UNIT-VI

VIBRATIONS

When a elastic bodies such as spring, wheel. and shafts are displaced from the equilibrium forces by applying of external forces and then released they exert a vibratary motion this is due to the reason that when a body is displaced the internal forces in the from strain energy are present in the body. when the body reaches equilibrium position the hole of strain energy converted into kinetic energy due to which the body continues to move in opposite direction

The terms used in vibrations

1. Period of vibration (or) time period:-

   It is the time interval after which the motion repeated itself it is expressed in sec

2. Cycle: it is the motion completed during one cycle

3. frequency:-

   It is the number of cycles described in one second. in S.I units the frequency express in hertz (H).

Types of vibratory motions:-

1. Free (or) natural vibrations:-

When no external forces acts on body or the giving it an initial displacement, then the body is said to be under natural vibrations. The frequency of natural vibration is called as natural frequency.

2. Forced vibration system:-

When the body vibrates under the influence of external force then the body is said to be forced vibration system. the external force applied to the body the periodic disturbing force created by unbalanced. the vibrations have the same frequency of same applied force.

3. Damped vibration system:-

When there is a reduction in amplitude over a every cycle of vibration the motion is said to be damped vibration. this is due to the fact that a certain amount of energy process by vibrating system it alway dissipating in over coming frictional resistance to motion

1. Types of free (or) natural vibration:-

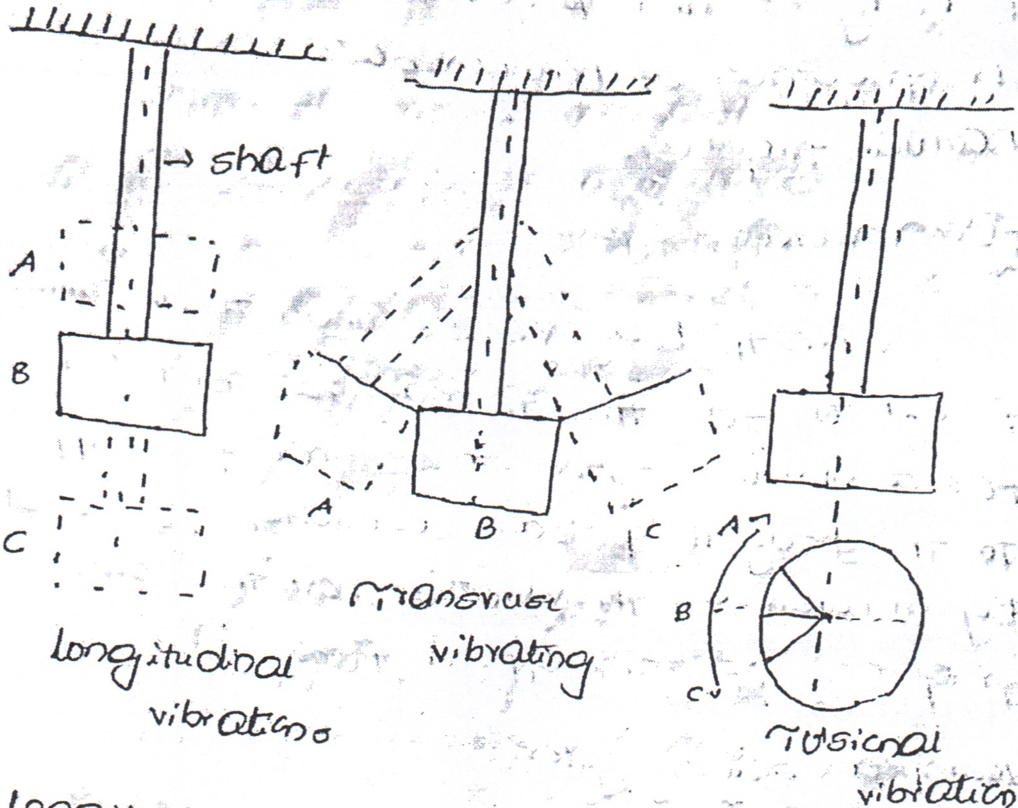
There are 3 type of free vibration

i. longitudinal

ii. transverse

iii. torsional

Consider a weight less constraint spring (or) shaft whose one end is fixed and other end carrying disc. the system may execute of the following vibrations-



1. longitudinal vibrations:-

When the particles of the shaft (or) disc moves parallel to the axis of shaft shown in fig (a). then the vibrations are called longitudinal vibrations.

