\epsilon = \left[-\frac{q_1}{2} + \frac{q_2}{2}\right]\left[\frac{2}{l^e}\right] = \underbrace{\begin{bmatrix} \frac{-1}{l^e} & \frac{1}{l^e} \end{bmatrix}}_{B} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$[K^e] = \int_V B^T D B \, dv = \int_l \begin{bmatrix} -1/l \\ 1/l \end{bmatrix} [E] \begin{bmatrix} -1/l & 1/l \end{bmatrix} A^e \, dx$$

$$K^e = \frac{A^e E^e}{l^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Axial Bar Element - Higher Order Element [HOE]

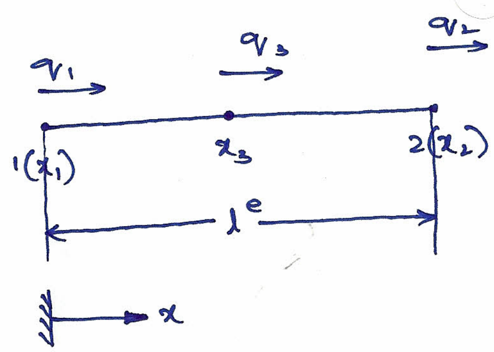
Quadratic I.F

$q(x) = a_0 + a_1 x + a_2 x^2$ ——— (1)

$q_1 = a_0 + a_1 x_1 + a_2 x_1^2$ ——— (2)

$q_2 = a_0 + a_1 x_2 + a_2 x_2^2$ ——— (3)

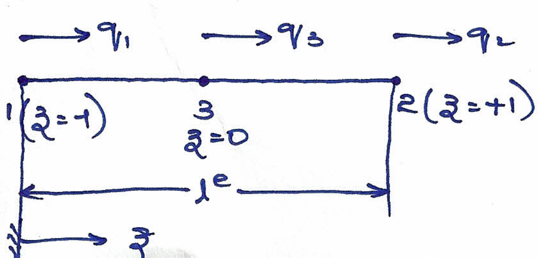
$q_3 = a_0 + a_1 x_3 + a_2 x_3^2$ ——— (4)



Difficult to solve (2), (3) & (4) $a_0, a_1 \,\&\, a_2$

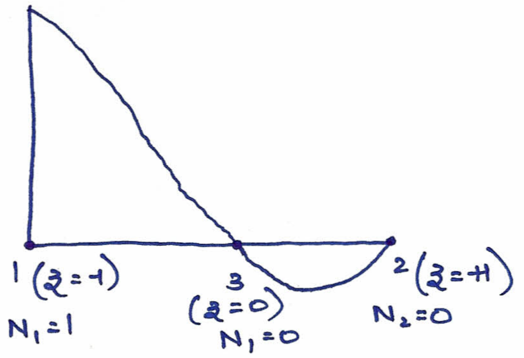
$x \longrightarrow \xi$ (Intrinsic co-ordinate system)

$$\xi = \frac{2(x-x_1)}{x_2-x_1} - 1$$



The shape functions are $N_1, N_2 \,\&\, N_3$

I.F is $q(\xi) = N_1 q_1 + N_2 q_2 + N_3 q_3$



Eq for N_1

$N_1 = C_1 \xi (1-\xi)$

at Node (1) $\xi=-1, N_1=1$

$1 = C_1(-1)(1-(-1))$

$1 = C_1(-2)$

$C_1 = -1/2$

$N_1 = \frac{-\xi}{2}(1-\xi)$

Eq for N_2

$N_2 = C_2 \xi (1+\xi)$

at Node (2) $\xi=+1$

$N_2 = 1$

$1 = C_2(1)(1+1)$

$C_2 = 1/2$

$N_2 = \frac{\xi}{2}(1+\xi)$

Eq for N_3

$N_3 = C_3 (1+\xi)(1-\xi)$

at Node (3) $\xi=0$ $N_3=1$

$1 = C_3 (1+0)(1-0)$

$C_3 = 1$

$N_3 = (1+\xi)(1-\xi)$

$$N_1 + N_2 + N_3 = -\frac{\xi}{2}(1-\xi) + \frac{\xi}{2}(1+\xi) + (1+\xi)(1-\xi) = 1$$

$$\text{Strain } \epsilon = \frac{\partial q}{\partial x} = \frac{\partial q}{\partial \xi}\cdot\frac{\partial \xi}{\partial x} = \left[\frac{\partial N_1}{\partial \xi}q_1 + \frac{\partial N_2}{\partial \xi}q_2 + \frac{\partial N_3}{\partial \xi}q_3\right]\frac{2}{l^e}\left[\frac{\partial \xi}{\partial x}\right]$$

$$\epsilon = \underbrace{\frac{2}{l^e}\begin{bmatrix}\frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \frac{\partial N_3}{\partial \xi}\end{bmatrix}}_{B}\begin{bmatrix}q_1\\ q_2\\ q_3\end{bmatrix}$$

