UNIT-6

UNIT-5

⇒ **Kinematics** :- Rectilinear & Curvelinear motion - velocity & Acceleration - motion of rigid Body - Types & their Analysis in planar motion.

⇒ **Kinetics** :- Analysis as a particle & Analysis as a rigid body in translation - central force motion - Equations of plane motion - Fixed axis rotation - Rolling bodies.

Dynamics :- Dynamics is the branch of mechanics which deals with the study of bodies in motion. Thus we can divide dynamics of the bodies into dynamics of particle & dynamics of rigid bodies

⇒ The dynamics of particle or rigid bodies can further be divided into two parts, namely kinematics & kinetics

⇒ **Kinematics** :- without considering the force causing them.

⇒ **Kinetics** : with considering the force causing them

Types of Motion :-

The motion of a body can be classified into three types they are

(1) Rectilinear (or) Translatory motion

(2) Rotatory motion

(3) plane motion

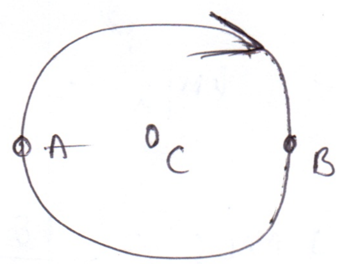
(1) Rectilinear (or) Translatory motion :- If the path traced by the body is a straight line, then it is called a rectilinear motion

The motion of the body from A to B in the fig is straight line and it is a rectilinear motion.

Eg :- A train moving on a straight track, An apple falling from a tree, A man walking on a flat road etc.,

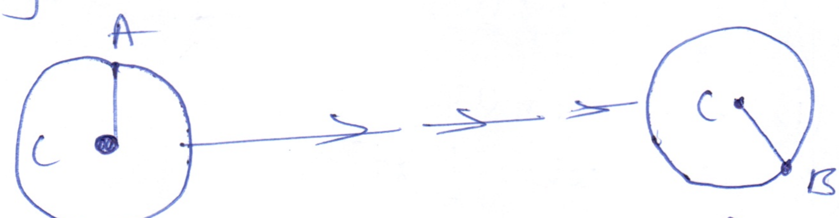
Rotatory motion :- If the path traced by the body is a circle, w.r.to an imaginary axis, then it is called a rotatory motion

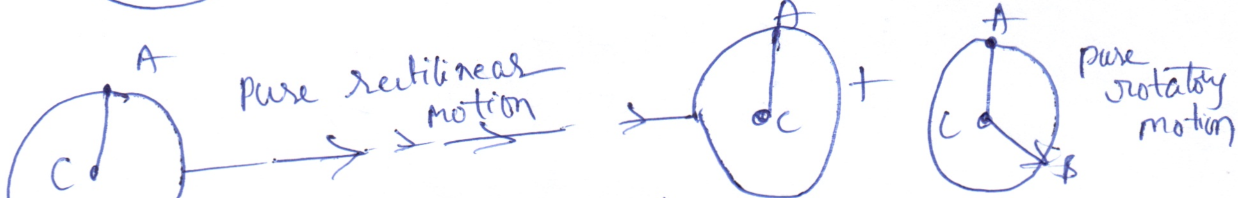
The motion of the body from A to B in fig is along a circular path w.r.to an imaginary axis passing through the centre of the circle "C" So the motion is known as rotatory motion



Exp : The motion of giant wheel, The blades of a ceiling fan, In a clock the hour, minute and second hands rotate about a fixed point etc..

plane motion :- A combination of rectilinear & rotatory motion is known as plane motion





Hence this motion is known as plane motion

Eg: The wheel of a moving car,
The motion of a spinning cricket ball

⇒ Rectilinear Motion :-

(i) Displacement (S) :- The shortest distance b/w two points is known as displacement

unit: m

(ii) velocity (v) :- The rate of change of displacement is known as velocity

$$v = \frac{ds}{dt} \quad \text{unit : } m/s$$

$$\Rightarrow 1\ km/h = \frac{1000\ m}{3600\ s} = \frac{5}{18}\ m/s$$

$$1\ m/s = \frac{18}{5}\ km/h$$

(iii) Acceleration (a):

