[4261] – 107

F.E. (Semester – II) Examination, 2012
ENGINEERING MATHEMATICS – II
(2008 Pattern)

Time : 3 Hours Max. Marks : 100

Instructions: *1) Answer **three** questions from **Section I** and **three** questions from **Section II**.*
*2) Answers to the **two** Sections should be written in **separate** answer books.*
*3) Black figures to the **right** indicate **full** marks.*
*4) **Use** of electronic pocket calculator is **allowed**.*
*5) Assume suitable data, **if necessary**.*

SECTION – I

1. A) Form a differential equation whose general solution is $y = Ae^{-2x} + Be^{3x}$ where A and B are arbitary constants. **6**

   B) Solve **any two**. **10**

   i) $y\,(x^2y+e^x)\,dx - e^x\,dy = 0$

   ii) $\frac{dy}{dx} - y\tan x = y^4 \sec x$

   iii) $\frac{dy}{dx} = \frac{x+2y-3}{3x+6y-1}$.

OR

2. A) Form a differential equation whose general solution is $y = c_1x + \frac{c_2}{x}$ where c_1 and c_2 are arbitary constants. **6**

B) Solve **any two**. **10**

i) $(x^4 + y^4)\,dx - (xy^3)\,dy = 0$

ii) $\cos x \frac{dy}{dx} + y = \sin x$

iii) $(x^2y + y^4)\,dx + (2x^3 + 4xy^3)\,dy = 0$.

3. Solve **any three** : **18**

a) A body at temperature 100°C is placed in a room whose temperature is 20°C and cools down to 60°C in 5 minutes. Find its temperature after 8 minutes.

b) A voltage Ee^{-at} is applied at t = 0 to a circuit containing inductance L and resistance R. Show that current at any time t is $\frac{E}{R - aL}(e^{-at} - e^{-\frac{R}{L}t}$.

c) A bullet is fired into sand tank, its retardation is proportional to square root of its velocity $(K\sqrt{V})$. Show that the bullet will come to rest in time $\frac{2\sqrt{V}}{K}$, where V is initial velocity.

d) Find orthogonal trajectories of family of parabola $y^2 = 4ax$, where a is arbitary constant.

OR

4. Solve **any three** : **18**

a) The charge Q on the plate of a condenser of capacity 'C' charged through a resistance R by a steady voltage V satisfy the differential equation $R\frac{dQ}{dt} + \frac{Q}{C} = V$

If Q = 0 at t = 0, show that $Q = CV\left[1 - e^{-t/RC}\right]$

Find the current flowing into the plate.

b) A steam pipe 20 cm in diameter is protected with a covering 6 cm thick for which the coefficient of thermal conductivity is K = 0.0003 cal/cm.deg.sec. Find the heat lost per hour through a metre length of the pipe, if the surface of the pipe is at 200°C and outer surface of the covering is at 30°C.

c) A tank initially contains 50 litres of fresh water. Brine containing 2 gm per litre of salt flows into the tank at the rate of 2 litres per minute and the mixture kept uniform by stirring runs out at the same rate. How long will it take for the quantity of salt in the tank to increase from 40 to 80 gm.

d) A body starts moving from rest is opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2, where x and v are displacement and velocity of the particle at that instant. Show that the velocity of the particle is given by $v^2 = \frac{c}{2b^2}(1 - e^{-2bx}) - \frac{cx}{b}$.

5. A) Find Fourier series to represent the function $f(x) = \pi^2 - x^2$ in the interval $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$

Deduce that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots = \frac{\pi^2}{1^2}.$$ **9**

B) If $I_n = \int_0^{\pi/4} \sin^{2n} x \, dx$

prove that $I_n = \left(1 - \frac{1}{2n}\right) I_{n-1} - \frac{1}{n\,2^{n+1}}$ and hence find I_3. **7**

OR

6. A) Obtain the constant term and the coefficient of the first sine and cosine term in the Fourier expansion of y as given in the following table **8**

x	0	1	2	3	4	5
y	9	18	24	28	26	20

B) Evaluate :

$$\int_0^2 x\left(1-\frac{x^3}{8}\right)^{1/3} dx .$$ **4**

C) Show that

$$\int_0^1 (x \log x)^3 dx = \frac{-3}{128} .$$ **4**

SECTION – II

7. a) Trace the following curves **(any two)** : **8**

i) $r^2 = a^2 \cos 2\theta$

ii) $y^2 = \frac{a^3 x}{a^2 - x^2}, (a > 0)$

iii) $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1.$

b) Evaluate

$$\int_0^\infty \frac{\cos x}{x}\left\{e^{-ax} - e^{-bx}\right\} dx$$ using DUIS. **5**

c) Find arc length of the curve $r = \frac{2a}{1+\cos\theta}$ from $\theta = 0$ to $\theta = \frac{\pi}{2}$. **4**

OR

8. a) Trace the following curves **(any two)** : **8**

i) $r = a \sin 2\theta$

ii) $x^3 + y^3 = 3axy$ $(a>0)$

iii) $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

b) If $\alpha(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-t^2/2}\, dt$ then show that erf(x) = $\alpha(x\sqrt{2})$. **4**

c) Find arc length of the curve $x = a(\cos t + \log \tan \frac{t}{2})$, $y = a \sin t$ intercepted between $t = \pi/2$ to t. **5**

9. a) Find the equation of sphere passes through the circle

$x^2 + y^2 + z^2 – 2x + 3y – 4z + 6 = 0$;

$3x – 4y + 5z – 15 = 0$,

and cuts the sphere $x^2 + y^2 + z^2 + 2x + 4y – 6z + 11 = 0$ orthogonally. **6**

b) Find the equation of right circular cone whose vertex at origin, semivertical angle is 30° and Axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$. **6**

c) Find the equation of right circular cylinder whose axis is the line $\frac{x-1}{2} = \frac{y-3}{2} - \frac{z-5}{-1}$ and the radius is 3. **5**

OR

10. a) Find the equation of sphere which is tangential to the plane $x - 2y - 2z - 4 = 0$ at (4, 0, 0) and passes through the point (2, 2, –2). **6**

b) Find the equation of right circular cone with vertex at the point (1, –1, 1), semivertical angle is 45° and the axis whose dr's are 2, 1, –2. **6**

c) Find the equation of right circular cylinder of radius 4, whose axis passes through origin and makes constant angle with co-ordinate axes. **5**

11. Solve **any two** : **16**

a) Evaluate $\iint_R \frac{xy}{\sqrt{1-y^2}} dxdy$ over the positive quadrant of circle $x^2 + y^2 = 1$.

b) Find the total area included between the two cardiodes $r = a(1+\cos\theta)$ and $r = a(1-\cos\theta)$.

c) Find the centre of Gravity of the area enclosed by the curves $y^2 = x$ and $x + y = 2$.

OR

12. Solve **any two** : **16**

a) Evaluate $\iint_R \frac{x^2y^2}{x^2+y^2}dxdy$, where R is annulus between the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

b) Evaluate $\iiint_V \sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}}\,dxdydz$ throughout the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

c) Find the moment of inertia (M.I.) about the line $\theta = \frac{\pi}{2}$ of the area enclosed by $r = a(1-\cos\theta)$.

———————