An Improved Backward/Forward Sweep Load Flow Algorithm for Radial Distribution Systems

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***Abstract*—This letter presents an improved backward/ forward sweep algorithm for three-phase load-flow analysis of radial distribution systems. In the backward sweep, Kirchhoff's Current Law and Kirchhoff's Voltage Law are used to calculate the upstream bus voltage of each line or a transformer branch. Then, the linear proportional principle is adopted to find the ratios of the real and imaginary components of the specified voltage to those of the calculated voltage at the substation bus. In the forward sweep, the voltage at each downstream bus is then updated by the real and imaginary components of the calculated bus voltage multiplying with the corresponding ratio. The procedure stops after the mismatch of the calculated and the specified voltages at the substation is less than a convergence tolerance. The proposed algorithm is tested with three IEEE benchmark distribution systems. Results show that the algorithm is accurate and computationally efficient in comparing with two other commonly used methods.**

***Index Terms*—Backward/forward sweep, ladder network theory, load flow analysis.**

I. INTRODUCTION

IN the past, many approaches for distribution system load-flow analyses have been developed [1]–[3]. Among these approaches, the ladder network theory and the backward/forward (BW/FW) sweep methods are commonly used due to their computational efficiencies and solution accuracies. In this letter, the authors propose an improved BW/FW sweep approach for radial distribution system load-flow analysis. The proposed method includes two steps: the backward sweep and the decomposed forward sweep. In the backward sweep, Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) are used to find the calculated voltage for each upstream bus of a line or a transformer branch. In the decomposed forward sweep, the linear proportional principle is employed to update the voltage at each downstream bus. After performing each backward sweep, the mismatch of the calculated and specified voltages at the substation is checked. The solution algorithm repeats until the convergence tolerance of the substation voltage is satisfied.

II. LINEAR PROPORTIONAL PRINCIPLE OF A PASSIVE NETWORK

A. *Distribution Line and Transformer Models*

For a distribution line model containing series impedance or a distribution transformer with different connection types, voltage and current relations between sending and receiving ends can be described by (1) and (2), respectively [2]

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R \tag{1}$$

$$\mathbf{I}_S = \mathbf{D}\mathbf{I}_R \tag{2}$$

where $\mathbf{V}_S$ is the vector of three-phase voltages of the sending end, $\mathbf{V}_R$ is the vector of three-phase voltages of the receiving end, $\mathbf{I}_S$ is the vector of three-phase line currents of the sending end, and $\mathbf{I}_R$ is the vector of three-phase line currents of the receiving end. By substituting the entities of the corresponding matrix into $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in (1) and (2), then (3) and (4) can be obtained in terms of their real and imaginary components. It is noted that, for a distribution feeder, $\mathbf{A}$ and $\mathbf{D}$ are identity matrices; for a transformer, both matrices are with real-number entries which are dependent on transformer connection types

$$\begin{aligned}\begin{bmatrix} V_A^r + jV_A^i \\ V_B^r + jV_B^i \\ V_C^r + jV_C^i \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} V_a^r + jV_a^i \\ V_b^r + jV_b^i \\ V_c^r + jV_c^i \end{bmatrix} \\ &+ \begin{bmatrix} r_{11} + jx_{11} & r_{12} + jx_{12} & r_{13} + jx_{13} \\ r_{21} + jx_{21} & r_{22} + jx_{22} & r_{23} + jx_{23} \\ r_{31} + jx_{31} & r_{32} + jx_{32} & r_{33} + jx_{33} \end{bmatrix} \\ &\times \begin{bmatrix} I_a^r + jI_a^i \\ I_b^r + jI_b^i \\ I_c^r + jI_c^i \end{bmatrix} \end{aligned} \tag{3}$$

$$\begin{bmatrix} I_A^r + jI_A^i \\ I_B^r + jI_B^i \\ I_C^r + jI_C^i \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} I_a^r + jI_a^i \\ I_b^r + jI_b^i \\ I_c^r + jI_c^i \end{bmatrix}. \tag{4}$$

By observing (3), it is known that the phase voltage at the upstream bus of each feeder branch is determined by the real and imaginary components of the line current, the line impedance, and the downstream bus voltage. Therefore, (3) and (4) can be expressed by (5) and (6) corresponding to real and imaginary parts, respectively

$$\mathbf{V}_S^r = \mathbf{A}\mathbf{V}_R^r + \mathbf{B}^r\mathbf{I}_R^r - \mathbf{B}^i\mathbf{I}_R^i, \mathbf{V}_S^i = \mathbf{A}\mathbf{V}_R^i + \mathbf{B}^i\mathbf{I}_R^r + \mathbf{B}^r\mathbf{I}_R^i \tag{5}$$

$$\mathbf{I}_S^r = \mathbf{D}\mathbf{I}_R^r, \;\; \mathbf{I}_S^i = \mathbf{D}\mathbf{I}_R^i. \tag{6}$$

B. *Linear Proportional Principle*

Consider an M-bus resistive network shown in Fig. 1 with a specified constant voltage, V_S, at bus 1. Before employing the backward sweep to calculate each bus voltage, an initial flat voltage profile at all other busses is firstly assumed. During the backward sweep, the current flowing through the end bus M and the line current between busses M and $M-1$ are obtained

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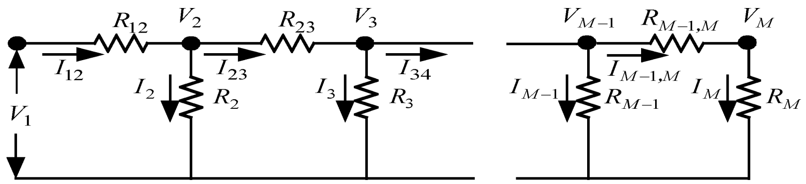

Fig. 1. M-bus resistive network.

