

probability and statistics.

Random means "haphazard" Hence any experiment happening under uncertain situations is called a random trial / ~~random experiment~~ (or) a random experiment (or) / experiment of chance.
Ex: result of tossing a coin / throwing a dice / drawing a card from a pack of 52 playing cards. - are random events (or) random experiments.

Event: Any question that we ask with regard to a random experiment defines an event.

Elementary events: Suppose that we have conducted a random experiment. It is completely defined when we know all the possible outcomes. Each outcome of this experiment is called an elementary event. Suppose a coin is tossed then either a head (H) or a tail (T) may turn up.
Therefore head or tail are two elementary events of the experiment of tossing a coin.

Sample space Sample space of a random experiment is the set of all possible outcomes that is the set of all elementary events of experiments and this set is finite.
 $S = \{H, T\}, \{1, 2, \dots, 6\}, \{HH, TT, HT, TH\}$, two coin

Note when two random experiment having m outcomes e_1, e_2, \dots, e_m and n outcomes $p_1, p_2, p_3, \dots, p_n$ respectively. are conducted simultaneously then the sample space consists of mn elementary events and hence is the set $S = \{(e_1, p_1), (e_1, p_2), \dots, (e_m, p_1), \dots, (e_m, p_n)\}$

$= \{(e_i, p_j) \mid i = 1, 2, \dots, m, j = 1, 2, \dots, n\}$
The order pair (e_i, p_j) means that e_i is the outcome of the first experiment and p_j is the outcome of the second experiment.

If three coins are tossed simultaneously -

$$S = \{ \dots \} \quad \text{throwing ^{two} dice} \quad S = \{ \dots \}$$

Two balls are to be drawn simultaneously from a set of red and 2 white balls.

$$\{ R_1 R_2, R_1 R_3, R_2 R_3, R_1 W_1, R_1 W_2, R_2 W_1, R_2 W_2, W_1 W_2 \} \quad \{ S_{1,2} = 10 \}$$

Equally likely events

Let S be the sample space of a random experiment. If all the elementary events of S have the same chance of occurring then the events are said to be equally likely events.

Mutually exclusive events

Let the set E_1 of outcomes of the event A is $\{ 2, 3, 4 \}$
 " " E_2 " " " " " " $B = \{ 4 \}$

$$S = \{ 2, 3, \dots, 12 \}$$

$E_1 \cap E_2 = \emptyset \Rightarrow$ The sets E_1, E_2 are disjoint. we then say that the events A and B are mutually exclusive that is one event has occurred the other event cannot occur or two events cannot occur together.

Mutually exhaustive events

Let S be the sample space of random experiment and $A_1, A_2, A_3, \dots, A_n$ be the events defined on the sample space

If $A_1 \cup A_2 \cup \dots \cup A_n = S$ and $A_i \cap A_j = \emptyset, i \neq j$ then the events are said to be exhaustive.

\therefore The events are said to be mutually exclusive and exhaustive.

Combination of events

Now we consider combination of ~~two~~ events in an experiment. This can be done by using the operation 'or' and 'not'.

Example Throwing two dice simultaneously and noting the total of the numbers that have turned up.

and define the events

$A: \text{sum is } \leq 5, B: 4 \leq \text{sum} \leq 8, C: 5 < \text{sum} < 8$

Let E_1, E_2 and E_3 denote the set of outcomes of the events A, B, C respectively. $E_1 = \{2, 3, 4, 5\}, E_2 = \{4, 5, 6, 7, 8\}, E_3 = \{6, 7\}$

while sample space $S = \{2, 3, \dots, 12\}$.

we define the ^{event} A or B as the event which occurs when either A or B or both occur. i.e. $(A \cup B)$

$E = E_1 \cup E_2 = \{2, 3, 4, 5, 6, 7, 8\}$

A and B \Rightarrow as the event which occurs when A and B both occur. $\Rightarrow A \cap B \Rightarrow E = E_1 \cap E_2 = \{4, 5\}$

we define the event not $A \Rightarrow$ the event which occurs when A does not occur.

If E_1 is the set representing the event A .

Then the set representing not A contains all elements of the sample space, which do not belong to E_1 .

That is A is represented by the complement in ' S ' of the set E_1 which denoted by E_1^c / \bar{E}_1

\therefore not A is also called the complementary event of A or ^{to} negation of A .

$\therefore A - B \Rightarrow$ The A must occur but not B .

