PJ – 577

I Semester M.C.A. Examination, January/February 2019
(CBCS)
COMPUTER SCIENCE
MCA – 104T : Discrete Mathematics

: 3 Hours Max. Marks : 70

Instruction : *Answer **any five** questions from Part – **A** and **any four** questions from Part – **B.***

PART – A

) Using Venn diagram prove that for any three sets A, B, and C
$\overline{(A \cup B) \cap C} = (\overline{A} \cap \overline{B}) \cup \overline{C}$

) Prove that for any three sets, A, B and C, $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **(3+3)**

et A = {1, 2, 3, 4} and let R be the relation on A defined by x R y if and only if x divides y"

) Write down R as a set of ordered pairs.

) Draw the digraph of R.

) Determine the in degrees and out degrees of the vertices in the digraph. **6**

) Simplify the compound proposition using the laws of logic $(p \vee q) \wedge \sim\{(\sim p) \vee q\}$.

) Let p, q and r be having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions

i) $(p \wedge q) \wedge r$

ii) $(p \wedge q) \rightarrow r$

iii) $p \wedge (r \rightarrow q)$. **(3+3)**

'rove the following :

) If A and B are countable sets, then $A \cup B$ is countable.

) If A and B are countable sets, then A – B and B – A are countable. **6**

'rove by Mathematical Induction that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **6**

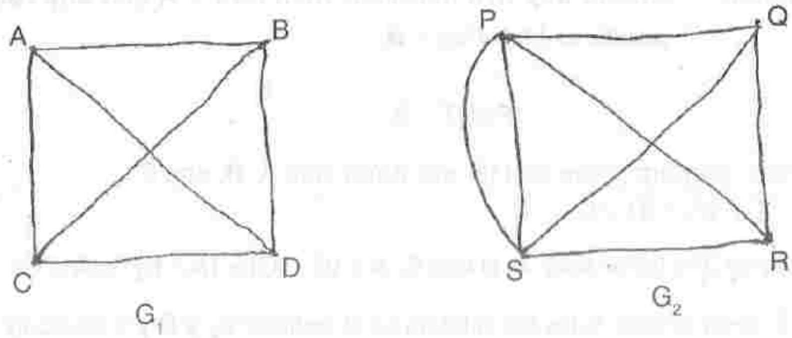
P.T.O.

6. A sequence {an} is defined recursively by $a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. **6**

7. a) State and prove Pigeonhole principle.

   b) Consider the function $f : R \to R$ defined by $f(x) = 2x + 5$, let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x - 5)$ prove that g is an inverse of f. **(3+3)**

8. Examine whether the following graphs are isomorphic (or) not **6**



PART – B

9. a) State and prove Addition theorem.

   b) In a survey of 260 college students, 64 had taken a Mathematics Course, 94 had taken a Computer Science, 58 had taken a Business Course, 28 had taken both Mathematics and Business Courses, 26 had taken both Mathematics and Computer Science Courses, 22 had taken both Computer Science and Business Courses and 14 had taken three types of courses

      i) How many of these students had taken none of the three courses ?

      ii) How many had taken only a Computer Science Course ? **(3+7)**

10. a) The matrix of a relation R on the set A = {1, 2, 3} is given by $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Show that R is an equivalence relation.

    b) Let A = {1, 2, 3, 4, 6, 12} on A defined the relation R by a R b if and only if "a divides b". Prove that R is a partial order on A. Draw the Halse diagram for this relation. **(5+5)**

11. a) Prove that the following compound proposition $[p \vee (q \wedge r)] \vee \sim[p \vee (q \wedge r)]$ is a tautology.

b) Test whether the following argument is valid

i) $p \to q$
$r \to s$
$p \vee r$
$\therefore p \vee s$

ii) $p \to q$
$r \to s$
$\sim q \vee \sim s$
$\therefore \sim (p \wedge r)$

(5+5)

