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Third Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain Fourier series for the function f(x) given by

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi}, & -\pi\le x\le 0\\ 1-\dfrac{2x}{\pi}, & 0\le x\le \pi\end{cases}$$

and hence deduce that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots\ldots\ldots\ldots$ **(07 Marks)**

b. Find the half-range cosine series for the function $f(x)=(x-1)^2$ in the interval $0<x<1$. **(07 Marks)**

c. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y as given in the following table. **(06 Marks)**

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

2 a. Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & |x|\le 0\\ 0, & |x|>\pi\end{cases}$.

Hence evaluate $\displaystyle\int_0^\infty \frac{x\cos x-\sin x}{x^3}.\cos\left(\frac{x}{2}\right)dx$ **(07 Marks)**

b. Find the Fourier cosine transform of e^{-x^2} **(07 Marks)**

c. Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\displaystyle\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi}{2}.e^{-m}, m>0.$ **(06 Marks)**

3 a. Form the partial differential equation by eliminating the arbitrary functions f and g from the relation $z=y^2+2f\left(\dfrac{1}{x}+\log y\right)$ **(07 Marks)**

b. Solve $\dfrac{\partial^3 t}{\partial x^2\partial y}+18xy^2+\sin(2x-y)=0$ **(07 Marks)**

c. Solve $(mz-ny)\dfrac{\partial z}{\partial x}+(nx-lz)\dfrac{\partial z}{\partial y}=(ly-mx)$ **(06 Marks)**

4 a. Derive one dimensional heat equation by taking u(x, t) as the temperature, x is the distance and t is the time. (Write the figure also.) **(07 Marks)**

b. Obtain the D'Almbert's solution of the wave equation $u_{tt}=C^2u_{xx}$, subject to the condition $u(x,o)=f(x)$ and $\dfrac{\partial u}{\partial t}(x,o)=o$. **(07 Marks)**

c. Obtain the various solutions of the Laplace's equation $u_{xx}+u_{yy}=0$, by the method of separation of variables. **(06 Marks)**

PART – B

5 a. Complete the real root of the equation $x\log_{10}x - 1.2 = 0$ by Regula-Falsi method, correct to four decimal places. **(07 Marks)**

b. Solve the system of equations using Gauss-Jordan method:

$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$
$$x + y + z = 9$$

(07 Marks)

c. Using the power method, find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **(06 Marks)**

6 a. The area of a circle (A) corresponding to diameter (D) is given below: **(07 Marks)**

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

b. Use Newton's divided difference formula to find f(9), given the data, **(07 Marks)**

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

c. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Weddle's rule, taking 7 ordinates. **(06 Marks)**

7 a. Derive Euler's equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = o$$

(07 Marks)

b. Find the curves on which the functional $\int_0^1 \left[(y')^2 + 12xy\right] dx$, with y(0) = 0 and y(1) = 1can be extremised. **(07 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is $S = \int_{x_1}^{x_2} \sqrt{x[1 + (y')^2]}\, dx$ **(06 Marks)**

8 a. Find the Z-transforms of the following :
i) cosh nθ ii) $(n + 1)^2$ **(07 Marks)**

b. Find the inverse z-transform of $\frac{z}{(z-1)(z-2)}$. **(07 Marks)**

c. Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u_n$ with $y_0 = 0$, $y_1 = 1$ and $u_n = 1$ for n = 0, 1, 2, 3, ... by z-transform method. **(06 Marks)**

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