$$B = \frac{2}{l^e}\left[\left(-\frac{1}{2}+\xi\right) \quad \left(1/2+\xi\right) \quad \left(-2\xi\right)\right], \quad [D] = [E]$$

$$[K^e] = \int_V B^T D B \, dV$$

$$[K^e] = \int_l \frac{2}{l^e}\begin{bmatrix}-1/2+\xi\\ 1/2+\xi\\ -2\xi\end{bmatrix}[E^e]\frac{2}{l^e}\left[(-1/2+\xi) \quad (1/2+\xi) \quad (-2\xi)\right] A^e dx$$

$$[K^e] = \int_{\xi=-1}^{\xi=+1}\frac{4A^eE^e}{(l^e)^{\cancel{4}}}\begin{bmatrix}(-1/2+\xi)^2 & (-1/2+\xi)(1/2+\xi) & (-1/2+\xi)(-2\xi)\\ (1/2+\xi)(-1/2+\xi) & (1/2+\xi)^2 & (1/2+\xi)(-2\xi)\\ (-2\xi)(-1/2+\xi) & (+1/2+\xi)(-2\xi) & (-2\xi)^2\end{bmatrix}\frac{\cancel{l^e}}{2}d\xi$$

$$\left[\frac{d\xi}{dx} = \frac{2}{l^e} \qquad dx = d\xi\,\frac{l^e}{2}\right]$$

$$[K^e] = \frac{2A^eE^e}{l^e}\begin{bmatrix} \int_{\xi=-1}^{\xi=+1}\left(-\frac{1}{2}+\xi\right)^2 d\xi & \int_{\xi=-1}^{\xi=+1}\left(-\frac{1}{2}+\xi\right)\left(\frac{1}{2}+\xi\right) & \int_{\xi=-1}^{\xi=+1}\left(-\frac{1}{2}+\xi\right)(-2\xi) \\ - & - & - \\ - & - & - \end{bmatrix}$$

$$[K^e] = \frac{2A^eE^e}{l^e}\begin{bmatrix} 7/6 & 1/6 & -4/3 \\ 1/6 & 7/6 & -4/3 \\ -4/3 & -4/3 & 8/3 \end{bmatrix}$$

Beam Element

Elemental Stiffness Matrix

$$[K^e] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

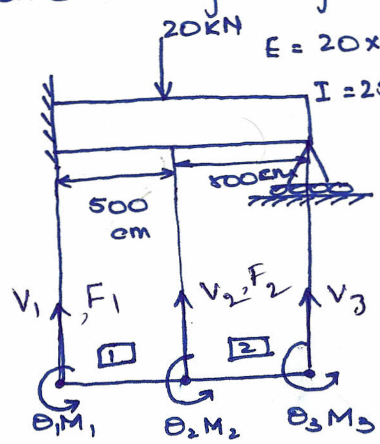
$$\{q\} = \begin{Bmatrix} V_1 \\ \theta_1 \\ V_2 \\ \theta_2 \end{Bmatrix}, \quad P = \begin{Bmatrix} F_1 \\ M_1 \\ F_2 \\ M_2 \\ F_3 \\ M_3 \end{Bmatrix}$$

l = length of the element

→ A beam fixed at one end and supported by a roller at other end has a 20KN concentrated load applied at the center of the beam span as shown in fig. Calculate the deflection under the load and construct the shear force and bending moment diagram of the beam.

$E = 20\times10^6\ N/cm^2$

$I = 2500\ cm^4$



$$[K] = 4\begin{bmatrix} 12 & 30 & -12 & 30 & 0 & 0 \\ 30 & 100 & -30 & 50 & 0 & 0 \\ -12 & -30 & 24 & 0 & -12 & 30 \\ 30 & 50 & 0 & 200 & -12 & 50 \\ 0 & 0 & -12 & -30 & -30 & -30 \\ 0 & 0 & 30 & 50 & 12 & 100 \end{bmatrix}$$

$$\begin{Bmatrix} F_1 \\ M_1 \\ -20\times10^3 \\ 0 \\ F_3 \\ 0 \end{Bmatrix} \begin{matrix} \\ \\ \to F_2 \\ \to M_2 \\ \\ \to M_3 \end{matrix}$$

$V_1 = 0$, $\theta_1 = 0$, $V_3 = 0$

Eliminate 1st, 2nd, 5th rows & columns

$$4\times10^4 \begin{bmatrix} 24 & 0 & 30 \\ 0 & 200 & 50 \\ 30 & 50 & 100 \end{bmatrix} \begin{bmatrix} V_2 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -20\times10^3 \\ 0 \\ 0 \end{bmatrix}$$

$4\,(24V_2 + 30\theta_3) = -20\times10^3$ ——— (1)

$4\,(200\theta_2 + 50\theta_3) = 0$ ——— (2)

$4\,(30V_2 + 50\theta_2 + 100\theta_3) = 0$ ——— (3)