- A coin is tossed twice. If event A denotes the number of heads is odd and the event B denotes the number of tails is odd then find the cases favourable to $A \cap B$, $A \cup B$,

$$S = \{HHH, HHT, HTH, \dots\}$$

$$A = \{HHH, HHT, THT, TTH\} \quad B = \{HHT, HTH, THT, TTT\}$$

$$A \cup B = \dots \quad A \cap B = \{HTH\} = \emptyset,$$

$$A \cap \bar{A} = \emptyset,$$

Probability of event

Let ~~the set~~ E be the set of all outcomes of the event A . If the experiment produces an outcome which belongs to E then the event A is said to have occurred

and $n(E) = n(S)$.

total number in sample space is n
and total number in E is m .

Then probability of the event of A is denoted

$$P(A) = \frac{\text{number of elementary events favourable to } A}{\text{total number of equally likely elementary events}}$$

$P(A) = \frac{m}{n}$ This result is called the statistical definition of probability.

$$P(\emptyset) = 0, \quad P(S) = 1$$

$$0 \leq P(A), P(B) \leq 1$$

→ Two dice are tossed once. Find the probability of getting an even number on the first dice or a total of 8.

A: getting an even number on the first dice

B: getting a total of 8

$$A = \{(2,1), \dots, (2,6), (4,1), \dots, (4,6), (6,1), \dots, (6,6)\} \Rightarrow P(A) = \frac{18}{36} = \frac{1}{2}$$

$$B = \{(2,6), (3,5), (5,3), (6,2), (4,4)\} \Rightarrow P(B) = \frac{5}{36}$$

$$A \cap B = \{(2,6), (4,4), (6,2)\} \Rightarrow P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{5}{36} - \frac{1}{12} = \frac{7}{9}$$

→ From a pack of well shuffled cards one card is drawn. Find the probability that this card is either a king or an ace.

A: card drawn is a king, B: card drawn is an ace

∵ 52 — 4 king, 4 aces,

$$P(A) = \frac{4}{52} = \frac{1}{13}, P(B) = \frac{4}{52} = \frac{1}{13}, P(A \cap B) = \emptyset$$

$$P(A \cup B) = P(A) + P(B) = \frac{2}{13}$$

(or) total 52 cards, $\frac{8}{52}$

→ Four cards are drawn. Find the probability that there are two hearts and two diamonds.

total number of possibilities = $52C_4 =$

two hearts can be chosen in $13C_2$ ways

" " " " " " $13C_2$ ways

$$\text{required probability} = \frac{13C_2 \times 13C_2}{52C_4} =$$

→ A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls, one bag is selected at random. From the selected bag one ball is drawn. Find the probability that the ball drawn is red.

A: The ball is drawn from the first bag.

B: " " " " second bag.

Three that

Since both bags are equally likely to be selected.

we have

$$P(A) = P(B) = 1/2$$

Now probability of drawing a red ball from the first bag

$$P_1 = \frac{4}{7} = 4/7$$

"

"

"

$$\text{and } P_2 = \frac{1}{6} = 1/6$$

$$\therefore P(A)P_1 + P(B)P_2 = \frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{1}{6} = \frac{19}{42}$$

→ A bag contains 6 red and 5 blue balls and another bag contains 5 red & 8 blue balls. A ball is drawn from the first bag and without seeing its colour is put in the second bag. A ball is then drawn from the second bag. Find the probability that the ball drawn is blue in colour / red.

sol/ Two cases arise. The ball drawn from the first bag is either red or blue

Case 1) Let the ball drawn from the first bag be red. The probability of drawing red ball from this bag is $\frac{6}{11}$

The ball is now put in the second bag. The second bag has now 6 red and 8 blue. The probability of drawing a ball of blue colour is

$$P_2 = 8/14$$

Probability of these two events occurring together - conjly is $P = P_1 P_2 = 6/11 \times 8/14 = 24/77$

Case 2) Let the ball drawn from the first bag be blue. The probability of drawing a blue ball from this bag is

$$P_1 = 5/11$$

The ball put in 2nd bag. Now 5 red & 9 blue ball

$$P_2 = 9/14, \Rightarrow P = 5/11 \times 9/14 = 45/154$$

Three coins are tossed simultaneously, what is probability that at least two heads occur to be, $n(S) = 2^3 = 8$ at least one, at most 2,

→ what is the probability that a number selected from numbers 1, 2, ..., 20 is an even number, when each of the given numbers is equally likely to be selected.

