

B. Tech Degree IV Semester Examination, April 2009

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT
401 ENGINEERING MATHEMATICS III
(2006 Scheme)

Time : 3 Hours

Maximum Marks : 100

PART - A
(Answer ALL questions)

(8 x 5 = 40)

- I. (a) Show that the function $f(z) = \sin z$ is analytic.
 (b) What do you mean by conjugate harmonic function? Verify whether the function $e^x \sin y$ is harmonic.
 (c) State and prove Cauchy's integral formula.
 (d) Define the residue of a function at an isolated singularity and determine the poles of $\frac{z^2 - 2z}{(z+1)^2(z^2+1)}$ and the residue at each pole.
 (e) Form the partial differential equation by eliminating the arbitrary constants in $z = (x-a)^2 + (y-b)^2 + 1$.
 (f) Solve $p^2 + q^2 = x^2 + y^2$.
 (g) Derive one dimensional heat equation.
 (h) Solve the equation $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$, $u(0, y) = 8e^{-3y}$ by the method of separation of variables.

PART - B

(4 x 15 = 60)

- II. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin, even though Cauchy-Riemann equations are satisfied at the origin. (5)
 (b) Find the analytic function $w = u + iv$, given that $v = e^{2x}(x \cos 2y - y \sin 2y)$. (5)
 (c) Find the equation of the orthogonal trajectories of the family of curves given by $3x^2y + 2x^2 - y^3 - 2y^2 = a$, where a is an arbitrary constant. (5)
- OR**
- III. (a) What do you mean by conformal mapping? Also discuss about
 (i) Translation
 (ii) Magnification and rotation (5)
 (b) Discuss the transformation about $w = \sin z$. (5)
 (c) Show that the image of the hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$ is the lemniscate $R^2 = \cos 2\phi$. (5)

(Turn Over)

- IV. (a) Verify Cauchy's theorem for the integral of z^3 taken over the boundary of the rectangle with vertices $-1, 1, 1+i, -1+i$. (5)
- (b) Find the Laurent's series expansion of $\frac{z^2-1}{z^2+5z+6}$ about $z=0$ in the region $2 < |z| < 3$. (5)
- (c) Using Residue theorem, evaluate $\int_C \frac{3z^2+2}{(z-1)(z^2+9)} dz$ where $C: |z-2|=2$. (5)
- OR
- V. (a) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration where $a > b > 0$. (8)
- (b) Evaluate $\int_0^{\infty} \frac{\sin mx}{x} dx$ using contour integration where $m > 0$. (7)
- VI. Solve –
- (i) $p(1+q) = qz$ (4)
- (ii) $z^2(p^2x^2 + q^2) = 1$ (5)
- (iii) $(x^2 + y^2 + yz)p + (x^2 + y^2 - xz)q = z(x+y)$ (6)
- OR
- VII. Solve –
- (i) $(D^3 - 3D^2D^1 + 4D^1^3)z = e^{x+2y}$ (4)
- (ii) $(D^2 - 3DD^1 + D^1^2)z = \sin x \cos y$ (5)
- (iii) $(D^3 - 7DD^1^2 - 6D^1^3) = x^2y + \sin(x+2y)$ (6)
- VIII. (a) A string is stretched and fastened to two points ℓ apart. Motion is started by displacing the string in the form $y = a \sin(\pi x / \ell)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x, t) = a \sin(\pi x / \ell) \cos(\pi ct / \ell)$. (8)
- (b) Obtain D'Alembert's solution of the wave equation by the method of separation of variables. (7)
- OR
- IX. (a) A string is stretched and fastened to two points $x = 0$ and $x = \ell$ apart. Motion is started by displacing the string into the form $y = k(\ell x - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t . (8)
- (b) Obtain solution of Laplace's equation over a rectangular region by the method of separation of variables. (7)