$\theta_2 = -0.003125$ rad

$\theta_3 = 0.0125$ rad

$V_2 = -3.646\times10^{-2}$ m

__Result__ $V_1 = 0$, $V_2 = -3.646\times10^{-2}$, $V_3 = 0$

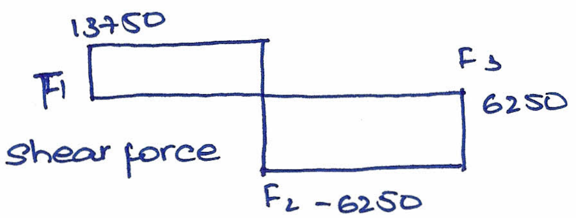
$\theta_1 = 0$, $\theta_2 =$, $\theta_3 =$

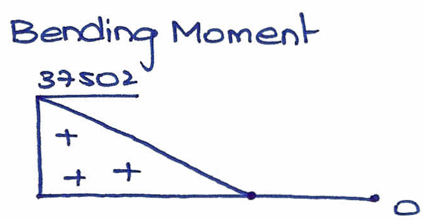
Shear force & Bending moment

$F_1 = 4\,(-12V_2 + 30\,\theta_2) = 13750$ N

$M_1 = 37502$ N-m

$F_3 = 6250$ N



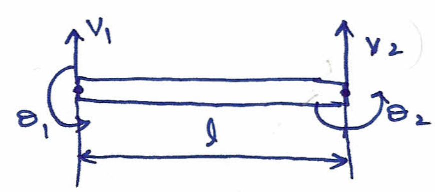


Shape Function Derivation for Beam Element

For Nodal Displacements

V_1 & V_2 are vertical

θ_1 & θ_2 are rotational



Polynomial function for polynomial coeff

$v(x) = a_1 + a_2x + a_3x^2 + a_4x^3 + \cdots\cdots$ (1)

$\frac{dv}{dx} = a_2 + 2a_3x + 3a_4x^2 + \cdots\cdots$ (2)

Applying baundary conditions

$v = v_1$ and $\frac{dv}{dx} = \theta_1$ as $x = 0$

$v = v_2$ and $\frac{dv}{dx} = \theta_2$ as $x = l$

These values in eq (1) & (2) we get

$$v_1 = a_1 \text{ —— } (3) \quad , \quad \theta_1 = a_2 \text{ —— } (4)$$

$$v_2 = a_1 + a_2 l + a_3 l^2 + a_4 l^3 \text{ —— } (5)$$

$$\theta_2 = a_2 + 2a_3 l + 3a_4 l^2 \text{ —— } (6)$$

eq (5) $\longrightarrow$ $v_2 = v_1 + \theta_1 l + a_3 l^2 + a_4 l^3$

or $a_3 l^2 + a_4 l^3 = v_2 - v_1 - \theta_1 l \times (3)$

eq (6) $\quad 2a_3 l + 3a_4 l^2 = \theta_2 - a_2 = \theta_2 - \theta_1 \times (l)$

$$3a_3 l^2 + 3\cancel{a_4} l^3 = 3v_2 - 3v_1 - 3\theta_1 l$$

$$2a_3 l^2 + \cancel{3}a_4 l^3 = \theta_2 l - \theta_1 l$$

$$a_3 l^2 = 3v_2 - 3v_1 - 3\theta_1 l - \theta_2 l + \theta_1 l$$

$$= 3v_2 - 3v_1 - 2\theta_1 l - \theta_2 l$$

$$a_3 = \frac{3}{l^2}(v_2 - v_1) - \frac{1}{l}(\theta_1 + \theta_2)$$

$$a_4 = \frac{2}{l^3}(v_1 - v_2) + \frac{1}{l^2}(\theta_1 + \theta_2)$$

Substitute values of a_1, a_2 & a_3, a_4 in eq (1)

$$v = v_1 + \theta_1 x + \left[\frac{3}{l^2}(v_2 - v_1) - \frac{1}{l^2}2(\theta_1 + \theta_2)\right] x^2 + \left[\frac{2}{l^3}(v_1 - v_2) + \frac{1}{l^2}(\theta_1 + \theta_2)\right] x^3$$

$$v = \underbrace{\left[1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}\right]}_{N_1} v_1 + \underbrace{\left[x - \frac{2x^2}{l} + \frac{x^3}{l^2}\right]}_{N_2} \theta_1 + \underbrace{\left[\frac{3x^2}{l^2} - \frac{2x^3}{l^3}\right]}_{N_3} v_2 + \underbrace{\left[\frac{-x^2}{l} + \frac{x^3}{l^2}\right]}_{N_4} \theta_2$$

That is $\quad N_1 v_1 + N_2 \theta_1 + N_3 v_2 + N_4 \theta_2$

Dynamic Analysis

→ Mechanical vibration eq $\Rightarrow$ $m\ddot{x} + c\dot{x} + Kx = F$ → Load

Stiffness

Acceleration of vibration

velocity

Damping coeff

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + Kx = F$$ (By considering undamped vibrations c term neglected)

