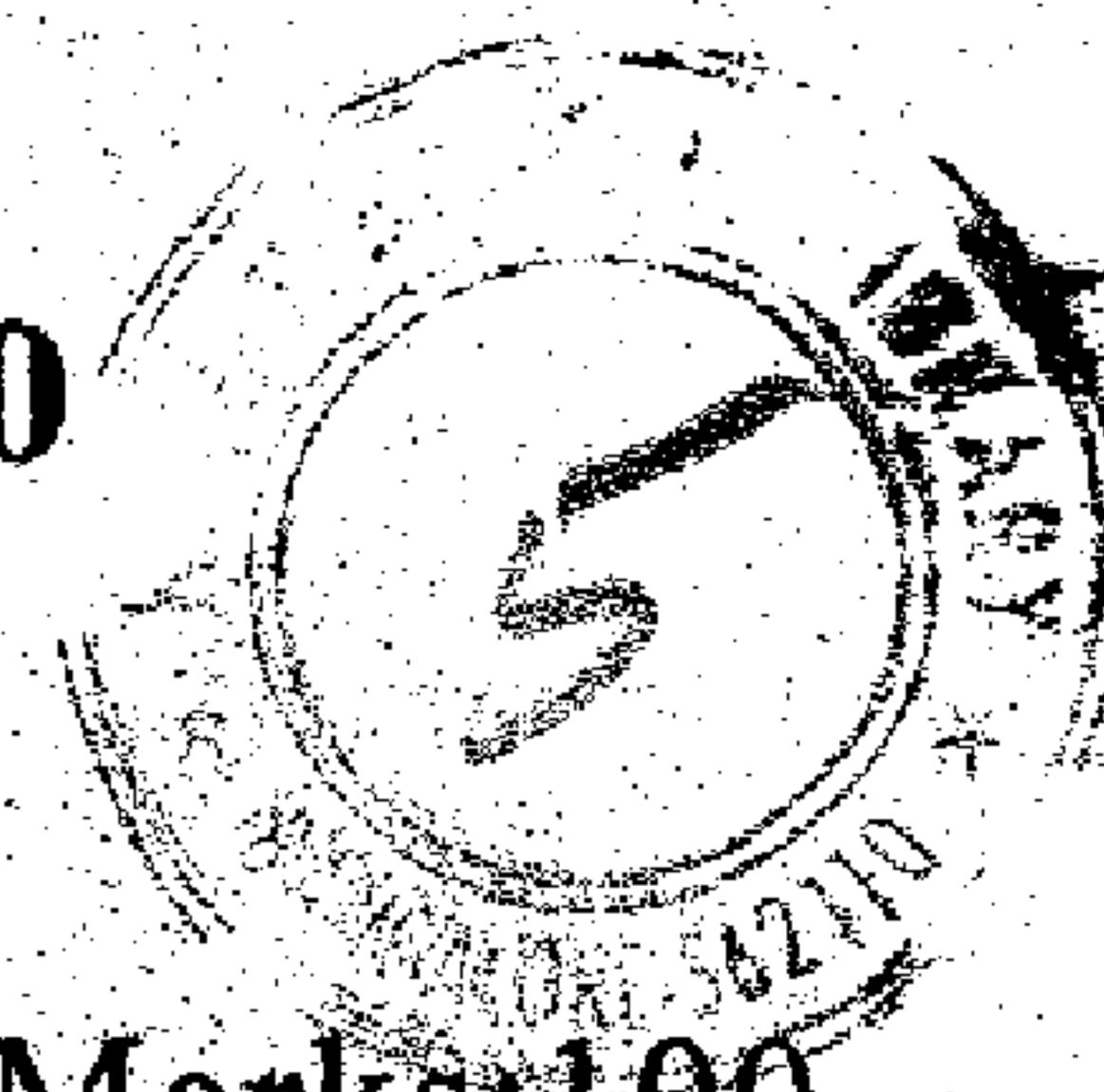


Third Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - III



Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART - A

- 1 a. Obtain Fourier series for the function
- $f(x)$
- given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi \end{cases}$$

and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

(07 Marks)

- b. Find the half-range cosine series for the function
- $f(x) = (x-1)^2$
- in the interval
- $0 < x < 1$
- .

(07 Marks)

- c. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of
- y
- as given in the following table.

(06 Marks)

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

- 2 a. Find the Fourier transform of
- $f(x) = \begin{cases} 1-x^2, & |x| \leq \pi \\ 0, & |x| > \pi \end{cases}$

Hence evaluate $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cdot \cos\left(\frac{x}{2}\right) dx$

(07 Marks)

- b. Find the Fourier cosine transform of
- e^{-x^2}

(07 Marks)

- c. Find the Fourier sine transform of
- $e^{-|x|}$
- . Hence show that
- $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$
- ,
- $m > 0$
- .

(06 Marks)

- 3 a. Form the partial differential equation by eliminating the arbitrary functions
- f
- and
- g
- from the relation
- $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

(07 Marks)

- b. Solve
- $\frac{\partial^3 t}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$

(07 Marks)

- c. Solve
- $(mz - ny) \frac{\partial z}{\partial x} + (nx - lz) \frac{\partial z}{\partial y} = (ly - mx)$

(06 Marks)

- 4 a. Derive one dimensional heat equation by taking
- $u(x, t)$
- as the temperature,
- x
- is the distance and
- t
- is the time. (Write the figure also.)

(07 Marks)

- b. Obtain the D'Alembert's solution of the wave equation
- $u_{tt} = C^2 u_{xx}$
- , subject to the condition
- $u(x, 0) = f(x)$
- and
- $\frac{\partial u}{\partial t}(x, 0) = 0$
- .

(07 Marks)

- c. Obtain the various solutions of the Laplace's equation
- $u_{xx} + u_{yy} = 0$
- , by the method of separation of variables.

(06 Marks)

PART – B

- 5 a. Complete the real root of the equation $x \log_{10} x - 1.2 = 0$ by Regula-Falsi method, correct to four decimal places. (07 Marks)

- b. Solve the system of equations using Gauss-Jordan method:

$$2x + 5y + 7z = 52$$

$$2x + y - z = 0$$

$$x + y + z = 9$$

(07 Marks)

- c. Using the power method, find the largest Eigen value and the corresponding Eigen vector of

$$\text{the matrix } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(06 Marks)

- 6 a. The area of a circle (A) corresponding to diameter (D) is given below: (07 Marks)

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 105 using an appropriate interpolation formula.

- b. Use Newton's divided difference formula to find $f(9)$, given the data, (07 Marks)

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

- c. Evaluate $\int_4^{5.2} \log_e x \, dx$ using Weddle's rule, taking 7 ordinates. (06 Marks)

- 7 a. Derive Euler's equation in the form

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$$

(07 Marks)

- b. Find the curves on which the functional $\int_0^1 [(y')^2 + 12xy] dx$, with $y(0) = 0$ and $y(1) = 1$ can be extremised. (07 Marks)

- c. Find the geodesics on a surface given that the arc length on the surface is

$$S = \int_{x_1}^{x_2} \sqrt{x[1+(y')^2]} dx$$

(06 Marks)

- 8 a. Find the Z-transforms of the following :

i) $\cosh n\theta$ ii) $(n+1)^2$

(07 Marks)

- b. Find the inverse z-transform of $\frac{z}{(z-1)(z-2)}$. (07 Marks)

- c. Find the response of the system $y_{n+2} - 5y_{n+1} + 6y_n = u_n$, with $y_0 = 0$, $y_1 = 1$ and $u_n = 1$ for $n = 0, 1, 2, 3, \dots$ by z-transform method. (06 Marks)

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