Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.Sc(H)/BT/SEM-2/BMT-204/2013

2013

BIO-MATHEMATICS - II

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) In Rolle's theorem $f'(x)$ should exist in

a) open interval

b) closed interval

c) semi-open interval

d) none of these.

ii) The eigenvalues of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

a) 6, 1

b) – 6, 1

c) – 6, – 1

d) 6, – 1.

iii) The order of the differential equation $\left\{ 1 + \frac{d^2y}{dx^2} \right\}^{\frac{1}{2}} = x^2$ is

a) 1

b) 2

c) 3

d) $\frac{1}{2}$.

iv) The complementary function of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0 \text{ is}$$

a) $Ae^{3x} + Be^{-3x}$

b) $(A + Bx)e^{-3x}$

c) $(A + Bx)e^{3x}$

d) none of these.

v) The value of $\Gamma\left(\frac{5}{2}\right)$ is

a) $\frac{1}{4}\sqrt{\pi}$

b) $\frac{3}{4}\sqrt{\pi}$

c) $\frac{1}{2}\sqrt{\pi}$

d) $\sqrt{\pi}$.

vi) The limit of the sequence $\left\{ \frac{2n}{n + 3\sqrt{n}}, n \in N \right\}$ is

a) $\frac{2}{3}$

b) 3

c) 2

d) $\frac{3}{2}$.

vii) Let $f : R \to R$ be defined by $f(x) = \frac{1}{3}x$, $x \in R$ and $g : R \to R$ be defined by $g(x) = 9x^2$, then $(g\,o\,f)(x)$ is

a) $\frac{1}{3}x^2$

b) $9x^2$

c) x^2

d) $\frac{1}{9}x^2$.

viii) The value of λ for which the vector $\lambda (6i + 2j - 3k)$ may be of unit length is

a) $\pm \frac{1}{11}$

b) $\pm \frac{1}{5}$

c) $\pm \frac{1}{7}$

d) $\pm \frac{1}{3}$.

ix) The values of λ and μ, for which the vectors $-3i + 4j + \lambda k$ and $\mu i + 8j + 6k$ are collinear are

a) $\lambda = 3, \mu = 6$

b) $\lambda = -3, \mu = -6$

c) $\lambda = -3, \mu = 6$

d) $\lambda = 3, \mu = -6$.

x) The straight line $\frac{x-5}{2} = \frac{y+2}{-2} = \frac{z-3}{2}$ meets the xy plane at

a) (1, 2, 0)

b) (– 1, 2, 0)

c) (1, – 2, 0)

d) (2, 1, 0).

xi) The direction ratio of the straight line

$$2x + y - 3z - 2 = 0 = 3x + 2y + 5z + 7 \text{ is}$$

a) $-11, 19, 1$

b) $11, 19, 1$

c) $-11, -19, -1$

d) $11, -19, 1$.

xii) The series $\sum \frac{1}{\sqrt{n^3}}$ is

a) convergent

b) divergent

c) oscillatory

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the the equation of the straigght line passing through the point (1, – 2, 3) and perpendicular to the plane $2x + y + 3z = 4$.

3. Define a group. Show that the set $G = \{ 1, \omega, \omega^2 \}$ form a group with respect to multiplication, where ω is the cube root of unity.

4. Find the the rank of the matrix $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

5. Find the equations of the straight line passing through the point $(-1, 1, -3)$ and perpendicular to the straight line $\frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{-4}$.

6. Position vectors of P and Q referred to the origin o are $(-i + 2j + k)$ and $(-3i + 5j + 2k)$. Find the scalar area of the triangle OPQ.

7. Show that the mapping $f : N \to N$, defined by $f(x) = x + 1$, where N is the set of all natural numbers, is injective but not surjective.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Let $G = \{ (a, b) \in Q \times Q;\ a \neq o \}$, where Q is the set of all rational numbers. Prove that (G, o) is a non-commutative group, where 'o' is defined by $(a, b)\ o\ (c, d) = (ac, ad + b)$, for $(a, b), (c, d)$ in G.

   b) Prove that a group (G, o) is Abelian if and only if $(a\ o\ b)^{-1} = a^{-1}\ o\ b^{-1}$ for all $a, b \in G$.

   c) Let $f : R \to R$ be defined by $f(x) = 3x,\ x \in R$ and $g : R \to R$ be defined by $g(x) = \frac{x}{3},\ x \in R$. Find $g\ o\ f$ and $f\ o\ g$ and hence show that $f\ o\ g = g\ o\ f$.

9. a) In the mean value theorem

$f(b) - f(a) = (b - a) f'(c)$, $a < c < b$, find c if $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \frac{1}{2}$.

b) Prove that every convergent sequence $\{x_n)$ is bounded. Give an example to show that the converse is not.

c) If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three vectors such that $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{O}$ and $|\vec{\alpha}| = 3$, $|\vec{\beta}| = 5$ and $|\vec{\gamma}| = 7$, find the angle between $\vec{\alpha}$ and $\vec{\beta}$.

10. a) Find the values of b and c for which the straight line $\frac{x-1}{2} = \frac{y-2}{7} = \frac{z+3}{3}$ lies on the plane $9x + by + cz = 30$.

b) Show that the straight lines $x = nz + a$, $y = mz + b$ and $x = z + 1$, $y = z + 2$ will be coplanar, if

$(a-1)(m-1) = (b-2)(n-1)$.

c) If $\vec{e_1}$ and $\vec{e_2}$ be two unit vectors and θ be the angle between them, then show that $2 \sin \frac{\theta}{2} = | e_1 - e_2 |$.

11. a) Test the convergence of the series
$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots$

b) Test the convergence of the seires $\sum \left(\frac{n}{n+1} \right)^{n^2}$.

c) Using the definition of the limit of a sequence, show that the limit of the sequence $\{ S_n \}$, where $S_n = \frac{2n}{n+3}$ is 2.

12. a) Define Gamma function and use it to evaluate

$$\int_0^{\infty} x^9 e^{-x^2} \, dx$$

b) Solve any *two* of the following :

i) $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^{2x}$

ii) $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 5y = 10 \sin x$

iii) $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x$

iv) $l \frac{d^2 \theta}{dt^2} + g \theta = 0, \; \theta = \alpha$ and $\frac{d\theta}{dt} = 0$ when $t = 0$.