12. a) By mathematical induction, prove that for every integer n, the number $A_n = 5^n + 2.3^{n-1} + 1$ is a multiple of 8.

b) Let $X \to Y$ be a function and A and B be arbitrary non-empty subsets of X. Then

i) If $A \subseteq B$ then $f(A) \subseteq f(B)$

ii) $f(A \cup B) = f(A) \cup f(B)$. **(5+5)**

13. a) Let n is a positive integer, then prove that

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0 \text{ and } \sum_{k=0}^{n} (2)^k \binom{n}{k} = 3^n.$$

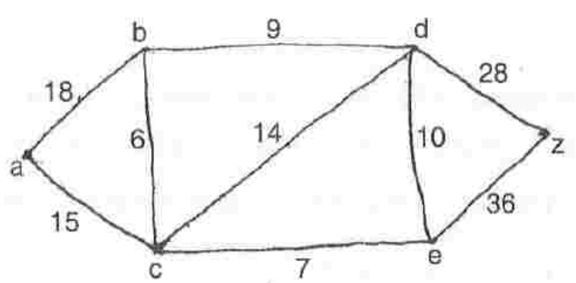
b) The probability distribution of a discrete random variable is given below :

x	−2	1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

Find (i) K (ii) Mean and (iii) Variance. **(5+5)**

14. a) Define Euler and Hamiltonian graph. Give an example of a graph which is Hamiltonian but not Eulerian and viceversa.

b) Using Dijkstra's algorithm, find the shortest path between a to z in the weighted graph.



(4+6)

PG – 430

I Semester M.C.A. Degree Examination, January/February 2018
(CBCS)
COMPUTER SCIENCE
MCA 104T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

***Instruction**: Answer **any five** questions from Part – **A** and **any four** from Part – **B**.*

PART – A

Answer **any five** questions. **Each** question carries **six** marks. **(5×6=30)**

1. a) For any two sets A and B prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. 3

   b) What is difference of sets ? If A = {4, 5, 6} and B = {5, 6, 9} find A – B and B – A. 3

2. a) Find inverse converse and contrapositive of the following implication "If Ramanujan can solve the puzzle, then Ramanujan can solve the problem". 3

   b) Show that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$. 3

3. a) Define one-to-one and on-to function with example. 3

   b) Let $f : R \to R$ determine whether f is invertible and if so, determine f^{-1}, where $f = \{(X, Y) \mid 2X + 3Y = 7\}$. 3

4. Use mathematical induction to prove that $n^3 - n$ is divisible by 3. Where n is a positive integer.

5. a) In how many ways can the letters of the word "MATHEMATICS" be arranged such that the words starts and ends with M ? 3

   b) Find the number of triangles formed by joining the vertices of a polygon of 12 sides. 3

6. Let R and S be the relations on a set prove that

   i) R is symmetric if and only if R^{-1} is symmetric.

   ii) If R and S are transitive then $R \cap S$ is transitive.

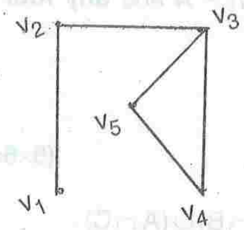
P.T.O.

7. a) Let f, g : R $\to$ R^+ defined by $f(x) = 2x + 3$ and $g(x) = x^2$ find $gof(x)$ and $fog(x)$. 3

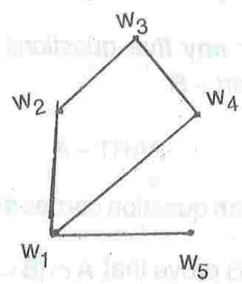
   b) Let f, g : R $\to$ R defined by $f(x) = 3x - 5$ for $x > 0$ and $f(x) = -3x + 1$ for $x \leq 0$. What are the values of $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-3)$. 3

