CS/B.Tech(OLD)/SEM-2/M-201/2013

2013
MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

    $$10 \times 1 = 10$$

    i) $\dfrac{1}{1-D}\, x^2 =$

        a) $x^2 + 2x + 1$ b) $x^2 + 2x$

        c) $x^2 - 2x + 1$ d) $x^2 + 2x + 2.$

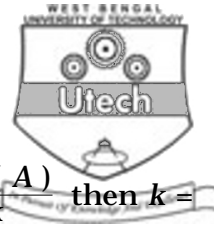   ii) The value of $\begin{bmatrix} 1 & 1 & -ac & bc \\ 1 & 1 & +ad & bd \\ 1 & 1 & +ae & be \end{bmatrix} =$

        a) 1   b) *abcd*

        c) 0   d) none of these.

iii) If $A = \begin{bmatrix} 3 & -2 & 4 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \dfrac{adj\,(A)}{k}$ then $k =$

    a)   15   b)   16

    c)   0   d)   1.

iv) The eigenvalues of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

    a)   1, 1   b)   6, 1

    c)   1, 0   d)   0, 6.

v) If A is an orthogonal matrix then det $(A) =$

    a)   1   b) -1

    c) ± 1 d) 0.

vi) Integrating factor of $x\,\dfrac{dy}{dx} + y = \log x$ is

    a) e^x b) x

    c) $\log x$ d)   none of these.

vii) Order and degree of the differential equation $\dfrac{d^2 y}{dx^2} = \sqrt{1 + \dfrac{dy}{dx}}$ are

a) 2, 2

b) 2, 1

c) 1, 2

d) 1, 1.

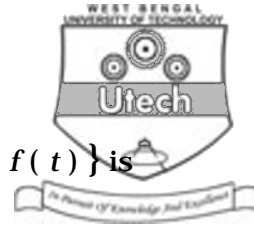viii) The Wronskian of the functions $\cos 2x$ and $\sin 2x$ is

a) 1

b) 2

c) – 2

d) none of these.

ix) The vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0), (0, 1, a, b) are linearly independent if

a) $a \neq 0$, $b \neq 2$

b) $a \neq 2$, $b \neq 0$

c) $a \neq 0$, $b \neq -2$

d) $a \neq -2$, $b \neq 0$.

x) $T : R^2$ is defined by $T(x, y) = (2x - y, x + y)$ then kernel of T is

a) $\{(1, 2)\}$

b) $\{(1, -1)\}$

c) $\{(0, 0)\}$

d) $\{(1, 2)\}, \{(1, -1)\}$.

xi) If $L\{f(t)\} = \tan^{-1}\left(\dfrac{1}{p}\right)$ then $L\{t, f(t)\}$ is

a) $\tan^{-1}\left(\dfrac{1}{p^2}\right)$

b) $\dfrac{1}{1 + p^2}$

c) $\dfrac{1}{1 + p}$

d) $\tan^{-1}\left(\dfrac{2}{\pi p}\right)$.

xii) $(\Delta - \nabla)\, x^2$ is equal to

a) h^2

b) $-2h^2$

c) $2h^2$

d) none of these,

where h is equal interval.

xiii) If E_a is the absolute error in a numerical calculation whose true and approximate values are X_t and X_a then the relative error is given by

a) $\left|\dfrac{E_a}{X_a}\right|$

b) $\left|\dfrac{E_a}{X_t}\right|$

c) $\left|\dfrac{E_a}{X_t - X_a}\right|$

d) $\left|\, E_a\,\right|$.

