Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS / B.TECH(NEW)BME / ECE / EE / EIE / PWE / ICE / EEE / SEM-3 / M(CS)-301 / 2012-13

2012

NUMERICAL METHODS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) In the problem "Find the area of a circle having radius 2; given $\pi = 3.14$", the kind of error of the approximation 3·14 for π is

      a) Truncation error b) Round-off error

      c) Inherent error d) Relative error.

   ii) The number 9.6506531 when round-off to 4 places of decimal will give

      a) 9.6506 b) 9.6507

      c) 9.6505 d) none of these.

iii) If $f(3) = a + \Delta f(1) + \Delta^2 f(1)$ then $a =$

a) $f(0)$ b) $f(1)$

c) $f(2)$ d) $f(3)$.

iv) Which of the following is ture ?

a) $E = 1 - \Delta$ b) $E = 1 + \Delta$

c) $\Delta = E + 1$ d) $E = \Delta^{-1}$.

v) It cannot be recommended to construct an interpolation polynomial for a function $f(x)$ if

a) $f(x)$ is not a polynomial

b) $f(x)$ is not derivable somewhere

c) $f(x)$ has abrupt changes

d) graph of $f(x)$ is unknown.

vi) The degree of precision of Simpson's $\frac{1}{3}$ rd rule is

a) 1 b) 2

c) 3 d) 4.

vii) In evaluating $\int_a^b f(x)\,dx$, the error in Trapezoidal rule is of order

a) h^3 b) h^4

c) h^2 d) h.

viii) When Gauss-Elimination method is used to solve $AX = B$, A is transformed to a

a) null matrix

b) upper-triangular matrix

c) identity matrix

d) diagonally-dominant matrix.

ix) If $\frac{dy}{dx} = x^2 + y$ and $y(0) = 1$, then $y(0.02)$ according to Euler's method is [$h = 0.01$]

a) 1.02 b) 1.04

c) 1.00 d) 0.99.

x) The finite difference method is used to solve

a) a system of ordinary differential equation

b) a Boundary Value Problem

c) a partial differential equation

d) a system of transcendental equation.

xi) The local truncation error in Euler's method for the solution of ODE of first order is

a) $O(h^2)$ b) $\frac{h^2}{2} y''(x_m + \theta h)$

c) $h^2 y''(x_m + \theta h)$ d) none of these.

xii) One root of the equation $x^2 + 2x - 2 = 0$ lies between

a) 1 and 2 b) 0 and 0.5

c) 0.5 and 1 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find $f(5)$ from the following data :

x	0	2	3	4	7	8
$f(x)$	4	26	58	112	466	668

3. Find the value of $\int_0^{\pi/2} \sqrt{1 - 0.162 \sin^2 x}\, dx$ using Simpson's $\frac{1}{3}$ rd rule taking six equal subintervals.

4. Using Newton-Raphson method find $\sqrt[4]{32}$ correct upto 4 places of decimal.

5. Find the inverse of the following matrix using Gauss Elimination method.

$$\begin{pmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{pmatrix}$$

6. Use Fourth order Runge-Kutta method to find an approximate value of $y(0.2)$ given that $y(0) = 0$ and $\frac{dy}{dx} = 1 + y^2$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Round off 31. 5218 to one significant figure. 1

b) What is the percentage error in approximating $\frac{4}{3}$ to 1.3333 ? 2

c) Prove that $E \equiv e^{hD}$, where E, h and D are the shift operator, the step length and the differential operator respectively. Hence, show that $hD \equiv \sin h^{-1}(\mu\delta)$, where, μ and δ have their usual meanings. 3 + 2

d) Define interpolation and extrapolation. Deduce the Newton's forward interpolation formula. 2 + 5

8. a) Find by Lagrange's formula, the interpolation polynomial which corresponds to the following data : 5

x :	0	1	2	3	4
$f(x)$:	3	6	11	18	27

b) Find a real root of the equation $x^3 - 2x - 5 = 0$ by using Regula-Falsi method. 5

c) Derive Simpson's one-third rule from Newton-Cote's quadrature formula. 5

9. a) Solve the system of linear equation by LU Factorization Method : 8

$2x - 6y + 8z = 24$

$5x + 4y - 3z = 2$

$3x + y + 2z = 16$

b) Compute y (0.5), by Milne's predictor-corrector method from $\frac{dy}{dx} = 2e^{x} - y$,

given that y (0.1) = 2.0100, y (0.2) = 2.0401, y (0.3) = 2.0907, y (0.4) = 2.1621. 7

10. a) Find the root of the equation $x \tan x = 1.28$, that lies in the interval (0, 1), correct to 4 decimal places, using Bisection method. 6

b) Find the solution of the following differential equation by Euler's method for $x = 1$ by taking $h = 0.2$, $dy/dx = x + y$ with $y = 1$ when $x = 0$. 5

c) Show that $(1 + \Delta) (1 - \nabla) \equiv 1$. 4

11. a) Obtain the Newton's Divide Difference interpolating polynomial, hence find $f(3)$: 8

x :	0	1	2	4	5	6
$f(x)$:	1	14	15	5	6	19

b) Solve the following system of equations using Gauss elemination method : 7

$-x + y + 10z = 35{\cdot}61$

$x + 10y + z = 20{\cdot}08$

$10 + y - x = 11{\cdot}19$