8. a) Define complete graph and bipartite graph with an example. 3

   b) Check whether graphs G_1 and G_2 are isomorphic. 3



Graph G_1



Graph G_2

PART – B

Answer **any four** questions. **Each** question carries **10** marks. **(4×10=40)**

9. a) A survey among 100 students show that of the three ice cream flavour Vanilla, Chocolate and Strawberry, 50 students like Vanilla, 43 like Chocolate, 28 like Strawberry, 13 like Vanilla and Chocolate, 11 like Chocolate and Strawberry, 12 like Strawberry and Vanilla and 5 like all of them. Find the number of students surveyed who like each of the following flavours. **(5+5)**

   i) Chocolate but not Strawberry.

   ii) Chocolate and Strawberry but not Vanilla.

   b) Let A, B and C be three finite sets prove that

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$$

10. a) A three digit number is formed with the digits 1, 3, 6, 4 and 5 at random find the number of numbers. **(5+5)**

    i) Divisible by 2 ?

    ii) Not divisible by 2 ?

    iii) Divisible by 5 ?

    b) Define the terms with example

    i) Rules of Syllogism ii) Modus Pones iii) Modus Tollens.

11. a) Prove for any proposition p, q, r following compound proposition.

$[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology. (5+5)

b) Find Explicit formula for $a_n = a_{n-1} + n$ for $n \geq 2$ $a_1 = 4$.

12. a) Show that the set of all integers is countable. (5+5)

b) State and prove the extended pigeon hole principle.

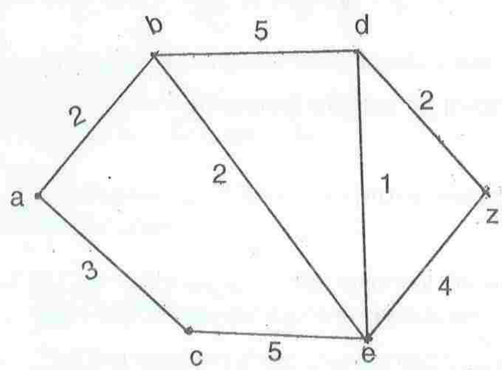
13. a) If $f : A \to B$ and $B_1 . B_2 \subseteq B$ then show that $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$. (5+5)

b) Let R be a relation represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ find the Matrix representing

a) R^{-1} b) $\overline{R}$ c) R^2

14. a) State and prove Hand shaking theorem. (5+5)

b) Find the length of a shortest path between a and z in the given weighted graph.



PG – 448

I Semester M.C.A. Degree Examination, January 2017
(CBCS)
COMPUTER SCIENCE
MCA – 104T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **any 5** questions from Part – **A** and **any 4** from Part – **B**.*

PART – A

Answer **any five** questions. **Each** question carries **six** marks. **(5×6=30)**

1. a) Determine the sets A and B, given that $A - B = \{1, 3, 7, 11\}$, $B - A = \{2, 6, 8\}$, $A \cap B = \{4, 9\}$.

   b) For any three sets A, B and C, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. **(3+3)**

2. Let $A = \{1, 2, 3, 4, 6\}$ and R be the relation on A defined by aRb if and only if "a is a multiple of b".
   i) Write down R as a set of ordered pairs
   ii) Represent R as a matrix
   iii) Draw the diagraph of R. **6**

3. a) Define the terms (i) Rule of Syllogism (ii) Modus pones (iii) Modus Tollens.

   b) Let p and q be primitive statements for which the conditional $p \rightarrow q$ is false. Determine the truth value of the following compound propositions
   i) $p \wedge q$ ii) $\sim p \vee q$ iii) $\sim q \rightarrow \sim p$ **(3+3)**

4. Prove by mathematical induction that for every positive integer $n > 2$, "$n! > 2^{n-1}$". **6**

5. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's selection is 1/5. What is the probability that (i) Both of them will be selected (ii) Only one of them will be selected (iii) None of them will be selected. **6**

6. Prove that the set of all real numbers in the open interval (0, 1) is uncountable. **6**

P.T.O.