2. Transverse vibrations:-

When the particles of shaft (or) disc moves approximately perpendicular to axis of shaft then the vibration are called transverse vibrations shown in fig (B)

iii. Torsional vibrations :-

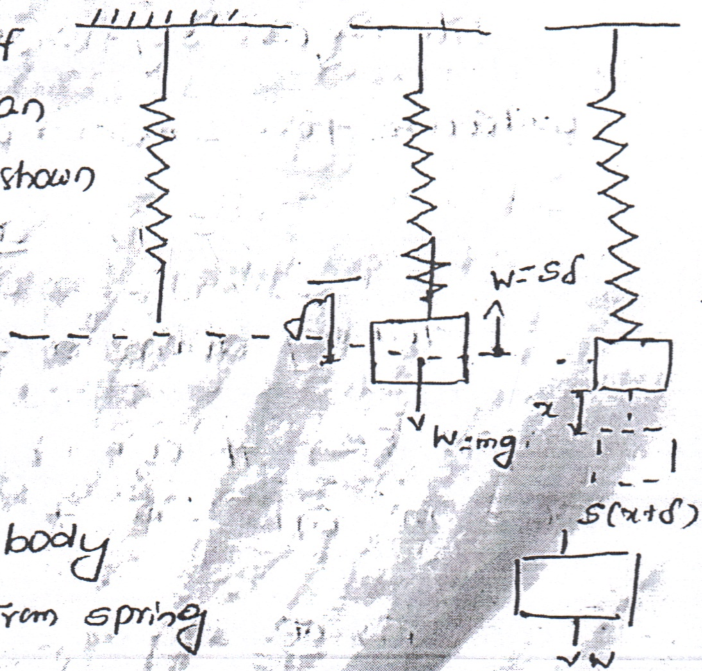
When the particles of shaft (or) disc moves in a circle about axis of shaft shown in fig C. then the vibrations are known as torsional vibration, here torsional shear stress develop in shaft

Natural frequency of free (or) natural longitudinal vibrations?

The natural frequency of the free longitudinal vibration may be determined by

1. equilibrium method

1. equilibrium method:



Consider a spring of negligible mass in an unstrained position shown in fig A.

let

s = stiffness of spring in N/m

m = mass of the body suspended from spring in (kg)

W = weight of the body in (N)

δ = static deflection of spring in (m) due to weight (W).

x = displacement given to the body when external force in (m)

In the equilibrium position as shown in fig b the gravitational pull $W = m \times g$ is balanced by a force spring $W = s\delta$

since the mass is now displaced from it's equilibrium position by a distance 'x' shown in fig. c. and then released the force after time (t)

restoring force $= W - s(x+\delta)$

$= W - sx - s\delta$

$= -sx \longrightarrow (1)$ $\quad W = s\delta$

The negative sign indicates taking upward

acceleration force = mass x acceleration

$= m \times \dfrac{d^2x}{dt^2} \longrightarrow (2)$

taking downward force positive

eqn(1) & (2) the eqn of motion of the body mass 'm' after time 't'

$$m \times \frac{d^2x}{dt^2} = -sx$$

$$m \frac{d^2x}{dt^2} + sx = 0$$

$$\frac{d^2x}{dt^2} + \frac{s}{m}x = 0 \longrightarrow (3)$$

W.K.T the fundamental eqn of SHM

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \longrightarrow (4)$$

Comparing eqn (3) & (4)

$$\omega^2 x = \frac{s}{m} x$$

$$\omega = \sqrt{s/m}$$

$\therefore$ time period $t_p = \frac{2\pi}{\omega}$

$$= 2\pi \sqrt{m/s}$$

natural frequency $f_n = \frac{1}{2\pi}\sqrt{s/m}$

$$F_n = \frac{1}{2\pi}\sqrt{g/\delta}$$

since $W = s\delta$

$$mg = s\delta$$

$$m = \frac{s\delta}{g}$$

sub 'g' value as 9.81 then

$$F_n = \frac{1}{2\pi}\sqrt{9.81/\delta}$$

$$= \frac{0.4984}{\sqrt{\delta}}$$

The value of static deflection 'δ' may be found out from given for longitudinal vibrations it may be obtained by relation $\frac{\text{stress}}{\text{strain}} = E$

$$\frac{W}{A} \times \frac{l}{\delta} = E$$

$$\delta = \frac{Wl}{AE}$$

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Energy method:

W.k.T the kinetic energy due to the motion of the body and potential energy with a respect to datum position which is equal to amount of work required from datum position

According to laws of Conservation of energy the total energy is Constant; therefore

$\Rightarrow K.E + PE = \text{Constant}$

$K.E = \frac{1}{2}mv^2$

$= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2$

P.E having two type

1. Gravitational potential energy
2. strain energy

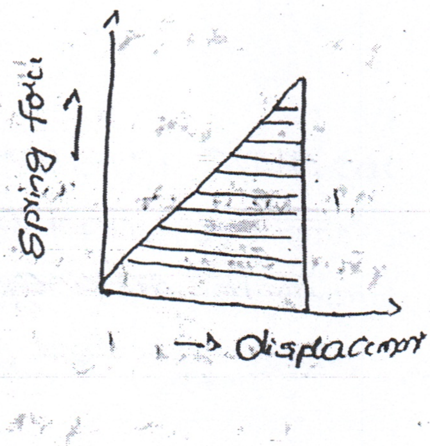
$\therefore$ P.E = gravitational potential energy + strain energy

W.K.T PE = mean force × displacement

$= \frac{1}{2}F \times x$

Spring force $F = sx$

$= \frac{1}{2} \times s \times x^2$



$K.E + P.E = C$

$\Rightarrow \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2} \times sx^2 = 0 \rightarrow (1)$

W.K.T SHM

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \longrightarrow (2)$$

$$\therefore \frac{d}{dt}\left[K.E + P.E\right] = 0$$

$$\frac{d}{dt}\left[\frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + \frac{1}{2} x^2 \times s\right]$$

$$\frac{1}{2} m\, 2\left(\frac{dx}{dt}\right) \frac{d^2x}{dt^2} + \frac{1}{2} s\, 2x \frac{dx}{dt}$$

$$\frac{dx}{dt}\left[m \frac{d^2x}{dt^2} + sx\right] = 0.$$

$$\frac{d^2x}{dt^2} + \frac{s}{m} x = 0 \longrightarrow (3)$$

Comparing eqn (2) & (3)

$$\omega^2 \cancel{x} = \frac{s}{m} \cancel{x}$$

$$\omega = \sqrt{s/m}$$

$\therefore$ natural frequency $= \omega/2\pi$

$$= \frac{1}{2\pi}\sqrt{s/m}$$

time period $(t_p) = 1/F_n$

$$= 2\pi \sqrt{m/s}$$

A shaft of 100mm dia and 1m long is fixed at one end and other end carries fly wheel of mass 1 tonne taking $E = 200G$ find natural frequency of longitudinal vibrations

Given $d = 100mm$

$= 0.1m$

$m = 1$ tonne

$= 1000\ kg$

$l = 1m$

$E = 200 GN/m^2$

$= 200 \times 10^9 N/m^2$

∴ static deflection $\delta = \frac{W \times L}{EA}$

$$= \frac{(1000 \times 9.81)(1)}{(200 \times 10^9)(\frac{\pi}{4})(0.1)^2}$$

$$= 6.24 \times 10^{-6}$$

natural frequency of longitudinal

$$F_n = \frac{0.4984}{\sqrt{6.24 \times 10^{-6}}}$$

$$= 199.61\ Hz$$

Rayleigh's method :-

In this method the maximum kinetic energy at mean position is equals to the maximum potential energy at extreme position assuming that the motion excited by vibration is in SHM than

$x = X \sin\omega t \longrightarrow (1)$

$x =$ Displacement of the body from mean position after time 't' sec

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x = maximum displacement from mean position to extreme position