SO From the numbers 1, 2, ..., 20, the even numbers are 2, 4, 6, ..., 20, or total 10 even numbers.

Since each number is equally likely to be selected.

∴ the required probability $\frac{10}{20} = \frac{1}{2}$

(Law of addition)

If A, B are any two events associated with a random experiment then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Consider the Venn diagram A, B subset of S

$A \cup B = I + II + III$ these three are mutually exclusive theorem. $(P(A \cup B) = P(A) + P(B) - P(A \cap B))$



$$P(A \cup B) = P(I) + P(II) + P(III)$$

$$I = A - B = A - (A \cap B)$$

$$A \cup B = (A - B) \cup (B - A) \cup A \cap B$$

$$II = B - A$$

$$P(A \cup B) = P(A - B) + P(B - A) + P(A \cap B)$$

$$III = A \cap B$$

$$= P(A) - P(A \cap B) + P(B) - P(A \cap B) + P(A \cap B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

mutually excl. -

0

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$P(\text{not } A) = P(\bar{A}) = P(A^c) = 1 - P(A)$$

$$A \cup A^c = S \Rightarrow P(S) = P(A) + P(A^c) = 1$$

$$A \subseteq B \Rightarrow P(A) \leq P(B)$$

→ Two dice are tossed once find the probability of getting an event number on the first dice or a total of 8,

$$S = 36$$

Conditional probability

The probability of happening of an event A under the condition that another event B is already known to happen is called conditional probability. It is denoted by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(OR) Let 'S' be the sample space of an experiment having n elementary. Let n_1 denote the number of elementary events favourable to B which are also favourable to A and n_2 denote the number of elementary events in the sample space which are favourable to B. ($n_2 \geq n_1$)

$$n_1 \leq n_2 \quad B \supset A \quad A \cap B$$

$$P(A|B) = \frac{n_1}{n_2} = \frac{n_1/n}{n_2/n} = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

$P(B) \neq 0,$

Similarly the conditional probability of B when A has occurred is given by

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

A dice is thrown twice and the sum of the numbers appearing is noted to be 8. What is the conditional probability that the number 5 has appeared at least once

$$A = \{ (5,1), (5,2), \dots, (5,6), (1,5), (2,5), (3,5), (4,5), (6,5) \}$$

= The number 5 appears at least once

$$B = \text{The sum of the number 5 appearing is 8}$$

$$= \{ (2,6), (3,5), (5,3), (6,2) \}$$

$$P(A) = \frac{11}{36}, \quad P(B) = \frac{5}{36}$$

$$P(A \cap B) = \frac{2}{36}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{5/36} = \frac{2}{5}$$

$$P(A) = 0.5$$

$$P(B) = 0.6$$

$$P(A \cap B) = 0.8$$

$$P(A|B)$$

$$P(B|A)$$

$$A \cap B = \{ (5,3), (3,5) \}$$

$$P(A \cap B) = 0.7, \quad P(A) = 0.4$$

$$P(A \cap \bar{B}) = 0.1, \quad P(A|B) = 3/4$$

Mutually ~~excl~~ exhaustive event

Let S be the sample space of a random experiment and $A_1, A_2, A_3, \dots, A_n$ be the events defined on the sample space, $\forall A_1 \cup A_2 \cup A_3 \dots \cup A_n = S$, and $A_i \cap A_j = \emptyset$, if $i \neq j$, then events are said to be an mutually exclusive and exhaustive.

Probability of an Event.

To every event in random experiment we attach a numerical value which is called its probability. we denote the probability of the event A by $P(A)$. and is defined as $P(A) = \frac{\text{number of elementary events favourable to } E}{\text{total number of equally likely elementary events}}$

where E represents an event A .

This result is called the statistical definition of probability. $P(\emptyset) = 0$, and $P(S) = 1$,

A - any event, then $0 \leq P(A) \leq 1$,

This result is also an axiom of calculus of probability

Ex Three coins are tossed simultaneously. what is the probability that at least two tails occur.

$$S = \{ \dots \}$$

$A =$ at least two tails occur.

$$P(A) = \frac{4}{8} = \frac{1}{2}$$

→ what is the probability that a number selected from the numbers 1, 2, 2, ..., 20 is an even number $P(A) = \frac{10}{20} = \frac{1}{2}$

→ Three dice are thrown simultaneously find the probability of appearing on the faces the sum is ≥ 6 ,

→ Two dice thrown or —
(b) setting sum is 8 (11) setting a sum of at least 9
(10) at most 5, (1) setting sum is even.

