Mathematics

Paper 1: Differential Equations, Abstract Algebra and Real Analysis

Time: 3 hours Max.Marks:80

SECTION - A

Answer ALL questions. 4 x 15 = 60

1. a) i) Solve $x \cos x \frac{dy}{dx} + \left(x \sin x + \cos x\right) y = 1$ (8marks)

   ii) Solve $p^2 + 2 p y \cot x = y^2$ (7marks)

or

   b) i) Solve $\frac{dy}{dx}(x^2 y^3 + xy) = 1$ (8marks)

   ii) Solve $y + px = p^2 x^4$ (7marks)

2. a) i) Solve $(D^2 - 3D + 2) y = \cos(e^{-x})$ by the method of variation of parameters. (8marks)

   ii) Solve $(D^2 + 9)y = \cos^3 x$ (7marks)

or

   b) i) Solve $(D^2 + 4) y = x \sin x$ (8marks)

   ii) Solve $3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$ (7marks)

3. a) i) State and prove Lagrange's theorem on Groups (8marks)

   ii) Show that a finite semi group satisfying cancellation laws is a group. (7marks)

or

   b) i) State and prove Cayley's theorem on Permutation Groups (8marks)

   ii) Show that $G = \left\{x = 2^a 3^b / a, b \in Z\right\}$ is a group under multiplication. (7marks)

4. a) i) Prove that the sequence $\{S_n\}$ defined by

$S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\cdots\cdots + \frac{1}{n!}$ is convergent. (8 marks)

ii) If $\{S_n\}$ is a Cauchy sequence then show that $\{S_n\}$ is convergent. (7marks)

or

b) i) State and prove Cauchy's n^{th} root test. (8marks)

ii) Test for convergence $\sum \frac{x^n}{x^n + a^n}$ (x > 0, a >0) (7marks)

SECTION – B

Answer any FOUR Questions 4x5=20

5. Solve y (xy + 2 $x^2 y^2$) dx + x (xy - $x^2 y^2$) dy = 0.

6. Find the orthogonal trajectories of the family of rectangular hyperbolas xy = a^2 where 'a' is the parameter.

7. Solve (D^2 -3D+2) y = cosh x.

8. Solve (D^2 -2D +1) y = $x^2 e^{3x}$

9. State and prove fundamental theorem on groups.

10. If f is a homomorphism of a group G in to a group G' then prove that kernel of f is a normal sub group of G.

11. If $\{a_n\}$ is a sequence defined by $a_1 = 1$, $a_{n+1} = \frac{2a_n + 3}{4}$ for $n \geq 1$. Show that $\{a_n\}$ is increasing sequence and find its limit.

12. Test for convergence $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$