The rate of change of velocity is known as acceleration

$$a = \frac{dv}{dt} = \frac{d^2 s}{dt^2} \quad \text{unit: } m/s^2$$

Motion under uniform acceleration :-

(a) Eqⁿs of motion along a straight line

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$

(iv) $S_n = u + a\left(n - \frac{1}{2}\right)$

where u = initial velocity
v = final velocity
t = time
s = displacement in 't' seconds
S_n = Displacement in the n^{th} second

Note : If a body starts from rest $u=0$
& if a body comes to rest $v=0$

(b). Eqn of motion under acceleration due to gravity (g):

(1) For a freely falling body

Here $a = +g$, $u = 0$ ($\because$ starting from rest)
$s = h$ (height)

(i) $v = 0 + gt \Rightarrow v = gt$

(ii) $h = 0(t) + \frac{1}{2}gt^2$

$\Rightarrow h = \frac{1}{2}gt^2$

(iii) $v^2 - 0^2 = 2gh \Rightarrow v^2 = 2gh$

(iv) $S_n = 0 + g(n - \frac{1}{2})$

$S_n = g(n - \frac{1}{2})$

(2) For a body vertically projected upwards

Here $a = -g$, $v = 0$

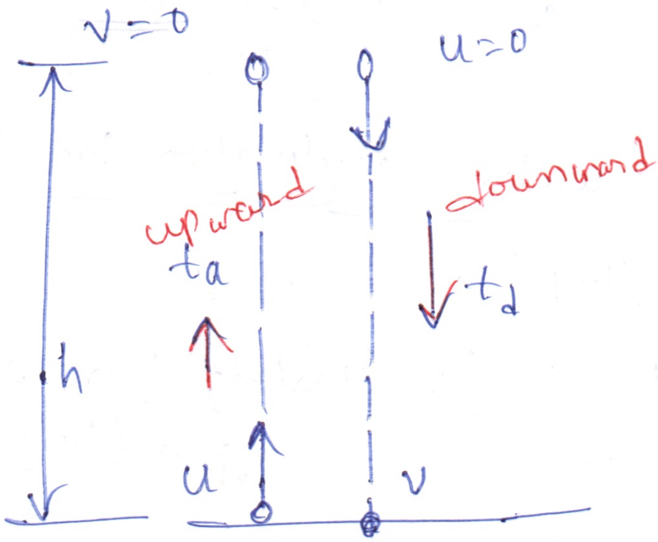
($\because$ Final velocity is zero after reaching max height) $s = h$

(v) $0 = u - gt \Rightarrow u = gt$

(vi) $h = ut - \frac{1}{2} gt^2$

(vii) $0^2 - u^2 = -2gh \Rightarrow u^2 = 2gh$

(viii) $S_n = u - g(n - \frac{1}{2})$



(i) Eqn for final velocity :-

change of velocity = Final velocity − Initial velocity
$(v - u)$

Rate of change of velocity = change of velocity / time

$$= \frac{(v - u)}{t}$$

But rate of change of velocity = Acceleration = a

$$a = \frac{v - u}{t}$$

$$at = v - u$$

$$v = u + at$$

(ii) Eq^n of motion for distance covered (S)

we know avg velocity = $\frac{\text{Initial velocity} + \text{Final velocity}}{2}$

$$= \frac{u+v}{2}$$

Distance covered (S) = Avg velocity × Time

$$= \frac{(u+v)}{2} \times t$$

Substituting the value of "v" from eq^n ($v = u + at$)

$$S = \frac{(u + u + at)}{2} \times t = \frac{(2u + at)}{2} \times t$$

$$\boxed{S = ut + \frac{1}{2}at^2} \quad \text{——— } (1)$$

∴ ⇒ The value of $t = \frac{v-u}{a}$ ——— in (1)

$$S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

$$S = (v-u)\left[\frac{u}{a} + \frac{(v-u)}{2a}\right]$$

$$S = (v-u)\left[\frac{(2u + v - u)}{2a}\right]$$

$$S = \frac{(v-u)(v+u)}{2a} = \frac{v^2 - u^2}{2a}$$

$$\boxed{v^2 - u^2 = 2as}$$

Distance travelled in the n^{th} second :-

u = Initial velocity of a body
a = Acceleration of the body
S_n = Distance covered in "n" seconds
S_{n-1} = Distance covered in $(n-1)$ seconds

