Code : 303202

BCA 2nd Semester Exam., 2014

MATHEMATICS (NUMERICAL TECHNIQUES)

Time : 3 hours *Full Marks* : 60

Instructions :

(i) All questions carry equal marks.

*(ii) There are **SEVEN** questions in this paper.*

*(iii) Attempt **FIVE** questions in all.*

*(iv) Question Nos. **1** and **2** are compulsory.*

1. Choose the correct options (any *six*) :

(a) If three approximate values of the number $\frac{1}{3}$ are given as 0·30, 0·33 and 0·34, then which of these three is the best approximation?

(i) 0·30

(ii) 0·33

(iii) 0·34

(b) If a function f is continuous between a and b and $f(a)$ and $f(b)$ are of opposite signs, then the number of roots of $f(x) = 0$ lying between a and b is

(i) exactly one

(ii) at least one

(iii) exactly two

(iv) at least two

(c) If $y_0, y_1, y_2, \cdots, y_n$ denote a set of values of y, then which of the following represents the second forward difference $\Delta^2 y_0$?

(i) $y_2 + 2y_1 - y_0$

(ii) $y_2 - 2y_1 + y_0$

(iii) $y_2 - 2y_1 - y_0$

(iv) None of the above

(d) Which of the following is correct, where the operators have their usual meanings?

(i) $\delta = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$

(ii) $\mu = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$

(iii) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

(iv) $\mu = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

(e) The $(n+1)$th divided difference of a polynomial of degree n is

(i) 0

(ii) 1

(iii) n

(iv) None of the above

(f) Which of the following formulae should be used, if interpolation is required near the beginning of a set of tabular values?

(i) Newton's forward interpolation formula

(ii) Newton's backward interpolation formula

(iii) Stirling's formula

(iv) Bessel's formula

(g) While using Simpson's $\frac{1}{3}$ rule, the number of equal subintervals, into which the given interval must be divided, should be a multiple of

(i) 2

(ii) 3

(iii) 4

(iv) 6

(h) Gauss elimination method for solving a system of linear equations reduces the system to an equivalent system which is

(i) diagonal

(ii) upper triangular

(iii) lower triangular

(iv) None of the above

(i) The system

$$2x+y=2$$
$$2x+1\cdot01y=2\cdot01$$

is

(i) not consistent

(ii) ill-conditioned

(iii) well-conditioned

(iv) neither ill-conditioned nor well-conditioned

(j) If the interval of differencing is 1, then the value of $\Delta\left(\frac{1}{x}\right)$ is

(i) $\frac{1}{x}$

(ii) $\frac{1}{x+1}$

(iii) $\frac{1}{x(x+1)}$

(iv) $-\frac{1}{x(x+1)}$

2. Answer any *three* of the following :

(a) Define absolute, relative and percentage errors. Evaluate the sum $\sqrt{3}+\sqrt{5}+\sqrt{7}$ to four significant digits.

(b) Define the operators Δ and ∇. Prove that

$$\Delta(y_k^2) = (y_k + y_{k+1})\Delta y_k$$

(c) Derive the Newton-Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(d) Prove that the third divided difference of the function $f(x) = \frac{1}{x}$ with arguments p, q, r, s is

$$-\frac{1}{pqrs}$$

(e) Given that $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$. Obtain the Taylor series for $y(x)$.

3. Find a root between 0 and 1 of the equation $x^3 + x - 1 = 0$ by the method of bisection correct to three decimal places.

4. Applying Lagrange's interpolation formula, find a cubic polynomial which approximates the following data :

x	-2	-1	2	3
$y(x)$	-12	-8	3	5

5. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by—

(a) Simpson's $\frac{1}{3}$ rule;

(b) Simpson's $\frac{3}{8}$ rule;

using six subintervals.

6. Solve the system of equations

$$5x - 2y + z = 4$$
$$7x + y - 5z = 8$$
$$3x + 7y + 4z = 10$$

by Gauss elimination method.

7. Use Runge-Kutta method to find $y(0 \cdot 1)$, given that

$$\frac{dy}{dx} = \frac{1}{x+y}, \; y(0) = 1$$

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