

Name : .....  
Roll No. : .....  
Invigilator's Signature : .....

**CS/B.Sc(H)/BT/SEM-2/BMT-204/2013**  
**2013**  
**BIO-MATHEMATICS - II**

Time Allotted : 3 Hours Full Marks : 70

*The figures in the margin indicate full marks.*  
*Candidates are required to give their answers in their own words*  
*as far as practicable.*

**GROUP - A**  
**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :  
10 × 1 = 10

- i) In Rolle's theorem  $f'(x)$  should exist in
  - a) open interval
  - b) closed interval
  - c) semi-open interval
  - d) none of these.



ii) The eigenvalues of the matrix  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$  are

- a) 6, 1
- b) -6, 1
- c) -6, -1
- d) 6, -1.

iii) The order of the differential equation  $\left\{ 1 + \frac{d^2y}{dx^2} \right\}^{\frac{1}{2}} = x^2$  is

- a) 1
- b) 2
- c) 3
- d)  $\frac{1}{2}$ .

iv) The complementary function of the differential equation

$$\frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 9y = 0 \text{ is}$$

- a)  $Ae^{3x} + Be^{-3x}$
- b)  $(A + Bx) e^{-3x}$
- c)  $(A + Bx) e^{3x}$
- d) none of these.



v) The value of  $\Gamma\left(\frac{5}{2}\right)$  is

a)  $\frac{1}{4}\sqrt{\pi}$

b)  $\frac{3}{4}\sqrt{\pi}$

c)  $\frac{1}{2}\sqrt{\pi}$

d)  $\sqrt{\pi}$ .

vi) The limit of the sequence  $\left\{\frac{2n}{n+3\sqrt{n}}, n \in N\right\}$  is

a)  $\frac{2}{3}$

b) 3

c) 2

d)  $\frac{3}{2}$ .

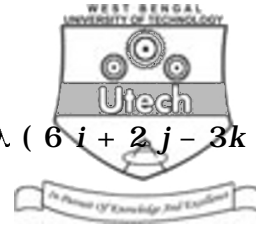
vii) Let  $f : R \rightarrow R$  be defined by  $f(x) = \frac{1}{3}x$ ,  $x \in R$  and  $g : R \rightarrow R$  be defined by  $g(x) = 9x^2$ , then  $(g \circ f)(x)$  is

a)  $\frac{1}{3}x^2$

b)  $9x^2$

c)  $x^2$

d)  $\frac{1}{9}x^2$ .



viii) The value of  $\lambda$  for which the vector  $\lambda ( 6 i + 2 j - 3k )$  may be of unit length is

a)  $\pm \frac{1}{11}$

b)  $\pm \frac{1}{5}$

c)  $\pm \frac{1}{7}$

d)  $\pm \frac{1}{3}$ .

ix) The values of  $\lambda$  and  $\mu$ , for which the vectors  $- 3i + 4j + \lambda k$  and  $\mu i + 8j + 6k$  are collinear are

a)  $\lambda = 3, \mu = 6$

b)  $\lambda = - 3, \mu = - 6$

c)  $\lambda = - 3, \mu = 6$

d)  $\lambda = 3, \mu = - 6$ .

x) The straight line  $\frac{x - 5}{2} = \frac{y + 2}{- 2} = \frac{z - 3}{2}$  meets the xy plane at

a)  $( 1, 2, 0 )$

b)  $( - 1, 2, 0 )$

c)  $( 1, - 2, 0 )$

d)  $( 2, 1, 0 )$ .



xi) The direction ratio of the straight line

$$2x + y - 3z - 2 = 0 = 3x + 2y + 5z + 7$$

- a) - 11, 19, 1
  - b) 11, 19, 1
  - c) - 11, - 19, - 1
  - d) 11, - 19, 1.
- xii) The series  $\sum \frac{1}{\sqrt{n}^3}$  is

- a) convergent
- b) divergent
- c) oscillatory
- d) none of these.

**GROUP - B**

**( Short Answer Type Questions )**

Answer any *three* of the following.  $3 \times 5 = 15$

2. Find the the equation of the straight line passing through the point ( 1, - 2, 3 ) and perpendicular to the plane  $2x + y + 3z = 4$ .
3. Define a group. Show that the set  $G = \{ 1, \omega, \omega^2 \}$  form a group with respect to multiplication, where  $\omega$  is the cube root of unity.