Now differentiating eqn (1)

$$\frac{dx}{dt} = \frac{d}{dt}(x \sin\omega t)$$

$$= x \cos\omega t (\omega)$$

since at mean position t

$\therefore$ maximum velocity at $v = \frac{dx}{dt} = \omega x$

$\therefore$ maximum kinetic energy at mean position

$$K.E_{max} = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m (\omega x)^2 \longrightarrow (2)$$

maximum potential energy at extreme

$$P.E = \frac{1}{2} S x^2 \longrightarrow (2)$$

equating eqn (2) & (3)

$$\frac{1}{2} m \omega^2 x^2 = \frac{1}{2} S x^2$$

$$\omega = \sqrt{S/m}$$

$$T_p = \frac{2\pi}{\omega}$$

$$= 2\pi \sqrt{m/S}$$

natural frequency $F_n = \frac{1}{T_p}$

$$= \frac{1}{2\pi} \sqrt{S/m}$$

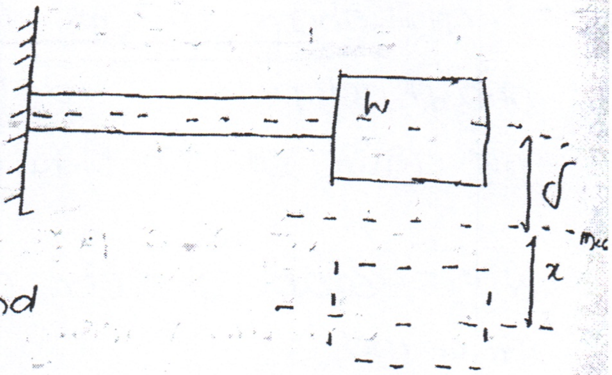
sub $mg = S\delta$

$$F_n = \frac{1}{2\pi} \sqrt{g/\delta}$$

$$f = \frac{0.4985}{\sqrt{\delta}}$$

natural frequency of transverse vibrations:-

Consider a shaft of negligible mass whose one end is fixed and other end carries of weight 'W' in other end



let

s = stiffness of shaft

δ = static deflection due to weight of

x = Displacement of a body from mean position after 't' sec

m = mass of the body

W.K.T

restoring force $= -sx$ ⟶(1)

accelerating force = m x acceleration

$$= m \times \frac{d^2x}{dt^2} \longrightarrow (2)$$

equating eqn (1) & (2)

$$m \times \frac{d^2x}{dt^2} = -sx$$

$$\frac{d^2x}{dt^2} + \frac{s}{m}x = 0$$

hence it is same as before

time period t_p for transverse vibration is same as longitudinal vibration

$T_p = 2\pi\sqrt{m/s}$

$F_n = \frac{1}{2\pi}\sqrt{s/m}$

Note :-
The shape of curve into which the vibrating shaft deflect is identical with the static deflection of curve at Cantilever beam at the end of load.

W.K.T static deflection of Cantilever beam at fixed end

$$= \frac{Wl^3}{3EI}$$

A Cantilever shaft of 50 mm dia and 300 mm long has a disc of mass 100 kg at it's free end. the young's modulus is 200 GN/m² determine the frequency of longitudinal & transverse vibration of shaft

Given $d = 50mm = 0.05m$

$l = 300mm = 0.3m$

$m = 100\,kg$

$E = 200 \times 10^9\ N/m^2$

$A = \frac{\pi}{4} d^2$

$= \pi/4\,(0.05)^2$

$= 0.0019\, m^2$

moment of inertia $I = \frac{\pi d^4}{64} = \frac{\pi (0.05)^4}{64}$

$= 3.6 \times 10^{-7}\ kg\text{-}m^2$

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Frequency of longitudinal vibration

$$\delta = \frac{Wl}{AE}$$

$$= \frac{(100 \times 9.81)(0.3)}{0.0019 \times 200 \times 10^9}$$

$$= 7.74 \times 10^{-7}$$

$$F_n = \frac{0.4984}{\sqrt{\delta}}$$

$$= \frac{0.4985}{\sqrt{7.74 \times 10^{-7}}}$$

$$= 566 \text{ Hz}$$

Transverse:

$$\delta = \frac{Wl^3}{3EI}$$

$$= \frac{(100 \times 9.81)(0.3)^3}{3(200 \times 10^9)(3.6 \times 10^{-7})}$$

$$= 1.22 \times 10^{-4}$$

$$F_n = \frac{0.4985}{\sqrt{\delta}}$$

$$= \frac{0.4985}{\sqrt{1.22 \times 10^{-4}}}$$

$$= 45 \text{ Hz}$$