Law of addition of probability.

→ If A and B are any two events associated with a random experiment then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Note ① If A and B are mutually exclusive (or) exhaustive events $A \cap B = \emptyset$, $\Rightarrow P(A \cap B) = P(\emptyset) = 0$.

$$P(A \cup B) = P(A) + P(B)$$

② Let A, B & C are events then
 $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

$$③ P(\text{not } A) = P(\bar{A}) = 1 - P(A), \quad (\bar{A} \cup A = S)$$

$$④ P(A \cup \bar{A}) = P(S) = 1$$

$$⑤ \text{ If } A \subset B \Rightarrow P(A) \leq P(B)$$

→ Two dice are tossed once. Find the probability of getting an even number on the first dice or total no. 8.

Let define A: getting even number on dice B: getting total 8.

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

$$P(B) = \frac{5}{36}$$

$$P(A \cap B) = \frac{3}{36} = \frac{1}{12}$$

$$P(A \cup B) = \frac{1}{2} + \frac{5}{36} - \frac{1}{12} = \frac{17}{36}$$

→ From a pack of well shuffled card. four cards are drawn. Find probability three are two hearts and two diamonds.
 required propab. $\frac{13C_2 \times 13C_2}{52C_4}$

→ what is the chance that leap year selected at random will contain 53 Sundays.

leap year 366 \rightarrow 52 full weeks, \Rightarrow 52 Sundays.
 we have extra two days, - Mon, Tue, Tue-Wed, Wed-Thy

we have seven cases the two cases are favourable.
 $= \frac{2}{7}$

Conditional Probability

the probability of the random event A under the condition that the random event B has occurred. ~~total~~ this probability is called as the conditional probability of A when B has occurred. That is the probability of happening of an event A under the condition that another event B is already known to happen.

and it is denoted by $P(A|B)$ and is defined as $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Let ~~n~~ be the total number of outcomes in the first trial and ~~m~~ be favourable to the event A $\Rightarrow P(A) = \frac{m}{n}$

Let S be the sample space of an experiment having 'n' elementary events. Let m_1 be the number of elementary events favourable to B which are also favourable to A and m_2 be the number of elementary events in 'S' favourable to B' obviously $m_2 \geq m_1$, which are favourable to B' obviously $m_2 \geq m_1$,

hence
$$P(A|B) = \frac{m_1}{m_2} = \frac{m_1/n}{\frac{m_2}{n}} = \frac{P(A \cap B)}{P(B)} > \frac{P(A \cap B)}{P(A)}$$

multiplication law of probability

Let $P(A|B)$ and $P(B|A)$ denote the conditionally probability of A when B has occurred and of B when A has occurred then $P(A \cap B) = P(B) (P(A|B)) = P(A) (P(B|A))$.

NOTE. Independent.

(1) The events A & B are independent $\Leftrightarrow P(A \cap B) = P(A)P(B)$

That is The events are independent means the probability of event A does not depend on occurrence or non-occurrence of the event B. hence in this case the conditional probability $P(A|B)$ is same as $P(A)$

$$P(A|B) = P(A), \text{ and } P(B|A) = P(B)$$

NOTE let A, B are independent then (i) $\bar{A} \cap \bar{B}$ (ii) $\bar{A} \cap B$

A dice is thrown twice and the sum of the numbers appearing is noted to be 8. What is that conditional probability that the number 5 has appeared at least once.

A: the number 5 appears at least once

$$= \{(5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (1,5), (2,5), (3,5), (4,5), (6,5)\} = 11$$

$$P(A) = 11/36,$$

B: The sum of the numbers appearing is 8

$$= \{(2,6), (3,5), (5,3), (4,4), (6,2)\} = 5 \Rightarrow P(B) = \frac{5}{36}$$

$$A \cap B = \{(3,5), (5,3)\} \Rightarrow P(A \cap B) = \frac{2}{36}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/36}{5/36} = \frac{2}{5}$$

$$\rightarrow P(A) = 0.5, P(B) = 0.6, P(A \cup B) = 0.8 \text{ and } P(A|B), P(B|A)$$

$$\rightarrow P(A) = 0.6, P(B) = 0.4, P(A \cap B) = 0.25, P(A|B), P(B|A)$$

$$\rightarrow P(A) = 0.4, P(A \cup \bar{B}) = 0.7, A, B \text{ independent and } P(B)$$

