BTS(C) – III– 09 – 065 - B

B. Tech Degree III Semester Examination, November 2009

IT/CS 303 DISCRETE COMPUTATIONAL STRUCTURES

(2006 Scheme)

Time : 3 Hours | Maximum Marks : 100

PART – A

(Answer all questions)

(8 x 5=40)

I. a. Prove that $(p \wedge q) \rightarrow (p \leftrightarrow q)$ is a tautology.

b. By mathematical induction, prove that

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1).$$

c. Define recurrence relation and explain its applications.

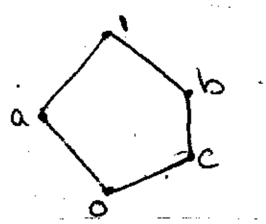
d. How many bit strings of length 10 contain (i) exactly four 1's (ii) atmost four 1's (iii) atleast four 1's?

e. Prove that if $G = (V, E)$ is an undirected graph with e edges, then $\sum_i \deg(V_i) = 2e.$

f. Define bipartite graph with an example.

g. Show that the set $Q^{\times}$ of all positive rational numbers forms a group under the operation defined by $a * b = \frac{ab}{2};\ a, b \in Q^{+}.$

h. Verify whether the lattice given by the Hasse diagram is distributive or not.



PART B

(4 x 15 = 60)

II. a. Construct the truth table for $(7p \leftrightarrow 7q) \leftrightarrow (q \leftrightarrow r)$. (7)

b. If R and S are equivalence relations on a set. Show that $R \cap S$ is also an equivalence relation. (8)

OR

III. a. Show that $p \rightarrow q$ and its contrapositive $\overline{q} \rightarrow \overline{p}$ are logically equivalent. (7)

b. Consider $A = B = C = R$ and let $f : A \rightarrow B$ and $g : B \rightarrow C$ be defined by $f(x) = x + 9$ and $g(y) = y^2 + 3$. Find the following composition functions.

(i) $fof\ (a)$ *(ii)* $gog(a)$ *(iii)* $fog(b)$ *(iv)* $gof\ (4)$. (8)

(Turn over)

IV. a. Show that the recurrence relation
$a_n = 2[a_{n-1} - a_{n-2}]; n \geq 2$ and $a_0 = 1;\ a_1 = 2.$ (6)

b. State the Pigeon Hole principle. (4)

c. From a club consisting of 6 men and 7 women, in how many ways we can select a Committee of 3 men and 4 women? (5)

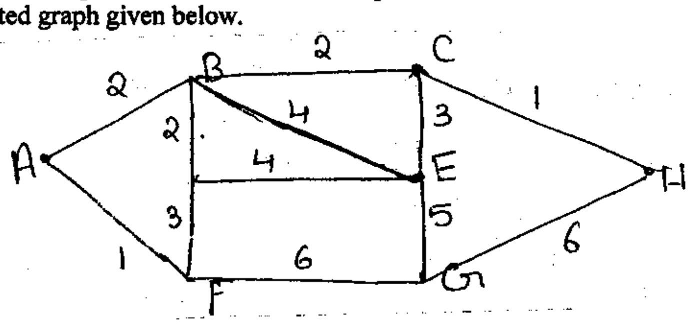
OR

V. a. Define recursive algorithm and explain the recursive algorithm for finding factorial of n. (7)

b. A box contains six red balls and four green balls. Four balls are selected at random from the box. What is the probability that two of selected balls will be red and two will be green? (8)

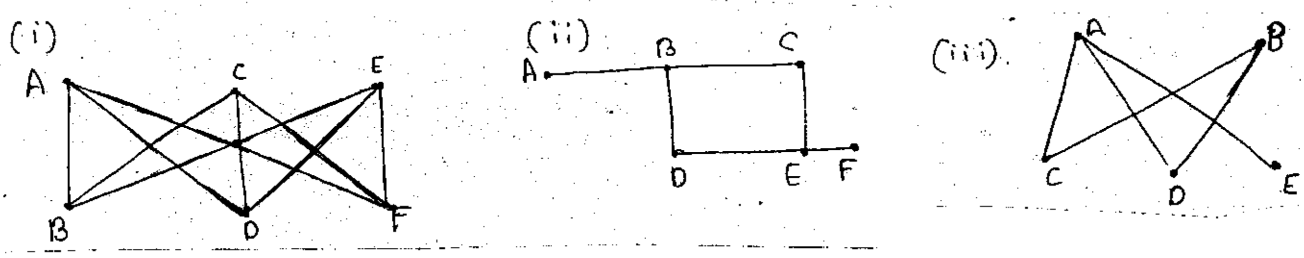
VI. a. Explain travelling salesman problem. (5)

b. Use Dijkstra's algorithm to find the shortest path between the vertices A and H in the weighted graph given below. (10)



OR

VII Find a Hamiltonian path or circuit, if it exists in each of the graphs given below. If it does not exist, explain why? (15)



VIII. Let A={a,b} which of the following tables defines a semi group on A? Which define a monoid on A? (15)

(i)

*	a	b
a	a	b
b	a	a

(ii)

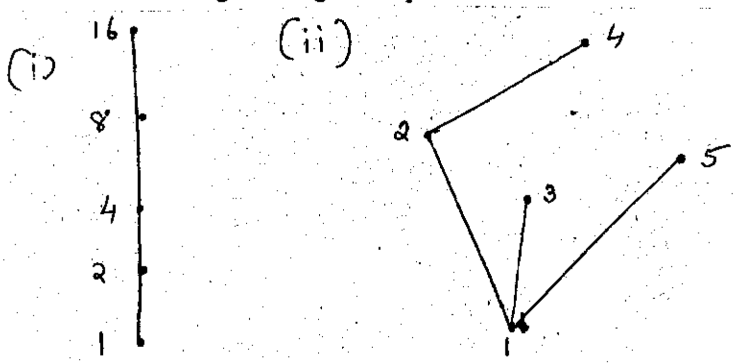
*	a	b
a	a	b
b	b	a

(iii)

*	a	b
a	b	b
b	a	a

OR

IX. Define a lattice. Which of the given diagrams represent lattice and which are not. Why? (15)



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