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First Semester B.E. Degree Examination, June/July 08
Engineering Mathematics - I

Time: 3 hrs.

Max. Marks: 100

Note : Answer any FIVE full questions, choosing atleast two from each part.

PART - A

- 1 a. Find the n^{th} derivative of $\frac{1}{(x+2)(2x+3)} + e^{2x} \cos x$. (07 Marks)
- b. If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2 - 1)y_{n+2} - (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. (07 Marks)
- c. Find the angle between the curves $r = \frac{a}{1 + \cos \theta}$, and $r = \frac{b}{1 - \cos \theta}$. (06 Marks)
- 2 a. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$. (07 Marks)
- b. If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x+y}\right)$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$. (07 Marks)
- c. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$. Find $J \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix}$. (06 Marks)
- 3 a. Obtain a reduction formula for $I_n = \int \operatorname{cosec}^n x \, dx$. Hence find I_3 . (07 Marks)
- b. Evaluate $\int_0^{\infty} \frac{dx}{(1+x^2)^n}$, $n > 1$. (07 Marks)
- c. Trace the curve $a^2 y^2 = x^2(a^2 - x^2)$. (06 Marks)
- 4 a. Find the length of the curve $y^2 = 4ax$ cutoff by the line $3y = 8x$. (07 Marks)
- b. Find the area between the curve $y^2(a+x) = x^2(a-x)$ and the asymptote. (07 Marks)
- c. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$, ($\alpha > -1$) using differentiation under integral sign. (06 Marks)

PART - B

- 5 a. Solve $\frac{dy}{dx} = \frac{y}{x + \sqrt{xy}}$. (07 Marks)
- b. Solve $\frac{x^2 dy}{dx} - 2xy - x + 1 = 0$; $y(1) = 0$. (07 Marks)
- c. For the family of curves $x^2 + 3y^2 = cy$ (C - parameter), find the orthogonal family of curves. (06 Marks)
- 6 a. Find the nature of the series, $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \dots$. (07 Marks)
- b. Test for convergence of the series, $\frac{1}{1+x} + \frac{1}{1+2x^2} + \frac{1}{1+3x^3} + \dots$. (07 Marks)
- c. Test the series for i) Absolute convergence ii) Conditional convergence.
 $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots$. (06 Marks)
- 7 a. Find the angle between any two diagonals of a cube. (07 Marks)
- b. Show that the points $(0, -1, 0)$, $(2, 1, -1)$, $(1, 1, 1)$ and $(3, 3, 0)$ are coplanar. (07 Marks)
- c. Find the shorter distance between the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$ and z - axis. (06 Marks)
- 8 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. (07 Marks)
- b. Find a , b , c , so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has maximum magnitude of 64 in the direction of z - axis. (07 Marks)
- c. Prove that $\operatorname{curl}(\phi \vec{F}) = \phi(\nabla \times \vec{F}) + \nabla \phi \times \vec{F}$ (06 Marks)