$\Rightarrow$ The distance travelled in the n^{th} second
= Distance travelled in "n" seconds − Distance travelled in $(n-1)$ seconds

$= S_n - S_{n-1}$

Distance travelled in "n" seconds is obtained by substituting $t = n$ in eq^n

$$\Rightarrow S_n = un + \frac{1}{2}an^2$$

$\Rightarrow$ Similarly $S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$

$\Rightarrow$ Distance travelled in the n^{th} seconds

$$\Rightarrow S_n - S_{n-1}$$
$$= \left(un + \frac{1}{2}an^2\right) - \left(u(n-1) + \frac{1}{2}a(n-1)^2\right)$$
$$= u + \frac{a}{2}(2n-1)$$

Eqn of motions due to gravity :-

⇒ The acceleration due to gravity "g". Hence when a body falls, the eqn of motions given by that eqns are modified by substituting "g" in place of "a". But when the body is moving vertically up the acceleration due to gravity is acting in the opposite direction. In that case the eqns are modified by substituting $(-g)$ in place of 'a'. The value of "g" is taken as $9.81\ m/s^2$ (or) $981\ cm/s^2$.

⇒ The distance "s" is replaced by "h". Hence the eqns of motions due to gravity in the downward directions & upward directions become as

1. For downward motion

$$a = +g$$
$$v = u + gt$$
$$s = h = ut + \frac{1}{2}gt^2$$
$$u^2 + v^2 = 2gh$$

(2) For upward motion

$$a = -g$$
$$v = u - gt$$
$$s = h = ut - \frac{1}{2}gt^2$$
$$v^2 + u^2 = -2gh$$

Points to be remembered :-

(i) If a body starts from rest, its initial velocity is zero, i.e., $u = 0$

(ii) If a body comes to rest its final velocity is zero i.e $v = 0$

(iii) If a body is projected vertically upwards the final velocity of the body at the highest points is zero i.e $v = 0$

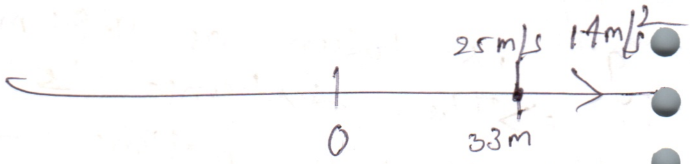
(iv) If a body starting moving vertically downwards, its initial velocity is zero i.e., $u = 0$

(v) Acceleration due to gravity is taken positive when a body is moving vertically downwards. But if the body is moving vertically upwards, the acceleration due to gravity is taken negative.

(1) The position of a particle in rectilinear motion is defined by the relation $x = t^3 - 2t^2 + 10t - 6$ where "x" is in meters and "t" is in seconds. (i) The position, velocity and acceleration of the particle at time $t = 3$. (ii) The Avg velocity during $t = 2s$ & $t = 3s$ & (iii) the Avg acceleration during the third second.

Sol: Given

$$x = t^3 - 2t^2 + 10t - 6$$



The expression for velocity & Acceleration can be obtained by differentiating the above expression successively w.r. to time

$$v = \frac{dx}{dt} = 3t^2 - 4t + 10$$

$$a = \frac{dv}{dt} = 6t - 4$$

(i) position, velocity, & Acceleration of the particle at time $t = 3s$

$$x(3) = (3)^3 - 2(3)^2 + 10(3) - 6 = 33m$$

$$v(3) = 3(3)^2 - 4(3) + 10 = 25 m/s$$

$$a(3) = 6(3) - 4 = 14 m/s^2$$

+ x-axis — so positive 'a' & 'v'

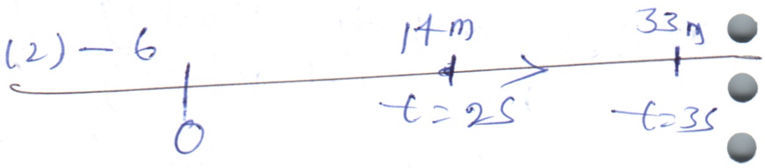
(ii) The Avg velocity during $t = 2s$ & $t = 3s$