4. Find the the rank of the matrix  $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

5. Find the equations of the straight line passing through the point  $( - 1, 1, - 3 )$  and perpendicular to the straight line  $\frac{x - 3}{- 2} = \frac{y + 1}{3} = \frac{z - 2}{- 4}$ .
6. Position vectors of  $P$  and  $Q$  referred to the origin  $o$  are  $( - i + 2j + k )$  and  $( - 3i + 5j + 2 k )$ . Find the scalar area of the triangle  $OPQ$ .
7. Show that the mapping  $f : N \rightarrow N$ , defined by  $f ( x ) = x + 1$ , where  $N$  is the set of all natural numbers, is injective but not surjective.

**GROUP - C**

**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) Let  $G = \{ ( a, b ) \in Q \times Q; a \neq 0 \}$ , where  $Q$  is the set of all rational numbers. Prove that  $( G, o )$  is a non-commutative group, where ' $o$ ' is defined by  $( a, b ) o ( c, d ) = ( ac, ad + b )$ , for  $( a, b ), ( c, d )$  in  $G$ .
- b) Prove that a group  $( G, o )$  is Abelian if and only if  $( a o b )^{-1} = a^{-1} o b^{-1}$  for all  $a, b \in G$ .
- c) Let  $f : R \rightarrow R$  be defined by  $f ( x ) = 3x, x \in R$  and  $g : R \rightarrow R$  be defined by  $g ( x ) = \frac{x}{3}, x \in R$ . Find  $g o f$  and  $f o g$  and hence show that  $f o g = g o f$ .



9. a) In the mean value theorem

$f(b) - f(a) = (b - a) f'(c)$ ,  $a < c < b$ , find  $c$  if  $f(x) = x(x - 1)(x - 2)$ ,  $a = 0$ ,  $b = \frac{1}{2}$ .

- b) Prove that every convergent sequence  $\{x_n\}$  is bounded.

Give an example to show that the converse is not.

- c) If  $\vec{\alpha}$ ,  $\vec{\beta}$ ,  $\vec{\gamma}$  are three vectors such that  $\vec{\alpha} + \vec{\beta} + \vec{\gamma} = \vec{O}$  and  $|\vec{\alpha}| = 3$ ,  $|\vec{\beta}| = 5$  and  $|\vec{\gamma}| = 7$ , find the angle between  $\vec{\alpha}$  and  $\vec{\beta}$ .

10. a) Find the values of  $b$  and  $c$  for which the straight line  $\frac{x-1}{2} = \frac{y-2}{7} = \frac{z+3}{3}$  lies on the plane  $9x + by + cz = 30$ .

- b) Show that the straight lines  $x = nz + a$ ,  $y = mz + b$  and  $x = z + 1$ ,  $y = z + 2$  will be coplanar, if

$$(a - 1)(m - 1) = (b - 2)(n - 1).$$

- c) If  $\vec{e}_1$  and  $\vec{e}_2$  be two unit vectors and  $\theta$  be the angle between them, then show that  $2 \sin \frac{\theta}{2} = |\vec{e}_1 - \vec{e}_2|$ .

11. a) Test the convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$$

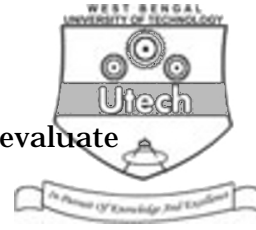
- b) Test the convergence of the series  $\sum \left( \frac{n}{n+1} \right)^{n^2}$ .

- c) Using the definition of the limit of a sequence, show that the limit of the sequence  $\{S_n\}$ , where  $S_n = \frac{2n}{n+3}$  is 2.

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12. a) Define Gamma function and use it to evaluate

$$\int_0^{\infty} x^9 e^{-x^2} dx$$



b) Solve any *two* of the following :

i)  $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = e^{2x}$

ii)  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 5y = 10 \sin x$

iii)  $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 2 \log x$

iv)  $l \frac{d^2\theta}{dt^2} + g\theta = 0, \theta = \alpha \text{ and } \frac{d\theta}{dt} = 0 \text{ when } t = 0.$

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