Roll No. ....

67041

MCA 1st Semester w.e.f. Dec. 2012 with new notes full and re-appear candidates Examination– December, 2013

Mathematical Foundation of Computer Science

Paper MCA-101

Time : 3 hours ***Max. Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all. **Question No. 1 is compulsory** and attempt **four** more questions by selecting **one** from each Unit. All questions carry equal marks.

1. (a) Find the domain and range of the function : $f(x) = \frac{1}{\sqrt{x-4}}$

(b) Define semi-group and coset.

(c) If p : It is cold and q: It is raining. Write simple verbal sentence which describes the following statements :

(i) $p \wedge \sim q$

(ii) $\sim p \vee \sim q$

(d) Let $A = \{1, 2, 3, 4, 5, 6\}$. Determine the truth value of the following statements :

(i) $(\forall x \in A)\ x + 4 < 8$

(ii) $(\exists x \in A)\ x + 4 = 9$

(e) Draw the Hasse diagram for the relation divisibility on the set $A = \{1, 2, 4, 5, 10, 20\}$.

(f) In the Boolean algebra (B, +, ., /), show that $(a . b. c)' = a' + b' + c'$ for all a, b, c $\in$ B.

(g) Let $\Sigma = \{0, 1\}$ be an alphabet, find Σ^3 and Σ^*.

(h) Describe the set represented by the regular expression $ab + c^*$.

UNIT – I

2. (a) Define properties of relation and show that the relation $R = \{(a, b) : a, b \in Z$ and $a + b$ is even$\}$ is an equivalence relation on the set Z of integers.

(b) Let $f(x) = \frac{ax}{x+1}, x \neq -1$. If $(fof)(x) = x$ find the value of a.

3. (a) Define group and show that the set Q^+ of positive rational numbers does not form a group for the binary operation * defined by

$$a * b = \frac{a}{b} \; \forall \, a, b \in Q^+$$

(b) Prove that the order of each sub-group of a finite group G is a divisor of the order of the group G.

UNIT - II

4. (a) Define tautology and verify that the proposition $p \wedge (p \wedge r) \Leftrightarrow (p \wedge q) \wedge r$ is a tautology.

(b) Using principle of mathematical induction show that $10^{2n-1} + 1$ is divisible by 11 for all positive integers n.

5. (a) Define modus pones and modus tollens and show that 't' is a valid conclusion from the premises : $p \Rightarrow q$, $q \Rightarrow r$, $r \Rightarrow s$, $\sim s$ and $p \vee t$

(b) Using law of algebra of propositions show that $p \Leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$

UNIT - III

6. (a) Define lattice and show that the set D_{30} of all positive factors of 30 forms a lattice with the relation divisibility.

(b) What is complemented lattice ? Show that if $(L, \wedge, \vee)$ is a complemented distributive lattice, then De Morgan's Laws $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$ holds for all a, b'∈ L.

7. (a) Let B = {1, 2, 3, 4, 6, 12} be the set of positive factor of 12. Two binary

operations '+' and '.' on B are defined as follows :

a + b = 1 cm (a, b) and a. b = gcd (a, b) for all a, b ∈ B

A unary operation '/' on B is defined as $a' = \frac{12}{a}$ for all a ∈ B.

Show that (B, +, . , /, a, 6) is a Boolean algebra.

(b) In the Boolean algebra (B, +, ., /), simplify the Boolean expression [a. (a + b) + (b/ + a).b]/.

UNIT - IV

(a) Explain regular expression and regular language. Find the language for the regular expressions $(a + b)^* (a + bb)$ and $a(a + b)^*ab$.

(b) Describe the deterministic and non-deterministic finite automaton. How deterministic finite automaton differs from non-deterministic finite automaton ?

9. (a) Compare the Moore and Mealy machine and prove that both machine have equivalent power.

(b) Construct a deterministic finite automata equivalent to $M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$ where δ is given as :

Transition function Table

State	Input	
	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
q_2	–	q_0, q_1

also draw the transition diagram of equivalent DFA.