TABLE I
MAXIMUM ABSOLUTE ERRORS OF BUS VOLTAGE MAGNITUDE AND PHASE ANGLE FOR THE TEST SYSTEMS

		Max. Error of Voltage Magnitude (Volt)			Max. Error of Voltage Phase Angle (Degree)		
Phase		A	B	C	A	B	C
Proposed vs. Ladder Network	13-bus	7.81	3.60	7.97	0.13	0.03	0.14
	37-bus	1.93	2.66	1.98	0.02	0.02	0.01
	123-bus	4.41	3.18	4.75	0.11	0.06	0.03
Proposed vs. BW/FW Sweep	13-bus	7.33	3.41	7.60	0.12	0.03	0.14
	37-bus	1.92	2.66	1.96	0.02	0.02	0.02
	123-bus	4.25	3.15	5.27	0.11	0.06	0.03

according to $I_M = V_M/R_M$ and $I_{M-1,M} = I_M$, respectively. The voltage at any other bus i becomes $V_i = I_{i,i+1} \times R_{i,i+1} + V_{i+1}$, where $I_i = V_i/R_i$ and $I_{i,i+1} = I_{i+1} + I_{i+1,i+2}$. Follow this procedure the voltage at bus 1, V_1 can be calculated. Since there is a specified voltage at bus 1, the ratio of the specified voltage to the calculated voltage becomes

$$\gamma = \frac{V_S}{V_1} \tag{7}$$

Then the final solution of each bus voltage can be determined by the calculated voltage at each bus multiplying with the ratio found in (7) based on the linear proportional principle.

III. SOLUTION ALGORITHM

In the proposed method, the backward sweep is based on KCL for finding each branch current and on KVL for calculating each bus voltage, as described in Section II-B. All currents and voltages are calculated in phasor forms during the backward sweep. For the forward sweep, the feeder network at each phase is decomposed into two independent resistive networks representing the real and the imaginary components according to Fig. 1, respectively. Therefore, the linear proportional principle is employed in the forward sweep for each decomposed network and to obtain the new three-phase voltage at each bus. The iteration repeats and terminates until the convergence criterion at the substation bus voltage is met.

Provided that the three-phase balanced substation voltages are specified and the initial flat voltages at all other busses are set. Listed below summarize major steps of the proposed solution algorithm.

1) Sort busses according to their distances to the substation.
2) Start from the end bus and perform backward sweep by employing KCL/KVL to find the calculated voltage of each upstream bus and the line current as described in Section II-B. The procedure proceeds until the substation voltage is calculated. Note that, if a junction bus is reached, the calculated voltage is recorded; the number of calculated voltages is the same as the number of connected braches (i.e., feeders/laterals). The upstream bus voltage of the junction bus is calculated based on the old junction bus voltage and the calculated branch current between the two busses.
3) Check if the mismatch of the specified and calculated voltages at the substation is less than the convergence tolerance. If yes, stop. Otherwise, proceed to next step.
4) The real and imaginary components of the specified substation voltages at each phase are then compare with the corresponding real and imaginary components of the calculated voltage and the six voltage ratios as that of (7) for the six decomposed networks, as shown in Fig. 1, of three phases are determined.
5) Apply the six voltage ratios determined at step 4 to perform the forward sweep by using linear proportional principle to find the updated voltage at each downstream bus of each corresponding network. The procedure for determining the proportional voltage at each downstream bus proceeds until the end bus is updated. Note that, if a junction bus is reached, the updated junction bus voltage is regarded as the source voltage and is used to compare with the corresponding component of the calculated voltage recorded at step 2 to determine a constant ratio; the calculated bus voltages of the corresponding downstream branch are updated by the new ratio accordingly.
6) Return to step 2.

IV. RESULTS

The proposed algorithm has been tested by the IEEE 13-, 37-, and 123-bus benchmark distribution systems of [4] on Pentium-IV PC with 1.6-GHz CPU. The convergence tolerance is 0.001 p.u. The feeders of the test systems possess different line configurations, transformer connections, and unbalanced spot/ distributed loads. Results obtained are also compared with the ladder network and the conventional BW/FW sweep methods [2], [3]. In Table I, it gives the maximum absolute errors of the bus voltage magnitude and phase angle for each phase by testing

TABLE II
EXECUTION TIME AND ITERATION NUMBER FOR THE TEST SYSTEMS.

Test System	Ladder Network		BW/FW Sweep		Proposed	
	Iter. No.	Time (sec)	Iter. No.	Time (sec)	Iter. No.	Time (sec)
13-bus	3	0.0782	4	0.0802	4	0.0640
37-bus	2	0.1565	3	0.2043	3	0.1540
123-bus	3	1.2680	4	1.6164	3	1.0398

the three benchmark systems. Table II shows comparisons of the execution time and the number of iterations. It is observed that the proposed method can be up to 35% more computationally efficient than the other two methods when the system size increases.

V. CONCLUSIONS

In this letter, the authors present an improved BW/FW sweep load-flow algorithm, which is suitable for three-phase radial distribution systems. The proposed method uses KVL and KCL to obtain the calculated voltage at each upstream bus in the backward sweep. Then, the linear proportional principle is employed to update the voltage at each bus in the forward sweep. The solution procedure terminates if the mismatch is less than the voltage tolerance at the substation. The proposed algorithm has been tested by three benchmark distribution systems. As shown in the results, it is concluded that proposed method is superior to the two commonly used methods in the computational efficiency, while the solution accuracy is maintained.

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