Total No. of Questions—12] [Total No. of Printed Pages—8+1

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S.E. (CIVIL) (I Semester) EXAMINATION, 2010

ENGINEERING MATHMATICS-III

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Answer **3** questions from Section I and **3** questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $(D^2 + 3D + 2)y = e^{e^x} + \cos e^x$

(*ii*) $(D^2 - 4D + 4)y = e^x \cos^2 x$

P.T.O.

(*iii*) $\frac{d^2y}{dx^2} + 4y = \tan 2x$ [By variation of parameters]

(*iv*) $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

(*b*) Solve : [5]

$$\frac{dx}{dt} + y = e^t$$

$$x - \frac{dy}{dt} = e^{-t},$$

given that $x = 1$, $y = 0$ at $t = 0$.

Or

2. (*a*) Solve any *three* : [12]

(*i*) $(D^2 + 6D + 9)y = \frac{1}{x^3 e^{3x}}$

(*ii*) $(D^5 - D)y = 2x + 2^x$

(*iii*) $(D^2 - 4D + 4)y = e^{2x} \sec^2 x$

[By variation of parameters]

(*iv*) $(x + a)^2 \frac{d^2y}{dx^2} - 4(x + a)\frac{dy}{dx} + 6y = x$

(*b*) Solve : [5]

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$$

3. (*a*) The differential equation satisfied by a beam, uniformly loaded with one end fixed and second subjected to a tensile force P is given by :

$$\mathrm{EI}\frac{d^2y}{dx^2} - \mathrm{P}y = \frac{-\mathrm{W}x^2}{2}$$

Show that the elastic curve for the beam under conditions

$y = 0$ and $\frac{dy}{dx} = 0$.

when $x = 0$, is given by :

$$y = \frac{\mathrm{W}}{2\mathrm{P}}\left[x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2}\right]$$

where, $\frac{\mathrm{P}}{\mathrm{EI}} = n^2$. [8]

(*b*) A homogeneous rod of conducting material of length 100 cm with ends kept at zero temperature satisfies the equation :

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

If the initial temperature is :

$$u(x, 0) = \begin{cases} x & ; \ 0 \le x \le 50 \\ 100 - x & ; \ 50 \le x \le 100. \end{cases}$$ [8]

Or

4. (*a*) It is found experimentally that a weight of 3 kg stretches a spring to 15 cm. If the weight is pulled down 10 cm below equilibrium position and then released

(*i*) find the amplitude, period and frequency of motion.

(*ii*) determine the position, velocity and acceleration as a function of time. [8]

(*b*) Solve the equation : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

subject to the following conditions :

(*i*) $u(x, \infty) = 0$

(*ii*) $u(0, y) = 0$

(*iii*) $u(1, y) = 0$

(*iv*) $u(x, 0) = x(1 - x)$ for $0 < x < 1$. [8]

5. (*a*) Solve the following system of equations by Gauss-Seidel iteration method : [9]

$$9x_1 + 2x_2 + 4x_3 = 20$$

$$x_1 + 10x_2 + 4x_3 = 6$$

$$2x_1 - 4x_2 + 10x_3 = -15$$

(*b*) Use Runge-Kutta method of fourth order to solve :

$$\frac{dy}{dx} = \sqrt{x + y} \quad ; \quad y(0) = 1$$

to find y at $x = 0.2$ taking $h = 0.1$. [8]

Or

6. (*a*) Solve the equation :

$$\frac{dy}{dx} = x^2 + y \quad ; \quad y\ (0) = 1$$

to find y at $x = 0.1$ using Euler's modified method taking $h = 0.05$. [9]

(*b*) Solve the following system of equations by Cholesky's method :

$$4x_1 - 2x_2 = 0$$
$$-2x_1 + 4x_2 - x_3 = 1$$
$$-x_2 + 4x_3 = 0.$$ [8]

SECTION II

7. (*a*) Compute the first four moments, coefficient of skewness and kurtosis for the following frequencies : [6]

No. of Jobs completed	**No. of Workers**
0—10	6
10—20	26
20—30	47
30—40	15
40—50	6