In simplest vibration model the damping effects are neglected (& the vibrations subside with time due to damping action)

→ The undamped free vibration model of a structure gives significant information about its dynamic behaviour

$$m\frac{d^2x}{dt^2} + Kx = F$$

$$[M]\ddot{q} + [K]q = \{P\} \rightarrow \text{Load vector}$$

$[K]$ → Stiffness matrix

$[M]$ → Mass matrix

Virtual Energy Principle

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\dot{q}^2 = \frac{1}{2}\dot{q}^T m \dot{q} \quad \text{——(1)} \qquad \dot{q} = \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix}$$

$$K = \frac{1}{2}\dot{u}^T m \dot{u} \quad \text{——(2)} \qquad \dot{u} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$$

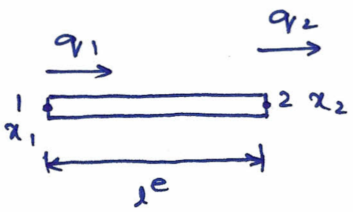
$$m = \int_V \rho\, dV$$

$$u = Nq$$
$$\dot{u} = N\dot{q}$$

$$KE = \frac{1}{2}\int_V \dot{u}^T(\rho\, dV)\dot{u} = \frac{1}{2}\int (N\dot{q})^T(\rho\, dV)(N\dot{q})$$

$$[M^{(e)}] = \int_V \rho N^T N\, dV \qquad KE = \int_V \dot{q}^T N^T \rho N \dot{q}\, dV = \frac{1}{2}\dot{q}^T \underbrace{\left[\int_V \rho N^T N\, dV\right]}_{m} \dot{q}$$

Axial Bar Element



$$u = N_1 q_1 + N_2 q_2$$
$$= [N_1 \quad N_2]\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$N = [N_1 \quad N_2]$$

$$[m^e] = \int \rho N^T N\, dV = \int_{l^e} \rho \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} [N_1 \quad N_2]\, A^e\, dl$$

$$\frac{P!\,q!\,\cancel{r!}}{(p+q+\cancel{r}+2)!} \times 2A$$

$$\frac{2!}{\cancel{6}\,3} \times \cancel{A} = \cancel{2 \times A}$$

$$= \rho A^e \int \begin{bmatrix} N_1^2 & N_1 N_2 \\ N_1 N_2 & N_2^2 \end{bmatrix} dl$$

$$= \rho A \begin{bmatrix} l^e/3 & l^e/6 \\ l^e/6 & l^e/3 \end{bmatrix} = \frac{\rho A^e l^e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$[m][\ddot{q}] + [K][q] = [P]$$

Let q may be the sinusoidal function with amplitude of 'v' $q = u \sin \omega t$

where ω = angular velocity $\omega = 2\pi f \longrightarrow$ Natural frequency

$$\dot{q} = u\omega \cos \omega t$$

$$\ddot{q} = -u\omega^2 \sin \omega t = -\omega^2 q$$

$$(m)(-\omega^2 q) + (K)(q) = (P)$$

$$(K - \omega^2 m)(q) = (P)$$

In order to estimate natural frequency

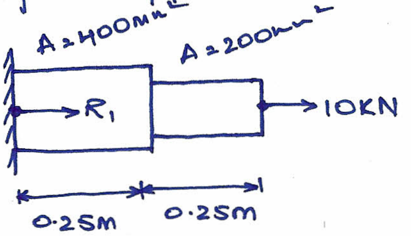
assume $[P] = 0$

$$\left[[K] - \omega^2 [m]\right] = 0$$

Let $\omega^2 = \lambda$

$K - \lambda m = 0$, λ which is vector, eigen vector

Calculate natural frequency, displacement of a stepped bar as shown in fig



$E = 2\times10^5 \, N/mm^2$

$\rho = 7200 \, kg/m^3$

$$[K^e] = \frac{A^e E^e}{l^e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad m^e = \frac{\rho A l}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$[K'] = \frac{400\times2\times10^5}{0.25}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[m'] = \frac{7200\times400\times10^{6}\times0.25}{6} \begin{array}{c} \begin{array}{cc} q_1 & q_2 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} q_1 \\ q_2 \end{array}$$

$$[P'] = \begin{bmatrix} R_1 \\ 0 \end{bmatrix} \begin{array}{c} q_1 \\ q_2 \end{array} \qquad [P^2] = \begin{bmatrix} 0 \\ 10^4 \end{bmatrix} \begin{array}{c} q_2 \\ q_3 \end{array}$$

$$[K^2] = \frac{200\times2\times10^5}{0.25}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad [m^2] = \frac{7200\times200\times10^{-6}\times0.25}{6} \begin{array}{c} \begin{array}{cc} q_2 & q_3 \end{array} \\ \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} q_2 \\ q_3 \end{array}$$

