1504

MCA 1st Semester Examination

Mathematics (N.S.)

MCA-104

Time : 3 Hours **Max. Marks : 60**

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/continuation sheet will be issued.

Note : Attempt five questions in all, selecting one question from each of the sections A, B, C & D, and all the subparts of the question in Section E. Use of scientific calculator is not allowed. All questions carry equal marks.

SECTION - A

1. (a) Show that the set A = {2, 3, 4, 6} is not a lattice with the relation of divisibility. Also draw the Hasse diagram of the poset A.

   (b) If B is a Boolean algebra and x, y $\in B$, then show that

   (x + y) + (x'.y') = 1 **(12)**

2. (a) A continuous random variable x has a probability function $f(x)=3x^2$, $0 \leq x \leq 1$. Find a and b such that

   (i) $P(x \leq a)=P(x > a)$ and (ii) $P(x>b) = 0.05$

   (b) Let G be the set of all non zero real numbers and let $a * b = \frac{ab}{2}, \forall a, b \in G.$ Show that (G,*) is an abelian group.

   (12)

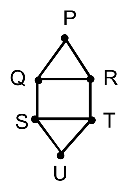
SECTION - B

3. (a) Show that an equation of the form $az\bar{z}+\bar{\alpha}z+\alpha\bar{z}+r=0,$ where a, r are real and $\alpha\bar{\alpha}-ar\geq 0,$ represents a straight line if a = 0 and a circle if $a\neq 0.$

   (b) Show that for an integer m & real $p\left[\frac{p-i}{p+i}\right]^m e^{2mi\cot^{-1}p}=1.$ **(12)**

4. (a) Show that V - E + R = 2 for any connected planer graph, where V, E and R are the number of vertices, edges and regions of the graph respectively.

   (b) Check whether the following graph is an isomorphic graph or not. **(12)**



SECTION - C

5. (a) Suppose that f(x) ≤ g(x) on some deleted neighborhood of the point C, and that $\lim_{x\to c} f(x)$ and $\lim_{x\to c} g(x)$ exist. Show that $\lim_{x\to c} f(x)\leq \lim_{x\to c} g(x).$

   (b) If $z(x+y) = x^2+y^2$

   Show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right).$ **(12)**

6. (a) Determine whether or not the following homogeneous system has a non zero solution:

   x+y–z=0, 2x+4y–z=0 and 3x+2y+2z=0.

   (b) Let $A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$

   Find all the eigen values of A. Is A diagonalization? If yes find P such that $D = P^{-1}AP$ is diagonal. **(12)**

SECTION - D

7. (a) Apply bisection method to obtain the smallest positive root of the equation $f(x)=x^3-5x+1=0$

   (b) Solve the equations $10x-y+2z=4$, $x+10y-z=3$ and $2x+3y+20z=7$ using the Gauss elimination method. **(12)**

8. (a) Show that the initial approximation x_0 for finding 1/N, where N is a positive integer, by the Newton-Raphson method must satisfy $0 < x_0 < 2/N$ for convergence.

   (b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using the Simpson's 3/8 rule. Also compare it with the exact solution. **(12)**

SECTION - E

9. (a) Write the truth table for the biconditional $p \leftrightarrow q$.

   (b) Define a modular lattice.

   (c) Show that $Re(iz)=-Im(z)$ and $Im(iz)=Re(z)$.

   (d) If A={1, 2, 3}, then find all permutations of A.

   (e) Evaluate $\int_{-\infty}^{0} e^x dx$

   (f) Write a 2×2 matrix A such that A^2 is diagonal but not A.

   (g) Find the probability of drawing one white ball from a bag containing 6 red, 8 black, 10 yellow and 1 green ball.

   (h) Write the limitations of bisection method.

   (i) find the local maxima and local minima, if any, of the function $f(x)=-x^3+12x^2-5$.

   (j) Evaluate $\lim_{n\to\infty} \frac{1+2+3+....+n}{n^2}$ **(12)**