The displacement at these two time instants are

$$x(3) = 33m$$

$$x(2) = (2)^3 - 2(2)^2 + 10(2) - 6$$

$$= 14m$$



∴ The Avg velocity during these time interval is given as

$$v_{ave} = \frac{\text{net displacement}}{\text{time placed}}$$

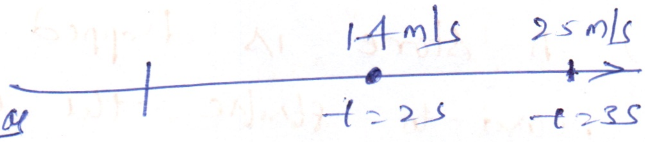
$$= \frac{33 - 14}{3 - 2} = 19 \text{ m/s}$$

(iii) Avg acceleration during ~~the gravity~~ $t = 2s$ & $t = 3s$

The instantaneous velocities at these two time instants are

$$v(2) = 3(2)^2 - 4(2) + 10 = 14 \text{ m/s}$$

$$v(3) = 25 \text{ m/s}$$



$\therefore$ Avg acceleration during this time interval is given as

$$a_{avg} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$= \frac{25 - 14}{3 - 2} = 11 \text{ m/s}$$

(1) The time variation of the position of a particle in rectilinear motion is given by $x = 2t^3 + t^2 + 2t$. If 'v' is the velocity and 'a' acceleration of the particle in consistent units, the motion started with at what velocity & acceleration

Sol:

$$x = 2t^3 + t^2 + 2t$$

$$v = \frac{dx}{dt} = 6t^2 + 2t + 2$$

$$a = \frac{dv}{dt} = 12t + 2$$

At $t = 0$

$v = 2$, & $a = 2$

(2) A stone is dropped into a well and found in heard to strike the water after 4 seconds. Find the depth of the well. If the velocity of sound is 350m/s

Sol:-

Let h = Depth of wall

t_1 = Time taken by the stone to reach the bottom of well (i.e., water surface)

t_2 = Time taken by the sound to travel from bottom to top of well

It is given that: $t_1 + t_2 = 4$ ——— (i)

Consider the motion of stone

Initial velocity $u = 0$

$a = +g$

Also $h = ut + \frac{1}{2}at^2$

$$h = 0 + \frac{9.81}{2} t_1^2$$

$h = 4.905 t_1^2$ ——— (ii)

Now consider the motion of sound

velocity of sound, $v = 350 \text{ m/s}$

$\therefore h = v \times t^2$

(or) $t_2 = \dfrac{h}{350}$ ———— (iii)

Substituting the value for "h" from eqn (ii) in eqn (iii)

$t_2 = \dfrac{4.905\, t_1^2}{350}$ ———— (iv)

Again substituting the value for "t" from eqn (iv) in eqn (i)

$$t_1 + \frac{4.905 t_1^2}{350} = 4$$

(or) $350 t + 4.905 t_1^2 = 350 \times 4$

(or) $4.905 t_1^2 + 350 t_1 - 1400 = 0$

Solving the above eqn in quadratic for t_1

$$t_1 = \frac{-350 \pm \sqrt{350 + 4 \times 4.905 \times 1400}}{2 \times 4.905}$$

$$t_1 = \frac{-350 \pm \sqrt{122500 + 27468}}{9.81}$$

$$t_1 = \frac{-350 \pm \sqrt{149968}}{9.81}$$

Considering + sign

$= 3.798$ seconds

Substituting $t_1 = 3.798$ in eqn (ii)

$h = 4.905 \times 3.798^2$

$h = \underline{70.75 \text{ m}}$

(3) A body starting from rest and covers 10 m in the 5th second of its journey. Find its uniform acceleration

Sol:- Given Initial velocity $u = 0$

Displacement in 5th sec = 10 m

using the relation

$$S_n = u + \frac{a}{2}(2n-1)$$

$$10 = 0 + \frac{a}{2}(2 \times 5 - 1)$$

$$4.5a = 10$$

(or) $a = \frac{10}{4.5} = 2.23 \ m/s^2$

Motional projectile :- projectile motion

A particle projected in the air in the direction other than vertical i.e., oblique to the direction of gravity is called projectile

