08/05/13 SEM III
E7RX Maths (III)
Engg. Mathematics

P3–upq–Feb.–13KL–31 A4 D

Con. 6381–13. **GS–6096**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.
(3) Figures to the **right** indicate **full** marks.

1. (a) Show that $f(z) = \frac{z}{|\bar{z}|^2}, |z| \neq 0$ is analytic function. Hence find f' (z). **5**

(b) Find Fourier series expansion for f (x) = sin x in $(-\pi, \pi)$ **5**

(c) Find Laplace transform of $t \cdot \sqrt{1 + \sin t}$ **5**

(d) Find z transformation of $\{\alpha^k \sin \alpha k\}, k \geq 0$. where α is constant. **5**

2. (a) Using Laplace transform evaluate $\int_0^\infty e^{-t} \frac{\sin 3t}{t} dt$. **6**

(b) Find the Fourier series expansion for f(x) = cos px where p is non-integer in $(0, 2\pi)$ **7**

(c) Find the matrix A, if adj $A = \begin{bmatrix} -2 & 1 & 3 \\ -2 & -3 & 11 \\ 2 & 1 & -5 \end{bmatrix}$. **7**

3. (a) Find inverse Laplace transform of — **6**

(i) $\log\left(\frac{s-2}{s-3}\right)$ (ii) $\frac{s+1}{(s^2-4)}$

(b) Find non-Singular maarices P and Q such that PAQ is in normal form. Also find rank of a matrix A where **7**

$$A = \begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 4 & -7 & 4 & -4 & 5 \end{bmatrix}$$

(c) Verify Green's theorem in the plane for $\oint_c (xy + y^2)dx + x^2 dy$ where c is the closed curve of the region bounded by $y = x$ and $y^2 = x$. **7**

4. (a) Obtain complex form of Fourier Series for the function $f(x) = e^{-ax}$ in $(-2, 2)$ where a is not an integer. **6**

[TURN OVER

(b) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix}$ compute A^{-1} and hence, Solve the system of equations 7

$x+ y+z = 9,\ 2x + 5y + 7z = 52,\ 2x + y - z = 0.$

(c) Find Laplace transform of — 7

$$f(t) = 1, \quad 0 \le t \le a$$
$$= -1, \quad a < t \le 2a$$
$$\& \ f(t + 2a) = f(t)$$

5. (a) Find the analytic function $f(z) = u + iv$ if $u = \left(r + \frac{a^2}{r}\right)\cos\theta$. 6

(b) Show that the equations. 7

$$ax + by + cz = 0$$
$$bx + cy + az = 0$$
$$cx + ay + bz = 0$$

has a non-trivial solution if a+b+c = 0 or if a=b=c. Find the non-trivial solution when the condition is satisfied.

(c) Find Fourier integral representing 7

$$f(x) = \begin{cases} 1-x^2 & , \ |x| \le 1 \\ 0 & , \ |x| > 1 \end{cases}$$

6. (a) Find the half range cosine series for $f(x) = 2x - x^2$ in (0, 2). 6

(b) Find the bilinear transformation which maps the points 2, i, –2 onto the points 1, i, –1. 7

(c) Using Laplace transform solve the differential equation 7

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 4,\ y(0) = 0 \ \& \ y'(0) = 1$$

7. (a) Find inverse z-transform of $F(z) = \frac{1}{(z-2)(z-3)}$ if ROC is $2 < |z| < 3$. 6

(b) Verify stoke's theorem for $\overline{F} = x^2\hat{i} + xy\hat{j}$ & C is the boundary of the rectangle 7

$x = 0,\ y = 0,\ x = 2,\ y = 3.$

(c) Using Divergence theorem evaluate $\iint_S \overline{F} \cdot \hat{n}\, ds$, where $\overline{F} = 4x\hat{i} + 3y\hat{j} - 4z^2\hat{k}$ and 7

S is closed surface bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $2x + 2y + z = 4$.