7. Let A and B be any two nonempty sets. (i) Define a function from A to B (ii) one-to-one function (iii) onto function. If $n(A) = 7$ and $n(B) = 4$, find the number of functions from A to B and one to one functions from A to B. **6**

8. a) Define simple graph, complete graph, regular graph with an example.

b) Show that the hyper cube Q_3 is a bipartite graph. **(3+3)**

PART – B

Answer **any four** questions. **Each** question carries **10** marks. **(4×10=40)**

9. a) Using Venn diagram, prove that $A \Delta (B \Delta C) = (A \Delta B) \Delta C$.

b) The MCA course of an University has 300 students. It is known that 180 can programme in Pascal, 120 can programme in Fortran, 30 in C++, 12 in Pascal and C++, 18 in Fortran and C++, 12 in Pascal and Fortran and 6 in all three languages.

i) A student is selected at random, what is the probability that the student can programme in exactly two languages ?

ii) Two students are selected at random what is the probability that both can programme in Pascal ? Both programme only in Pascal ? **(4+6)**

10. a) Define R on A ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} by $(x, y) \in R$ if $x - y$ is a multiple of 5.

i) Show that R is an equivalence relation on A.

ii) Determine the equivalence classes and partition of A induced by R.

b) Define the bijective function. If $f : R \to R$ and $g : R \to R$ where $f(x) = 3x + 7$ and $g(x) = x(x^3 - 1)$. Show that f is one to one but not g. **(5+5)**

11. a) Prove the validity of the following statement. If Ragini gets the supervisor's position and works hard, then she will get a raise. If she gets the raise, then she will buy a new car. She has not purchased a new car. Therefore either Ragini did not get the supervisor's position or she did not work hard.

b) Prove that $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (q \vee p)$. **(5+5)**

12. a) State and prove the extended pigeon hole principle.

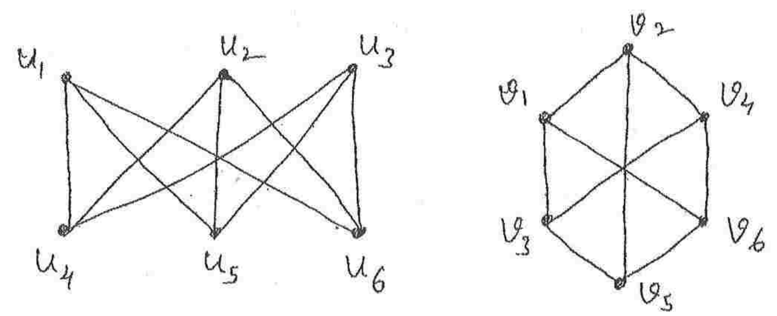
b) Shirts numbered consecutively from 1 to 20 are worn by students of a class. When any 3 of these students are chosen to be debating team from the class, the sum of their shirt numbers is used as the code number of the team. Show that if any 8 of the 20 are selected, then from these 8 we may form at least two different teams having the same code number. **(5+5)**

13. a) Solve the linear recurrence relation $a_n = 4a_{n-1} + 5a_{n-2}$ with $a_1 = 2$, $a_2 = 6$.

b) We must form a committee of eight people from two mathematicians and ten economists. In how many way can we do it, if the committee must include at least one mathematician ? **(5+5)**

14. a) Prove that, in any undirected graph, the number of odd degree vertices is even.

b) Verify that the two graphs shown below are isomorphic.