The path traced by a particle is called is ~~subjected~~ trajectory

* Since there is no horizontal force acting on the projectile and air resistance is neglected

velocity of projection :- (u) : The velocity with which the projectile is projected into space is called velocity of projection

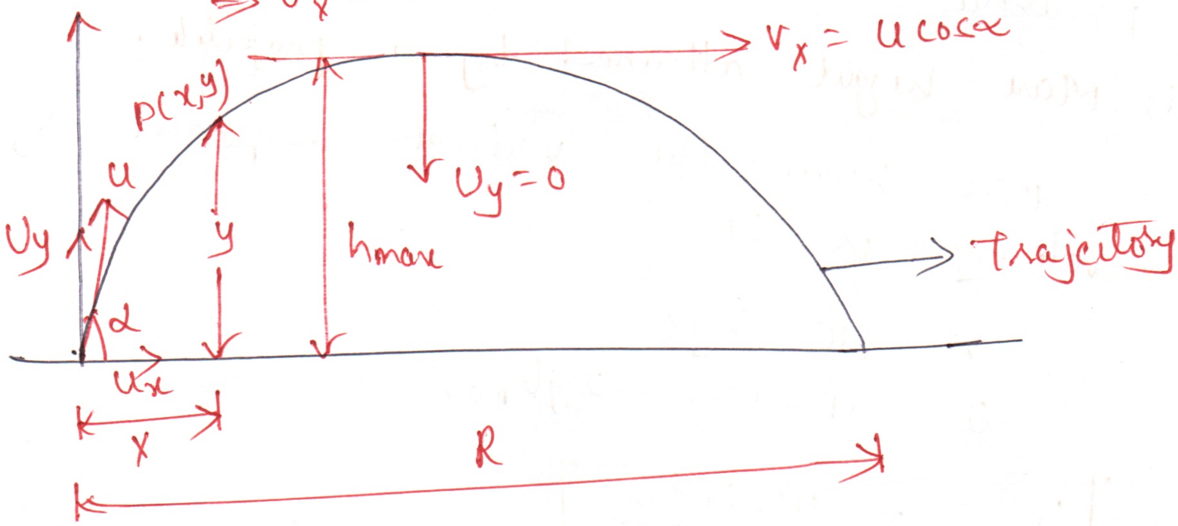
Angle of projection :- (α) :- The angle made by the direction of projection with the horizontal is called angle of projection

(3) <u>Horizontal Range (R)</u> : The horizontal distance travelled by the projectile during its flight is called horizontal range

(4) <u>Time of flight (T)</u> : Total time taken by the projectile to reach maximum height and to reach to the ground is called time of flight. Thus it is the time during which is the projectile is in motion.

$\Rightarrow U_y = u \sin\alpha$ vertical direction

$\Rightarrow U_x = u\cos\alpha \Rightarrow$ Horizontal direction



$\Rightarrow$ <u>Eqns of motion of projectile :-</u>

A particle projected in the space has the motion in vertical as well as in horizontal direction

$\Rightarrow$ <u>Eqns of trajectory:-</u>

The position of the projectile after time, t second is P, and its coordinates are 'x' & 'y'

$\Rightarrow$ Consider vertical motion

$$y = u \cdot t - \frac{1}{2} g t^2$$

$$y = u \sin\alpha \cdot t - \frac{1}{2} g t^2 \quad \text{———— (i)}$$

$\Rightarrow$ Consider Horizontal direction

$$x = U_x t = u\cos\alpha \cdot t \quad \text{———— (ii)}$$

(or) $t = \dfrac{x}{u\cos\alpha}$

Substituting the value of "t" in eqn (i)

$$y = u\sin\alpha \cdot \frac{x}{u\cos\alpha} - \frac{1}{2} g \cdot \frac{x^2}{u^2\cos^2\alpha}$$

$$y = x\tan\alpha - \frac{g x^2}{2u^2 \cdot \cos^2\alpha} \quad \text{—— (iii)}$$

The eqn (iii) define the coordinates of the projectile at any instant and is the eqn of parabola. Thus the trajectory of the projectile is a parabola

