

FACULTY OF ENGINEERING

B.E. 2/4 (ECE / M/ P/AE / CSE) II – Semester (Main) Examination, April / May 2013

Subject : Mathematics – IV

Time : 3 hours

Max. Marks : 75

Note: Answer all questions from Part-A. Answer any FIVE questions from Part-B.

PART – A (25 Marks)

1. From the following functions, which is analytic at $z = 0$. (2)
 - a) $|z|^2$ b) $1/z$ c) \bar{z} d) none of above

2. Find $\oint_c \frac{dz}{z^2 - 1}$, where c is the triangle with vertices at $0, a+bi, -a+bi, 0 < a < 1$ and $0 < b < 1$. (3)

3. Expand Cotz in Taylor's series about the point $z = 0$. (2)

4. State whether the following are true or false. (3)
 - a) $f(z) = \frac{1}{z(2-z)}$ has essential singularity at $z = 0$, and $z = 2$

 - b) $f(z) = \frac{z - \sin z}{z^2}$ has removable singularity at $z = 0$.

 - c) $f(z) = z e^{\frac{1}{z^2}}$ has no singularity at $z = 0$.

5. Identify the continuous distributions from the following. (2)

a) Normal distribution	b) Gamma distribution
c) Poisson distribution	d) Binomial distribution

6. The p.d.f. of a continuous random variable X is given by (3)

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{otherwise} \end{cases} \quad \text{where } \lambda > 0. \text{ Find mean and variance of } X.$$

7. Write the applications of F-test. (2)

8. The normal distribution is a limiting form of binomial distribution if (3)
 - a) $n \rightarrow \infty, p \rightarrow 0$ b) $n \rightarrow 0, p \rightarrow q$ c) $n \rightarrow \infty, p \rightarrow n$ d) $n \rightarrow \infty$ and neither p or q is small.

9. Write normal equations to fit the straight line $y = a + bx$ using method least squares. (2)

10. Write the equations of lines of regression y on x and x on y . (3)

PART – B (50 Marks)

11.a) If $f(z)$ is regular function of z then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2. \quad (5)$$

b) State and prove Cauchy's integral theorem. (5)

12.a) Define Bilinear transformation. Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = 0, 1, \infty$. (5)

b) Show that
$$\int_0^{2\pi} \frac{\cos 2\theta}{1 - 2a \cos \theta + a^2} d\theta = \frac{2\pi a^2}{1 - a^2}, (a^2 < 1). \quad (5)$$

13.a) Find the mean and variance of the uniform probability distribution given by (5)

$$f(x) = \frac{1}{n} \text{ for } x = 1, 2, \dots, n.$$

b) For the following probability distribution (5)

x :	-3	-2	-1	0	1	2	3
P(x) :	0.001	0.01	0.1	k	0.1	0.01	0.001

Find k and $E(x^2 + 2x + 5)$.

14.a) Find the mean and variance of Gamma distribution. (5)

b) The nicotine contents in milligrams in two samples of tobacco were found to be as follows: (5)

Sample A :	24	27	26	21	25	-
Sample B :	27	30	28	31	22	36

Can it be said two samples came from same normal population. Test at 5% level of significance.

15.a) Fit a parabola $y = a + bx + cx^2$ to the following data (5)

x :	2	4	6	8	10
y :	3.07	12.85	31.47	57.38	91.29

b) Find the angle between the two regression lines. (5)

16.a) Find the analytic function whose imaginary part is $\frac{2 \sin x \sin y}{\cos 2x + \cosh 2y}$ (5)

b) Find the correlation coefficient between x and y for the given values. Find also the two regression lines. (5)

x :	1	2	3	4	5	6	7	8	9	10
y :	10	12	16	28	25	36	41	49	40	50

17.a) Expand $f(z) = \frac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. (5)

b) For the following Poisson distribution data, test its goodness of fit at level of significance 0.05. (5)

x :	0	1	2	3	4
y :	419	352	154	56	19