$$\rightarrow P(A \cap \bar{B}) = \frac{1}{4}, P(A \cup B) = \frac{3}{4} \text{ and } P(B), P(A)$$

$$\rightarrow P(C)$$

Random Variable

Let S be the sample space corresponding to a random experiment E . A random variable (RV) on a sample space S defines a function that assigns a real number $X(s)$, $s \in S$. That is a random variable defines a function $X: S \rightarrow \mathbb{R}$, where S is the domain and range is $\mathbb{R} = (-\infty, \infty)$. The set of all possible values

of X which is a subset of \mathbb{R} is called the range space R_X

when the random variable X takes a value x and its corresponding probability is defined as $P(X=x) = P(x)$.

Ex Tossing a two coins, the random variable as the number of heads $x=0, 1, 2$, the corresponding probabilities.

$$P(x=0) = \frac{1}{4}, \quad P(x=1) = \frac{2}{4}, \quad P(x=2) = \frac{1}{4}$$

One Dimensional Random Variable

① Discrete random variable ② Continuous random variables.

Random variable / stochastic variable / variable / chance variable.

A random variable is of two types.

(1) D.R. (2) C.R.V.

Discrete random variable

A random variable X is said to be discrete if it takes a finite number or an infinite but countable number of values. we denote the possible values taken by X as $x_1, x_2, \dots, x_n, \dots$ which terminate in the finite case.

therefore, a real valued function X defined on a discrete sample space S is called a discrete random variable.

Ex Tossing of two coins, the possible values of the random variable (number of heads) are 0, 1, 2,

① Throwing a dice.

Note Discrete theoretical Distribution.

- 1) Binomial distribution
- 2) Poisson dist
- 3) Rectangular dist
- 4) multinomial distr
- 5) Negative Binomial, distr
- 6) Geometric distribution
- 7) Hyper-geometric distribution

probability function of D.P.V.

If for a discrete random variable X the real valued function $P(x)$ such that $P(X=x) = P(x)$. then $P(x)$ is called probability function / probability density function of a D.P.V. X .
 probability function $P(x)$ gives the measure of probability for different value of X .

and we know.

point probability / probability mass function

Let X be a D.P.V. which takes values x_1, x_2, x_3, \dots and let $P(X=x_i) = P(x_i) = P_i$ is probability function.

and it satisfies the following condition.

(i) $P_i \geq 0 \forall i$ (ii) $\sum_{i=1}^{\infty} P_i = 1$

also the collection of pairs $(x_i, P_i) \quad i=1, 2, 3, \dots$

X	x_1	x_2	x_3	...
$P(X=x_i)$	P_1	P_2	P_3	...

P_i is called the probability distribution / the discrete probability distribution of the discrete random variable X .

Ex From a lot of 12 items containing 3 defective items. a sample of 4 items are drawn at random, without replacement. Let a random variable X denote the number of defective items in the sample. Find the probability distribution of X .

$x=0 \Rightarrow P(x=0) = \frac{{}^9C_4}{{}^{12}C_4} = \frac{14}{55}$ $P(x=1) = \frac{{}^9C_3 \cdot {}^3C_1}{{}^{12}C_4} = \dots$ $P(x=2), P(x=3)$

probability mass function.

If X is discrete RV then its probability function $P(x)$ is a discrete probability function / probability mass function. In other words, let x_1, x_2, \dots, x_n are n different values of a discrete RV X and $P(x_1), P(x_2), \dots, P(x_n)$ be their respective probability such that:

$$(i) P(x_i) \geq 0 \quad (ii) \sum_{i=1}^n P(x_i) = 1 \quad i = 1, 2, 3, \dots, n.$$

then $P(x)$ is known as the probability mass function of the variable X . and the values $P(x_i), i = 1, 2, \dots, n$ are called as

the probability distribution of the random variable X .