(ii) Max height attained by a projectile :-

At max height, the vertical component of velocity is zero

$$v^2 = u^2 - 2gh$$

$$0 = u^2\sin^2\alpha - 2gh_{max}$$

$$\boxed{h_{max} = \frac{u^2 \cdot \sin^2\alpha}{2g}}$$

(iii) Time required to reach max height :-

$$v = u - gt$$

$$0 = u\sin\alpha - gt$$

or $\boxed{t = \frac{u\sin\alpha}{g}}$

(iv) Time of flight (T) :-

At the end of T, h = 0

$$h = ut - \frac{1}{2} g t^2$$

$$0 = u\sin\alpha \cdot T - \frac{1}{2} g t^2$$

$$\boxed{T = \frac{2u \cdot \sin\alpha}{g}}$$

(v) Horizontal range (R)

During the flight, the projectile moves in horizontal direction with constant velocity ($u\cos\alpha$)

$$Range = (u\cos\alpha)T = u\cos\alpha \times \frac{2u\sin\alpha}{g}$$

$$Range = \frac{u^2 \cdot \sin 2\alpha}{g} \quad \text{------------ (i)}$$

maximum Range :

For given velocity of projection, the range is more if $\sin 2\alpha = 1$

i.e $2\alpha = 90^\circ$ (or) $\alpha = \frac{90^\circ}{2} = 45^\circ$

Subs $\alpha = 45^\circ$ in the eq (1)

$$\Rightarrow R_{max} = \frac{u^2 \cdot \sin(2\times 45^\circ)}{g} = \frac{u^2}{g}$$

(2) $u = 0$, $v = 1.828$ m/sec

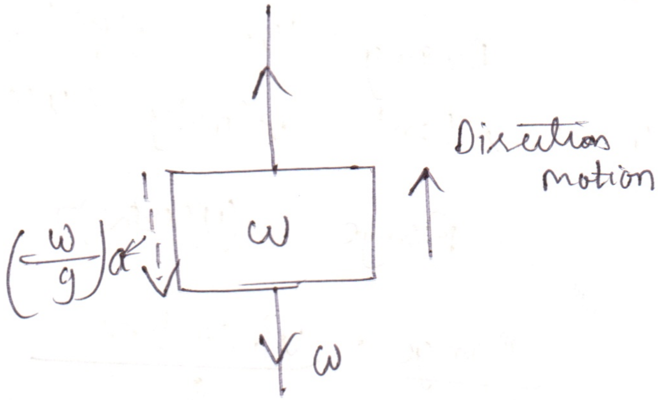
$S = 1.825$ m

$v^2 - u^2 = 2as$

$1.828^2 - 0 = 2a \times 1.828$

$a = \dfrac{1.828}{2}$

$a = 0.914$ m/sec^2



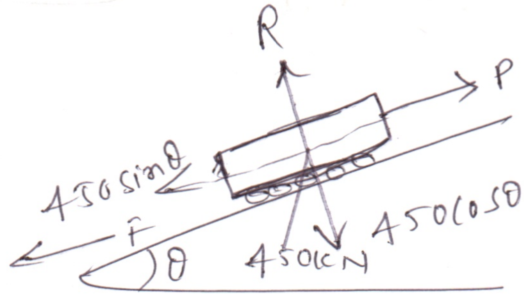
For equilibrium $\Sigma F_y = 0$

$T = W + \left(\dfrac{W}{g}\right)a$

$T = 4448 + \dfrac{4448}{9.81} \times 0.197$

$\underline{\underline{T = 4862.42\ N}}$

(3) $W = 450$ kN



Slope $= \tan\theta = \dfrac{1}{120} = \sin\theta$

Speed $= 72$ kmph

$= 72 \times \dfrac{5}{18} = 20$ m/s

⇒ Resistance $F = 5$ N/kN

$= 5 \times 450 = \underline{\underline{2250\ N}}$

⇒ Since the train moving with steady speed

$P - 6000 = 0$

$P = 6000$ N

$P = P \times V = 6000 \times 20 = 120000$ W

$\underline{P = 120kW}$

Now Net force exerted $= P - 450 \sin\theta - 2250$

$= P - 3750 - 2250$

$= \underline{(P - 6000)\ N}$