MATHEMATICS

Paper 2: Solid Geometry, Abstract Algebra and Real Analysis

Time: 3 hours Max Marks: 80

SECTION - A

Answer all the FOUR questions. Each question carries 15 marks. 4X15=60

1. a) i) Show that every finite integral domain is a field. (8marks)

   ii) Show that the characteristic of an integral domain is either zero or prime. (7 marks)

   (Or)

   b) i) Show that the ring of integers is a principal ideal ring. (8marks)

   ii) If f is a homomorphism of a ring R into the ring R^1 then show that **f** is an into isomorphism if and only if ker **f** $=\{0\}$ (7 marks)

2. a) i) If f:[a,b] $\rightarrow$ R is continuous on [a,b] then show that f is Uniformly continuous on [a,b]. (8marks)

   ii) Prove that $\frac{\pi}{6}+\frac{\sqrt{3}}{15}<\sin(0.6)<\frac{\pi}{6}+\frac{1}{8}$ (7 marks)

   (Or)

   b) i) State and prove Cauchy's mean value theorem. (8marks)

   ii) Determine the constants a and b so that the function defined by $f(x)=2x+1$ if $x \le 1$, ax^2+b if $1<x<3$, $5x+2a$ if $x \ge 3$ is continuous everywhere. (7 marks)

3. a) i) if f:[a,b] $\rightarrow$ R is monotonic on [a,b] then show that f is integrable on [a,b]. (8marks)

   ii) if f(x) = x^3 is defined on [0,a] show that f $\in$ R([0,a]) and $\int_0^a f(x)\,dx=\frac{a^4}{4}$ (7 marks)

   (or)

   b) i) State and prove fundamental theorem of integral calculus. (8marks)

   ii) Prove that $\frac{1}{\pi}\le\int_0^1\frac{\sin \pi x}{1+x^2}\,dx\le\frac{2}{\pi}$. (7 marks)

4. a) i) Find the S.D. between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{34}$.

Find also the equations and the points in which the S.D. line meets the givenlines. (8marks)

ii) A Variable plane is at a constant distance 3p from the origin and meets the coordinate axes in A, B, C. Show that the locus of the centroid of the Δ ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$ ` (7 marks)

(Or)

b) i) Show that the plane 14x -8y+ 13 = 0 bisects the obtuse angle between the planes 3x + 4y -5z +1 = 0& 5x + 12y – 13z = 0 (8marks)

ii) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at (1, -2, 1) and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$. (7 marks)

SECTION – B

Answer any FOUR Questions 4x5=20

5. Show that a field has no proper ideals.
6. If f is a homomorphism of a ring R in to a ring R' then prove that kernel of **f** is an ideal of R.
7. Examine the continuity of the function **f** defined by f(x) = $\lim_{n\to\infty} \frac{x^n}{1 + x^n e^x}$ $\forall x \geq 0$
8. Show that the function f(x) = x sin (1 / x) if x≠0, f(x) = 0 if x = 0 is continuous at x = 0 but not differentiable at x = 0
9. If $f(x) = x^2$ on [0,1] and $p = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}$ then compute $L(p, f)$ and $U(p, f)$.
10. State and prove first mean value theorem of integral calculus.
11. Find the image of the point (2, -1, 3) in the plane $3x - 2y + z = 9$.
12. Find the limiting points of the coaxial system defined by the Spheres $x^2 + y^2 + z^2 + 4x + 2y + 2z + 6 = 0$ and $x^2 + y^2 + z^2 + 2x - 4y - 2z + 6 = 0$