Variance standard deviation:

The standard deviation of the probability distribution of a random variable X is the positive square-root of the variance of that random variable.

$$S.D = \sigma = \sqrt{E(X^2) - (E(X))^2} = \sqrt{E(X - E(X))^2} = \sqrt{E(X) - \mu^2}$$

Note $V(cX) = c^2 V(X), V(c) = 0, V(X+c) = V(X).$
 $V(aX+b) = a^2 V(X), V(aX+b)$

If X, Y are two independent RV then

$$V(X \pm Y) = V(X) + V(Y)$$

Distribution function / cumulative distribution

Let X be a RV then the function $F(x)$ the distribution function is denoted by $F(x)$ and is defined as

$$F(x) = P(X \leq x) \quad \text{and it has the following properties.}$$
$$= \sum_{x_i < x} P(x_i) \quad (1)$$

$$(ii) 0 \leq F(x) \leq 1, (iii) \text{ If } x_1 < x_2 \Rightarrow F(x_1) < F(x_2)$$

$$P(a \leq X \leq b) = F(b) - F(a).$$

mathematical expectation / Expected value / mean of random variable

The mean of random variable is obtained by multiplying each probable value of the variable by its corresponding probability and then adding these products.

Let a random variable X assumes the values x_1, x_2, \dots, x_n with probabilities p_1, p_2, \dots, p_n resp. The mean of RV X denoted by $E(X)$ or μ .

$$E(X) = x_1 p_1 + x_2 p_2 + \dots + x_n p_n = \sum_{i=1}^n x_i p_i$$

Note ~~$E(c) = c$~~ , let X, Y are RV and a, b, c are constants

$$(1) \rightarrow E(c) = c, \rightarrow E(cX) = c E(X), \rightarrow E(aX \pm b) = aE(X) \pm b,$$

$$\rightarrow E\left(\frac{aX+b}{c}\right) = \frac{1}{c} [aE(X) + b],$$

$$\rightarrow E(X+Y) = E(X) + E(Y).$$

$$\rightarrow E(X \cdot Y) = E(X) \cdot E(Y) \text{ if } X, Y \text{ independent.}$$

$$\rightarrow \text{E} \left(\frac{1}{X} \right) \text{ and } \frac{1}{E(X)} \text{ are not same.}$$

$$\rightarrow E[X - E(X)] = 0.$$

Variance

Variance of the probability distribution of a random variable X is the mathematical expectation of $[X - E(X)]^2$

$$\begin{aligned} \text{Var}(X) &= E[X - E(X)]^2 \\ &= \sum_{i=1}^n [x_i - E(X)]^2 p_i \\ &= \sum_{i=1}^n [x_i^2 + [E(X)]^2 - 2x_i E(X)] p_i \\ &= \sum_{i=1}^n x_i^2 p_i + \sum_{i=1}^n [E(X)]^2 p_i - 2E(X) \sum_{i=1}^n x_i p_i \\ &= E(X^2) + [E(X)]^2 - 2E(X) \cdot E(X) \\ &= E(X^2) + [E(X)]^2 - 2[E(X)]^2 = E(X^2) - [E(X)]^2 \end{aligned}$$

→ mean and s.d of a RV X are 5 and 4 resp. Find $E(X^2)$ and s.d of $(5-3X)$

Find the variance of X from the following data

x	2.0	3.5	4.5	5.0	6.0	$E(X) = 4.3$, $E(X^2) = 19.55$
P	0.1	0.2	0.4	0.2	0.1	$V(X) = 1.06$

→ Daily demand for transistors is having the following probability distribution.

Demand	1	2	3	4	5	6
Probability	0.10	0.15	0.20	0.25	0.18	0.12

Determine the expected daily demand for transistor and find variance.

→ The probability distribution of a RV X is given below. Find $E(X)$, $V(X)$, $E(2X-3)$, $V(2X-3)$.

x	-2	-1	0	1	2
P	0.2	0.1	0.3	0.3	0.1

→ A RV X has the following probability distribution

$X = x_i$	0	1	2	3	4
$P(X = x_i)$	c	$2c$	$2c$	c^2	$5c^2$

Find 'c' $P(X < 3)$, $P(0 < X < 4)$

$Y = y_i$	0	1	2	3	4	5	6
$P(y_i)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

$P(X < 4)$, $P(Y \geq 5)$, $P(3 \leq X \leq 6)$

what will be the minimum value of k so that $P(X \leq 3) > 3$,

x	-2	-1	0	1	2	3
P	0.1	k	0.2	$2k$	0.3	k

$\frac{1}{6}$ m $\frac{1}{4}$ m $\frac{1}{6}$

611, V.

x	0	1	2	3	4	5	6	7
P	0	k	$2k$	$3k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find $P(X \geq 7) > \frac{1}{2}$, $P(X < 6)$, $P(0 < X < 5)$, $P(X > 6)$,

x	0	1	2
P	0	k	$2k$

x	1	2	3	4	5
P	c	c	$3c$	$\frac{1}{2}c$	$6c^2$