$$[K^e] = 1.6\times10^8 \begin{array}{c} \begin{array}{ccc} q_1 & q_2 & q_3 \end{array} \\ \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} q_1 \\ q_2 \\ q_3 \end{array}, \quad m = 0.06 \begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad P = \begin{bmatrix} R_1 \\ 0 \\ 10^4 \end{bmatrix}$$

$$[m][\ddot{q}] + Kq = [P]$$

$$a^2 = \lambda$$

$$[-a^2(m)+(K)](q) = [P]$$

$$1.6\times10^8\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \lambda\, 0.06\begin{bmatrix} 4 & 2 & 0 \\ 2 & 6 & 1 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} R_1 \\ 0 \\ 10^4 \end{bmatrix}$$

$$1.6\times10^8\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0.36\lambda & 0.06\lambda \\ 0.06\lambda & 0.12\lambda \end{bmatrix}\begin{bmatrix} q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10^4 \end{bmatrix}$$

To calculate natural frequency $(P) = 0$

Solve the eq^n for Eigen value problem

$$\begin{bmatrix} 4.8\times10^8 - 0.36\lambda & -1.6\times10^8 - 0.06\lambda \\ -1.6\times10^8 - 0.06\lambda & 1.6\times10^8 - 0.12\lambda \end{bmatrix} = 0$$

$$0.468\lambda^2 + 1.344\times10^8\lambda - 5.12\times10^{16} = 0$$

As $\lambda_1 = 0, \because q_1 = 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a^2 = \lambda$

$a = \sqrt{\lambda}$

$2\pi f = \sqrt{\lambda}$

$f = \frac{\sqrt{\lambda}}{2\pi}$

$\lambda_2 = 13.45\times10^5 \quad \lambda_3 = 54.417\times10^8$

$\lambda_1, \lambda_2, \lambda_3$ are also called eigen values

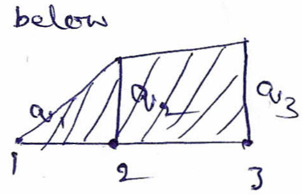
$$1.6\times10^8\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - 13.45\times10^8\begin{bmatrix} 0.36 & 0.06 \\ 0.06 & 0.12 \end{bmatrix}\begin{bmatrix} q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10^4 \end{bmatrix}$$

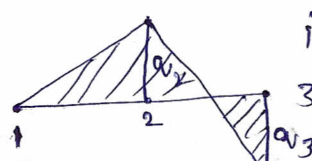
q_2, q_3

After getting the values of q_2 and q_3 draw Modes Shapes as shown below

$$1.6\times10^8\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - 54.419\times10^8\begin{bmatrix} 0.36 & 0.06 \\ 0.06 & 0.12 \end{bmatrix}\begin{bmatrix} q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10^4 \end{bmatrix}$$

q_2, q_3



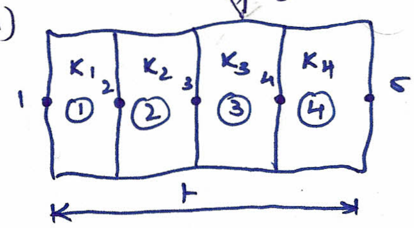


Steady State Heat Transfer Analysis

1-D (one Dimension)

conductivity matrix

$$[K_T] = \int_V [B_T]^T [D_T][B_T]\, dV$$



$T = a_0 + a_1 x$

Field variable temperature

at $x = x_1, \ T = T_1$

$x = x_2, \ T = T_2$

(10)

$$T = \left[\frac{x_2 - x}{x_2 - x_1}\right] T_1 + \left[\frac{x - x_1}{x_2 - x_1}\right] T_2$$

$$T = N_1 T_1 + N_2 T_2$$

$$\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left[N_1 T_1 + N_2 T_2\right]$$

$$= \underbrace{\begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} \end{bmatrix}}_{B} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

$$[B_T] = \left[-1/l^e \quad 1/l^e\right]$$

$$[K_T] = \int_l \begin{bmatrix} -1/l^e \\ 1/l^e \end{bmatrix} [K^e] \left[-1/l^e \quad 1/l^e\right] A^e \, dl^e$$

$$[K_T] = \frac{A^e K^e}{l^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Loads

(1) Internal heat generation $= \dddot{Q}$

(2) Heat flux $= \ddot{q}$

(3) Convection

Load Vector

$\dddot{Q}$ (Internal heat generation (or) Body force)

$$[P^e]_{\dddot{Q}} = \begin{bmatrix} \frac{\dddot{Q} A^e l^e}{2} \\ \frac{\dddot{Q} A^e l^e}{2} \end{bmatrix}$$

