SE / Comp / III 15/may 2012
Applied Mathematics-III

Con. 3567–12. GN–5306

(3 Hours) [Total Marks : 100

N.B. : 1) Question No. 1 is compulsory.
2) Solve any four questions out of remaining six questions.
3) Figures to the right indicate full marks

Q.1 a) Find the Laplace transform of $f(t) = e^{-4t} sinht\ sint$. (5)

b) Express the matrix A= $\begin{bmatrix} 3 & -2 & 6 \\ 2 & 7 & -1 \\ 5 & 4 & 0 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix. (5)

c) If the functions $f_1(x) = 1, f_2(x) = x$ and $f_3(x) = -1 + ax + bx^2$ are orthogonal in [-1,1] then determine the constants a and b. (5)

d) Find the Fourier transform of $f(x) = e^{-|x|}$ (5)

Q.2 a) Find the Laplace transform of $f(t) = sin^5 t$ (6)

b) For the matrix A= $\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$ Find the nonsingular matrices P and Q such that PAQ is in normal form. (6)

c) Find the Fourier series for $f(x) = x$ in $(0, 2\pi)$ (8)

Q.3 a) Find the Laplace transform of $f(t) = sin2t \quad ,0 < t < \pi/2$ (6)

$= 0 \quad ,\pi/2 < t < \pi$ and $f(t) = f(t + \pi)$.

b) Reduce the following the matrix to normal form and find its rank (6)

$$\begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

c) Find Fourier series expansion of $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $(0, 2\pi)$ and hence prove that (8)

$$\frac{\pi^4}{90} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + - - -$$

Q.4 a) Find Inverse Laplace transform of $\phi(s) = \frac{s^2+1}{s^3+3s^2+2s}$ (6)

b) Is the matrix A= $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ Unitary? If yes find A^{-1} (6)

c) Obtain the half range sine series in $(0, \pi)$ for $f(x) = x(\pi - x)$ and hence find the value of (8)

$$\sum \frac{(-1)^n}{(2n-1)^3}$$

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Q.5 a) Find Inverse Laplace transform of $\phi(s) = \frac{(s+1)e^{-s}}{s^2+s+1}$ (6)

b) Find the complex form of Fourier series for $f(x) = e^x$ in $(-\pi, \pi)$. (6)

c) Express the function $f(x) = \begin{cases} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$ as Fourier integral .Hence evaluate (8)

$$\int_0^\infty \frac{sin\omega sin\omega x}{cos\omega}\, d\omega.$$

Q.6 a) Using Convolution theorem find Laplace Inverse of $\phi(s) = \frac{1}{(s^2+4s+13)^2}$ (6)

b) Find Fourier series for $f(x) = 1 - x^2$ *in* $(-1,1)$ (6)

c) Solve the following system of equations (8)

$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11$$

Q.7 a) Evaluate $\int_0^\infty \frac{sin2t+sin3t}{te^t}\, dt$ (6)

b) Find Z-Transform of (6)

i) $f(k) = 1, k \geq 0, |z| > 1$

ii) $f(k) = a^k, k \geq 0, |z| > a$

iii) $f(k) = \frac{1}{2^k}, k \geq 0, |2z| > 1$

c) Solve $\frac{dx}{dt} + y = sint, \frac{dy}{dt} + x = cost$ where $x = 0, y = 2$ at $t = 0$. (8)