(*b*) Compute the coefficient of correlation between the supply and price : [6]

x	y	f
5	7	6
9	9	9
15	14	13
19	21	20
24	23	16
28	29	11
32	30	7

(*c*) There are 6 married couples in a room. It two persons are chosen at random, find the probability that :

(*i*) they are of different sex

(*ii*) they are married to each other. [5]

Or

8. (*a*) Obtain the correlation between population density (per square mile) and death rate (per thousand persons) from the data related to 5 cities. [6]

Population Density	Death Rate
200	12
500	18
400	16
700	21
300	10

(*b*) If two lines of regressions are $9x + y - \lambda = 0$ and $4x + y = \mu$ and the means of x and y and 2 and –3 respectively, find the values of λ and μ and coefficient of correlation between x and y. [6]

(*c*) Number of road accidents follows a Poisson's distribution with mean 5, find the probability that in a certain month number of accidents on the highway will be : [5]

(*i*) less than 3

(*ii*) between 3 and 5

(*iii*) more than 3.

9. (*a*) A particle describes the straight line $r = a \sec \theta$ with constant angular velocity ω. Find the radial and transverse components of velocity and acceleration. [5]

(*b*) If the directional derivatives of $\phi = a(x + y) + b(y + z) + c(x + z)$ has maximum value 12 in the directional parallel to the line

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{3}$$

find the values of a, b, c. [5]

(*c*) Establish any *two* : [6]

(*i*) If $\rho\overline{E} = \nabla\phi$, prove that $\overline{E} \cdot \text{curl } \overline{E} = 0$.

(*ii*) Show that curl curl curl curl $\overline{E} = \nabla^4 \overline{E}$, where $\overline{E}$ is solenoidal.

(*iii*) $\nabla \cdot (r^3 \overline{r}) = 6r^3$.

Or

10. (*a*) A particle moves along the curve $x = a \cos t$, $y = a \sin t$; $z = bt$ with constant angular velocity ω. Find the radial and transverse components of its linear velocity and acceleration at any time t. [5]

(*b*) Find the directional derivatives of ∇f at $(1, 2, -1)$ where $f(x, y, z) = x^2 y + xyz + z^3$ along normal to the surface $x^2 y^3 = 4xy + y^2 z$ at the point $(1, 2, 0)$. [5]

(*c*) Establish any *two* :

(*i*) $\nabla^4 e^r = e^r + \dfrac{4}{r} e^r$

(*ii*) $\overline{F} = \dfrac{\overline{a} \times \overline{r}}{r^n}$ is solenoidal field

(*iii*) $\nabla \times [\overline{a} \times (\overline{b} \times \overline{r})] = \overline{a} \times \overline{b}$

where $\overline{a}$ and are constant vectors. [6]

11. (*a*) Verify Green's theorem for $\overline{F} = xi + y^2j$ over first quadrant of the circle $x^2 + y^2 = 1$. [6]

(*b*) Evaluate $\iint_S (x\hat{i} + y\hat{j} + z^2\hat{k}) \cdot dS$

where S is the curved surface of the cylinder $x^2 + y^2 = 4$ bounded by planes $z = 0$ and $z = 2$. [6]

(*c*) Evaluate using Stokes' theorem $\int_C (ydx + zdy + xdz)$, where C is intersection of $x^2 + y^2 + z^2 = a^2$, $x + z = a$. [5]

Or

12. (*a*) Evaluate $\iint_S (2xy\overline{i} + yz^2\overline{j} + xz\overline{k})\, d\overline{S}$ over the surface of the region bounded by $x = 0$, $y = 0$, $y = 3$, and $x + 2z = 6$. [6]

(*b*) Obtain the equation of streamlines in case of steady motion of fluid defined by $\overline{q} = (y - xz)\overline{i} + (yz + x)\overline{j} + (x^2 + y^2)\overline{k}$. [6]

(*c*) Evaluate $\int_C \overline{F} \cdot d\overline{r}$ for $\overline{F} = (2y + 3)\overline{i} + xz\overline{j} + (4z - x)\overline{k}$ along the path $x^2 = 2t^2$; $y = t$; $z = t^3$ from $t = 0$ to $t = 1$. [5]