W.P

$$\dddot{Q} = \int_V T \dddot{Q} \, dV$$

$$= \int_V (N_1 T_1 + N_2 T_2) \dddot{Q} \, dV$$

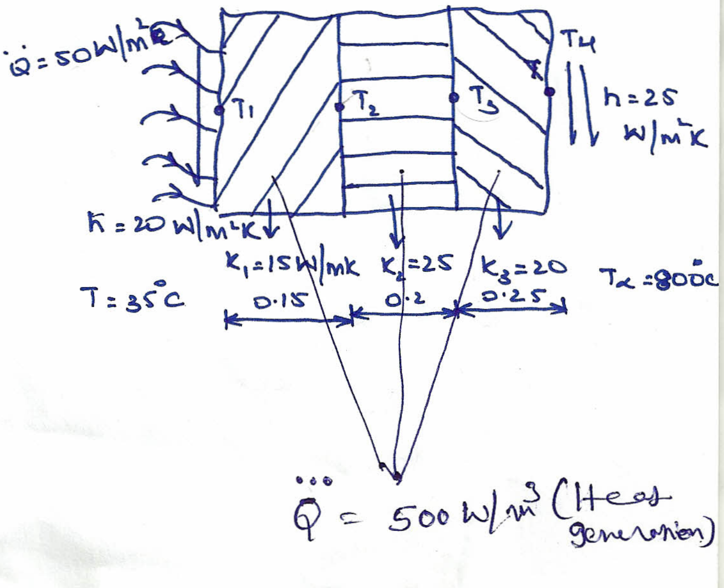
$$= \int_{l^e} (N_1 T_1 + N_2 T_2) \dddot{Q} A^e \, dl^e$$

Surface heat flux

$$[P^e]_{\ddot{Q}} = \begin{bmatrix} 0 \\ \ddot{Q} A^e \end{bmatrix}$$

Convection $= hA\,(T_1 - T_\infty)$

→ Calculate the temp at the junction points of a composite wall shown in fig



$$[K_T^e] = \frac{A^e K^e}{l^e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Stiffness matrix due to conduction

$$[K_T^e]_h = \begin{bmatrix} hA & 0 \\ 0 & 0 \end{bmatrix}$$

Convection occurs at Node 1

So hA is added to 1-1 location

$$[K_T^1] = \frac{15A}{0.15}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \begin{bmatrix}20A & 0\\ 0 & 0\end{bmatrix} = 100A \begin{array}{c c} \begin{array}{cc} T_1 & T_2 \end{array} & \\ \begin{bmatrix}1.2 & -1\\ -1 & 1\end{bmatrix} & \begin{array}{c} T_1\\ T_2\end{array}\end{array} \qquad \dddot{Q} = 500\ W/m^3$$

$$[K_T^2] = \frac{25A}{0.2}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} = 100A \begin{array}{c c} \begin{array}{cc} T_2 & T_3 \end{array} & \\ \begin{bmatrix}1.25 & -1.25\\ -1.25 & 1.25\end{bmatrix} & \begin{array}{c} T_2\\ T_3\end{array}\end{array}$$

$$[K_T^3] = \frac{20A}{0.25}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 25A\end{bmatrix} = 100A \begin{array}{c c} \begin{array}{cc} T_3 & T_4 \end{array} & \\ \begin{bmatrix}0.8 & -0.8\\ -0.8 & 1.05\end{bmatrix} & \begin{array}{c} T_3\\ T_4\end{array}\end{array}$$

Convection at node (4) so hA is added to 4-4 location

$$[K_T] = 100A \begin{array}{c c} \begin{array}{cccc} T_1 & T_2 & T_3 & T_4 \end{array} & \\ \begin{bmatrix}1.2 & -1 & 0 & 0\\ -1 & 2.25 & -1.25 & 0\\ 0 & -1.25 & 2.05 & -0.8\\ 0 & 0 & -0.8 & 1.05\end{bmatrix} & \begin{array}{c} T_1\\ T_2\\ T_3\\ T_4\end{array}\end{array}$$

Write global $[P_T]$ also

$[K_T][T] = [P_T]$ ← equilibrium

Solve for T_1, T_2, T_3 & T_4

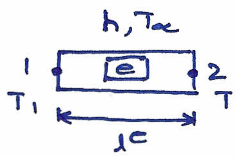
$$[K_T^1] = \frac{25A}{0.4}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{25\times0.4\times0.7}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

$$[K_T^2] = \frac{25\times0.025}{0.4}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{25\times0.4\times0.7}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$$

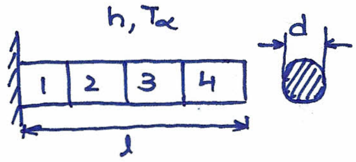
$$[K_T^3] = \frac{25\times0.025}{0.4}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix} + \frac{25\times0.4\times0.7}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 25\times0.7\times0.4\end{bmatrix}$$

$$P_T^1 = \begin{bmatrix}\dfrac{25\times0.7\times0.4\times50}{2}\\[2ex] \dfrac{25\times0.7\times0.4\times50}{2}\end{bmatrix} \quad P_T^2 = \begin{bmatrix}\dfrac{25\times0.7\times0.4\times50}{2}\\[2ex] \dfrac{25\times0.7\times0.4\times50}{2}\end{bmatrix} \quad P_T^3 = \begin{bmatrix}\dfrac{25\times0.7\times0.4\times50}{2}\\[2ex] \dfrac{25\times0.7\times0.4\times50}{2}\end{bmatrix} + \begin{bmatrix}0\\ 25\times0.025\\ \times50\end{bmatrix}$$

