9. *(a)* Verify Stokes' theorem for

$$\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$$

taken around the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$. 7

(b) Evaluate $\int_S \vec{F} \cdot dS$, where

$$\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$$

and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 7

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B.Tech 2nd Semester Exam., 2016

MATHEMATICS—II

Time : 3 hours *Full Marks* : 70

Instructions :

(i) The marks are indicated in the right-hand margin.

(ii) There are ***NINE*** *questions in this paper.*

(iii) Attempt ***FIVE*** *questions in all.*

(iv) Question No. ***1*** *is compulsory.*

1. Answer any *seven* of the following as directed : 2×7=14

(a) The series

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$$

converges, if

(i) $p > 0$

(ii) $p < 1$

(iii) $p > 1$

(iv) $p \leq 1$

(Choose the correct option)

(b) The series

$$1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$$

is

(i) oscillatory

(ii) conditionally convergent

(iii) divergent

(iv) absolutely convergent

(Choose the correct option)

(c) The period of $|\sin t|$ is $\underline{\pi}$.

(Fill in the blank)

(d) In the Fourier series expansion of

$$f(x)=|\sin x|$$

in $(-\pi, \pi)$, the value of b_n = $\underline{0}$.

(Fill in the blank)

(e) The function

$$f(x)=\begin{cases}1-x, & \text{in } -\pi<x<0\\ 1+x, & \text{in } 0<x<\pi\end{cases}$$

is an odd function.

(Write True or False)

(f) $\int_0^2\int_0^x (x+y)\,dx\,dy = \underline{4}$.

(Fill in the blank)

(g) $\int_0^{\pi/2}\int_0^{a\sin\theta} r\,dr\,d\theta = \underline{\frac{a^2\pi}{8}}$.

(Fill in the blank)

(h) On changing to polar coordinates

$$\int_0^{2a}\int_0^{\sqrt{(2ax-x^2)}} dx\,dy$$

becomes ______.

(Fill in the blank)

(i) If $\nabla\cdot\vec{F}=0$, then $\vec{F}$ is called Solenoidal

(Fill in the blank)

(j) If $\vec{A}$ is such that $\nabla\times\vec{A}=0$, then $\vec{A}$ is called Irrotational

(Fill in the blank)

2. (a) Discuss the convergence of the series

$$x+\frac{2^2\cdot x^2}{\lfloor 2}+\frac{3^3\cdot x^3}{\lfloor 3}+\frac{4^4\cdot x^4}{\lfloor 4}+\frac{5^5\cdot x^5}{\lfloor 5}+\cdots\infty$$

$(x>0)$ 7

(b) Prove that the series

$$\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}-\cdots$$

converges absolutely. 7

3. (a) Expand $f(x) = x \sin x$ as a Fourier series in the interval $0 < x < 2\pi$. 7

(b) Obtain Fourier expansion for the function

$$f(x) = \begin{cases} \pi + x, & \text{if } -\pi \le x \le 0 \\ \pi - x, & \text{if } 0 \le x \le \pi \end{cases}$$

and $f(x + 2\pi) = f(x)$. 7

4. (a) Find half-range cosine series for the function $f(x) = x^2$, $0 < x < 2$. 7

(b) Find the Laplace transform of $te^{-t} \sin 3t$. 7

5. (a) Find the value of

$$L\left\{\int_0^t \frac{e^{-t} \sin t}{t} dt\right\}$$

7

(b) Find the inverse Laplace transform of

$$\frac{s+2}{s^2 (s+1)(s-2)}$$

7

6. (a) By changing the order of integration in

$$I = \int_0^1 \int_{x^2}^{2-x} xy\,dx\,dy$$

evaluate the integral. 7

(b) Evaluate

$$\iint r \sin\theta\, dr\, d\theta$$

over the cardioid $r = a(1 - \cos\theta)$ above the initial line. 7

7. (a) Evaluate

$$\iint_R (x + y)^2\, dx\, dy$$

where R is the parallelogram in the xy-plane with vertices (1, 0), (3, 1), (2, 2), (0, 1) using the transformation $u = x + y$ and $v = x - 2y$. 7

(b) Find by triple integration, the volume of the sphere $x^2 + y^2 + z^2 = a^2$. 7

8. (a) Show that

$$\nabla^2 (r^n) = n(n+1) r^{n-2}$$

7

(b) Find the work done in moving a particle in the force field

$$\vec{F} = 3x^2 \vec{i} + (2xz - y)\vec{j} + z\vec{k}$$

along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$. 7