

- N.B. :** (1) Question No. 1 is compulsory.  
(2) Attempt any four questions out of the remaining six questions.  
(3) Figures to the right indicate full marks.

1. (a) If  $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$  is analytic, find the constants a, b, c, d, e. 5
- (b) Find the Fourier series expansion for  $f(x) = |\sin x|$ , in  $(-\pi, \pi)$  5
- (c) Find the Laplace transform of  $\sin t \cdot H\left(t - \frac{\pi}{2}\right) - H\left(t - \frac{3\pi}{2}\right)$  5
- (d) If  $\{f(k)\} = \begin{cases} 4^k, & \text{for } k < 0 \\ 3^k, & \text{for } k \geq 0 \end{cases}$  find  $Z\{f(k)\}$  5
2. (a) If  $\int_0^{\infty} e^{-2t} \sin(t + \alpha) \cos(t - \alpha) dt = 3/8$  then find  $\alpha$ . 6
- (b) Find the Fourier series expansion for  $f(x) = \sqrt{1 - \cos x}$  in  $(0, 2\pi)$   
Hence deduce that  $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}$  7
- (c) Find the inverse of A if  $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & -2 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  7
3. (a) Find Laplace Transform of following  
i)  $e^{-4t} \int_0^t u \sin 3u du$  ii)  $\frac{1}{t}(1 - \cos t)$ . 6
- (b) Find non-singular matrices P & Q s.t. PAQ is in Normal form. Also find rank of A &  $A^{-1}$ .  
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$
 7
- (c) Evaluate by Green's theorem  $\int_C \bar{F} \cdot d\bar{r}$  where  $\bar{F} = -xy(xi - yj)$  and C is  
 $r = a(1 + \cos \theta)$  7
4. (a) Obtain complex form of Fourier series for the functions  $f(x) = \sin ax$  in  $(-\pi, \pi)$  6
- (b) For what value of  $\lambda$ , the following system of equations possesses a non-trivial solution? Obtain the solution for real values of  $\lambda$ .  
 $3x_1 + x_2 - \lambda x_3 = 0, 4x_1 - 2x_2 - 3x_3 = 0, 2\lambda x_1 + 4x_2 + \lambda x_3 = 0$  7
- (c) Find inverse Laplace Transform of following  
i)  $2 \tanh^{-1} s$  ii)  $\frac{s^2}{(s^2 + 1)(s^2 + 4)}$  7

5. (a) Find the orthogonal trajectory of the family of curves  $3x^2y + 2x^2 - y^3 - 2y^2 = c$  6  
 (b) Find the relation of linear dependence amongst the rows of the matrix

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}.$$

- (c) Express the function  $f(x) = \begin{cases} -e^{kx}, & \text{for } x < 0 \\ e^{-kx}, & \text{for } x > 0 \end{cases}$  as Fourier Integral.

And prove that  $\int_0^{\infty} \frac{\omega \sin \omega x}{\omega^2 + k^2} d\omega = \frac{\pi}{2} e^{-kx}$  if  $x > 0, k > 0$

6. (a) Obtain half-range cosine series for  $f(x) = x$  in  $0 < x < l$ . 6

- (b) Show that under the transformation  $w = \frac{5-4z}{4z-2}$  the circle  $|z| = 1$  in the  $z$ -plane is transformed into a circle of unity in the  $w$ -plane. Also find the center of the circle. 7  
 (c) A vector field is given by  $\vec{F} = 3x^2y \mathbf{i} + (x^3 - 2yz^2) \mathbf{j} + (3z^2 - 2y^2z) \mathbf{k}$  is irrotational. Also find  $\phi$  such that  $\vec{F} = \nabla\phi$ . Also evaluate the line integral from  $(2,1,1), (2,0,1)$ . 7

7. (a) Find inverse Z-transform of  $F(z) = \frac{z}{[z - (1/4)][z - (1/5)]}$ ,  $\frac{1}{5} < |z| < \frac{1}{4}$  6

- (b) Find the analytic function  $f(z) = u + iv$  in terms of  $z$  if  $u - v = (x - y)(x^2 + 4xy + y^2)$  7  
 (c) Using laplace transform solve the following differential equation with given condition.  $(D^2 - 3D + 2)y = 4e^{2t}$ ,  $y(0) = -3, y'(0) = 5$  7