Finite Element ~~AND~~ (1-D)



Temp profile



conductivity matrix

$$[K_T^e] = \frac{A^e k^e}{l^e}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$[K_T^e]_h = \frac{hPl^e}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}, \quad P = \text{Wetted perimeter}, \quad A = c/s\ \text{Area}$$

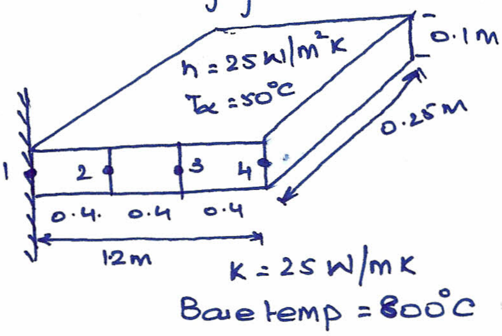
$$[P_T]_{\dddot{q}} = \begin{bmatrix}\dfrac{\dddot{q}Al}{2}\\[2ex] \dfrac{\dddot{q}Al}{2}\end{bmatrix} \qquad [P_T^e]_{h} = \begin{Bmatrix}\dfrac{hPlT_\infty}{2}\\[2ex] \dfrac{hPlT_\infty}{2}\end{Bmatrix}$$

If the tip is convective

For tip element

$$[K_T^4]_h = \begin{bmatrix}0 & 0\\ 0 & hPl\end{bmatrix} \qquad [P_T^4]_h = \begin{Bmatrix}0\\ hAT_\infty\end{Bmatrix}$$

Calculate the temp profile in a rectangular fin as shown in fig. Consider tip is also convective.



$P = (0.25 + 0.1)2 = 0.7 \text{ m}$

$A = 0.25 \times 0.1 = 0.025 \text{ m}^2$

$$[P_T^1] = [P_T^1]_h + [P_T^1]_{\dot{Q}} + [P_T^1]_{\ddot{Q}}$$

$$= \underbrace{\begin{Bmatrix} 20A \times 35 \\ 0 \end{Bmatrix}}_{hAT_\infty} + \begin{Bmatrix} \ddot{Q}A \\ 0 \end{Bmatrix} + \begin{bmatrix} \frac{\dot{q}Al}{2} \\ \frac{\dot{q}Al}{2} \end{bmatrix}$$

$$= \begin{Bmatrix} 700A \\ 0 \end{Bmatrix} + \begin{Bmatrix} 50A \\ 0 \end{Bmatrix} + \begin{bmatrix} \frac{500A \times 0.15}{2} \\ \frac{500A \times 0.15}{2} \end{bmatrix} = \begin{array}{c} \begin{Bmatrix} 787.5A \\ 37.5A \end{Bmatrix} \end{array} \begin{array}{l} T_1 \\ T_2 \end{array}$$

$$[P_T^2] = \begin{bmatrix} \frac{500A \times 0.2}{2} \\ \frac{500A \times 0.2}{2} \end{bmatrix} = \begin{bmatrix} 50A \\ 50A \end{bmatrix} \begin{array}{l} T_2 \\ T_3 \end{array}$$

$$[P_T^3] = \begin{bmatrix} \frac{500A \times 0.25}{2} \\ \frac{500A \times 0.25}{2} \end{bmatrix} + \begin{bmatrix} 0 \\ 25A \times 800 \end{bmatrix} = \begin{bmatrix} 62.5A \\ 20062.5A \end{bmatrix} \begin{array}{l} T_3 \\ T_4 \end{array}$$

$$[P_T] = \begin{bmatrix} 787.5A \\ 87.5A \\ 112.5A \\ 20062.5A \end{bmatrix} \begin{array}{l} T_1 \\ T_2 \\ T_3 \\ T_4 \end{array}$$

$$[K_T][T] = [P_T]$$

$$100A \begin{bmatrix} 1.2 & -1 & 0 & 0 \\ -1 & 2.25 & -1.25 & 0 \\ 0 & -1.25 & 2.05 & -0.8 \\ 0 & 0 & -0.8 & 1.05 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} 787.5A \\ 87.5A \\ 112.5A \\ 20062.5A \end{bmatrix}$$