GROUP – B
(Short Answer Type Questions)
Answer any *three* of the following. $3 \times 5 = 15$

2. Examine whether the transformation $T : R^2 \to R^2$ defined by $T(x, y) = (2x - y, x)$ is linear or not.

3. Apply convolution theorem to find Inverse Laplace Transform of $\dfrac{s}{\left(s^2 + 9\right)^2}$.

4. Prove (without expanding) that

$$\begin{vmatrix} 1 + a & 1 & 1 & 1 \\ 1 & 1 + b & 1 & 1 \\ 1 & 1 & 1 + c & 1 \\ 1 & 1 & 1 & 1 + d \end{vmatrix} = abcd \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} \right).$$

5. If $P_n(x)$ is the Legendre's polynomial or Legendre's function of 1st kind then prove the following :

   a) $(2n + 1) xP_n = (n + 1) P_{n+1} + nP_{n-1}$

   b) $(2n + 1) P_n = P'_{n+1} - P'_{n-1}$ where P'_{n+1} denotes the derivative of P_{n+1}

6. Find the inverse of the matrix $A = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{bmatrix}$

   and hence solve the system : $2x - 2y + 4z = -4$

   $$x + z = 0$$

   $$4z - y = 2$$

7. Evaluate $\displaystyle\int_0^1 \frac{dx}{1 + x}$ using Trapezoidal Rule, taking four equal sub-intervals.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve the following differential equation by Variation of Parameter Method :

$$\left(D^2 + 1 \right) y = \sec x \tan x$$

   b) Solve the following system using Cramer's rule :

$$x + y + z = 1$$

$$ax + by + cz = r$$

$$a^2 x + b^2 y + c^2 z = r^2$$

   where $a \neq b \neq c$

   c) Find the missing data from the following table :

x :	$- 2$	$- 1$	0	1	2
y :	6	0	?	0	6

9. a) A linear transformation $T : R^3 \rightarrow R^2$ is defined by $T (x, y, z) = (x + y, x - z)$. Find the Rank and Nullity.

   b) From the following table, construct the difference table and compute $f (19)$ by Newton's Backward Interpolation formula.

x :	0	5	10	15	20
$f (x)$:	1·0	1·6	3·8	8·2	15·4

c) Define the rank of a matrix. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 6 & 7 & 8 \\ 2 & 6 & 9 & 12 & 15 \\ 4 & 8 & 12 & 14 & 16 \end{pmatrix}.$$

10. a) Show that $(3, 1, -2)$, $(2, 1, 4)$, $(1, -1, 2)$ form a basis of R^3.

b) Find the Laplace Transform of $f (t) = \sin t$, $0 < t < \pi$

$$= 0 \quad , \quad t > \pi$$

c) Apply Simpson's $\frac{1}{3}$ rd rule to evaluate $\displaystyle\int_0^6 \frac{1}{(1 + x)^2} \, dx$

taking six equal sub-intervals from $[0, 6]$ and correct up to three decimal places.

11. a) Sove $\left(D^3 - 2D^2 + D - 2 \right) y = e^x + e^{-3x}$ where $D = \dfrac{d}{dx}$.

b) Solve : $\dfrac{dx}{dt} + 2x - 3y = t$

$$\dfrac{dy}{dt} - 3x + 2y = e^{2t}$$

c) Find the general solution and singular solution of

$$y = px + \sin^{-1} p \text{ where } p = \dfrac{dy}{dx}.$$

12. a) Solve the following Cauchy-Euler homogeneous differential equation :

$$\left(x^2 D^2 - 3xD + 4 \right) y = x^2 \text{ given that } y (1) = 1,$$
$$y' (1) = 0 \text{ where } D = \dfrac{d}{dx}.$$

b)   Solvve : $x \dfrac{dy}{dx} + y = y^2 x^3 \cos x.$

c)   Compute $f(0{\cdot}5)$ and $f(0{\cdot}9)$ from the following table :

$x :$	0	1	2	3
$f(x) :$	1	2	11	34

13. a)   Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}.$$

Hence diagonalise the above matrix.

b)   If $P_n(x)$ is Legendre's Polynomial then prove that

$$\int_0^6 P_n(x) P_m(x) \, dx = 0 \qquad , \quad m \neq n$$

$$= \dfrac{2}{2n+1} \quad , \quad m = n$$

OR

Find the Bessel's function, $J_n(x)$ of 1st kind from the Bessel's equation $x^2 \dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} + \left(x^2 - n^2 \right) y = 0.$

Hence prove that $J_{-\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x.$