write simultaneous eq and solve for T_1, T_2, T_3 & T_4

$100A(1.2T_1 - T_2 + 0 \times T_3 + 0T_4) = 787.5A \rightarrow (1)$

||ly write $\rightarrow (2)$

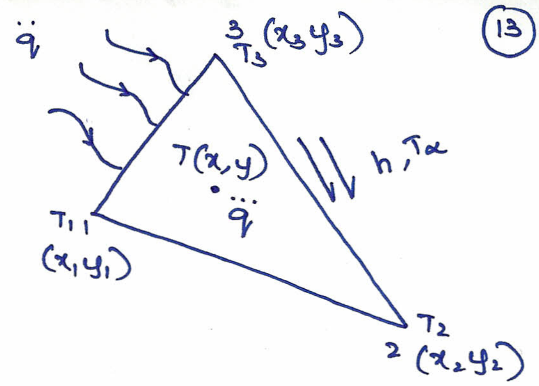
$\rightarrow (3)$

and

$\rightarrow (4)$

2-D Heat Transfer Analysis

$$[K_T^e] = \int_V B^T D B \, dV$$



$$B_T = \frac{1}{\det J} \begin{bmatrix} y_{23} & y_{31} & y_{12} \\ x_{32} & x_{13} & x_{21} \end{bmatrix}$$

$$[D_T] = \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix}$$

$$[K_T^e]_h = \frac{h t\, l_{2-3}}{6} \begin{array}{c} \begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \end{array} \begin{array}{c} \\ T_1 \\ T_2 \\ T_3 \end{array}$$

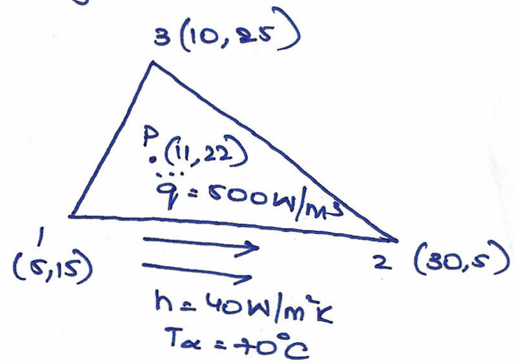
Load vector

$$\{P_T^e\}_{\dddot{q}} = \frac{\dddot{q} A^e t^e}{3} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{P_T^e\}_h = \frac{h t^e l_{2-3} T_\infty}{2} \begin{Bmatrix} 0 \\ 1 \\ 1 \end{Bmatrix}$$

$$\{P_T^e\}_{\dot{q}} = \frac{\ddot{q}\, t\, l_{1-3}}{2} \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix}$$

Calculate the shape functions, conductivity matrix, thermal load vector and temp values at the nodal points of a triangular element as shown in fig. $t = 10\,mm$, $K_x = K_y = 50\,W/mK$



$N_1 = 0.133$, $N_2 = 0.083$, $N_3 = 0.783$

$$\det J \quad J = \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix} = \begin{bmatrix} -5 & -10 \\ 20 & -20 \end{bmatrix}$$

$$\det J = 100 + 200 = 300$$

$$[B_T] = \frac{1}{\det J} \begin{bmatrix} y_{23} & y_{31} & y_{12} \\ x_{32} & x_{13} & x_{21} \end{bmatrix} = \frac{1}{300} \begin{bmatrix} -20 & 10 & 10 \\ -20 & -5 & 25 \end{bmatrix}$$

$$[D_T] = \begin{bmatrix} K_x & 0 \\ 0 & K_y \end{bmatrix} = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \qquad A = \frac{1}{2} \det J$$

$$[K_T^e] = B_T^T D_T B_T t^e A^e$$

$$[K_T^e] = \frac{1}{300} \begin{bmatrix} -20 & -20 \\ 10 & -5 \\ 10 & 25 \end{bmatrix} \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \frac{1}{300} \begin{bmatrix} -20 & 10 & 10 \\ -20 & -5 & 25 \end{bmatrix} \times 10 \times 150$$

$$\{P_T\} = \frac{\dddot{q} A t}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{500 \times 150 \times 10}{3 \times 10^6 \times 10^3}\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad [A \,\&\, t \text{ are in mm}]$$

$$[K_T]_h = \frac{h t\, l_{1-2}}{6}\begin{bmatrix}2 & 1 & 0\\1 & 2 & 0\\0 & 0 & 0\end{bmatrix} = \frac{40 \times 10 \times 10^{-3} \times 26.92 \times 10^{-3}}{6}\begin{bmatrix}2 & 1 & 0\\1 & 2 & 0\\0 & 0 & 0\end{bmatrix}$$

$$l_{1-2} = \sqrt{625+100} = 26.92$$

$$= \sqrt{(x_2-x_1)^2 + (y_1-y_2)^2}$$

$$[K_T] = [K_T^e] + [K_T]_h$$

$$[P_T]_h = \frac{40 \times 10 \times 10^{-3} \times 26.92 \times 10^{-3} \times 70}{2}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$[P_T]_{\dddot{q}} = \frac{\ddot{q}\, t\, l_{2-3}}{2}\begin{bmatrix}0\\1\\1\end{bmatrix} = \frac{400 \times 10 \times 10^{-3} \times 28.28 \times 10^{-3}}{2}\begin{bmatrix}0\\1\\1\end{bmatrix}$$

$$[P_T] = [P_T]_{\dddot{q}} + [P_T]_h + [P_T]_{\ddot{q}}$$