(5+5)

I Semester M.C.A. Degree Examination, January 2016
(CBCS)
COMPUTER SCIENCE
MCA – 104 T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **any five** questions from Part – **A** and **any four** from Part – **B**.*

PART – A

1. a) Determine the sets A and B, given that $A - B = \{1, 2, 4\}$, $B - A = \{7, 8\}$ and $A \cup B = \{1, 2, 4, 5, 7, 8, 9\}$.

   b) Prove that, for any three sets A, B and C (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$. **(3+3)**

2. a) Let x be the set of factors of 12 and let $\leq$ be the relation divisor i.e., $x \leq y$ if and only if x divides y. Draw the Hasse diagram of $(X, \leq)$.

   b) $f : Z \to N$ is defined by $f(x) = \begin{cases} 2x-1, & \text{if } x>0 \\ -2x, & \text{if } x \leq 0 \end{cases}$. Prove that f is one-to-one and onto. **(3+3)**

3. Prove the validity of the following arguments : **6**

   i) $p \to r$
   $\neg p \to q$
   $q \to s$
   $\therefore \neg r \to s$

   ii) $(\neg p \vee \neg q) \to (r \wedge s)$.
   $r \to s$.
   $\neg t$
   $\therefore p$

4. Obtain an explicit form for the following sequences $\{a_n\}$ defined recursively by $a_n = 2a_{n-1} + 1$ for $n \geq 2$, with $a_1 = 3$. **6**

5. The probability that an integrated circuit will have defective etching is 0.12, the probability that it will have a crack defect is 0.29, and the probability that it will have both defects is 0.07. What is the probability that a newly manufactured chip will have (i) an etching or crack defect ? (ii) neither defect ? **6**

P.T.O.

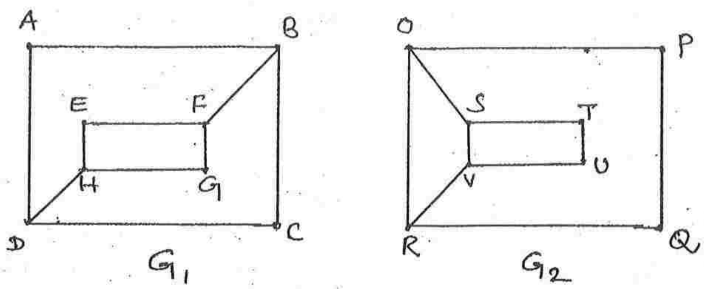
6. Prove the following :

   a) If A and B are countable sets, then $A \cup B$ is countable.

   b) If A and B are countable sets, then $A - B$ and $B - A$ are countable. **(3+3)**

7. Find the number and sum of all positive divisors of 24. **6**

8. Examine whether the following pair of graphs are isomorphic or not. Justify your answer. **6**



PART – B

9. a) Draw a Venn diagram for the following and solve $n(A) = 32$, $n(B) = 29$, $n(A \cap B) = 11$, $n(B \cap C) = 12$, $n(A \cap C) = 13$ and $n(A \cap B \cap C) = 5$. Hence $n(A \cup B \cup C)$, n(only A), n(only B) and n(only C).

   b) Thirty cars are assembled in a factory. The options available are a music system, an air conditioner and power windows. It is known that 15 of the cars have music systems, 8 have air conditioners and 6 have power windows. Further, 3 have all options. Determine atleast how many cars do not have any option at all. **(5+5)**

10. a) If I be the set of all integers and if the relation R be defined over the set I by xRy if $x - y$ is an even integer, where $x, y \in I$, show that R is an equivalence relation.

    b) Consider the functions $f, g : R \to R$, defined by $f(x) = 2x + 3$ and $g(x) = x^2 + 1$. Find the composition function $(g \circ f)(x)$ and $(f \circ g)(x)$. **(5+5)**

11. a) Prove that $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$.

    b) Prove the following : "If $\neg p \leftrightarrow q$ is true, $q \to r$ is true and $\neg r$ is true then p is true" by the method of contradiction. **(5+5)**

12. a) State and prove the pigeon hole principle.

b) For all positive integers n, prove that if $n \geq 24$, then n can be written as a sum of 5's and or 7's. **(5+5)**

13. a) Two cards are drawn from a pack of cards at random. What is the probability that it will be

i) a diamond and a heart

ii) a king and a queen

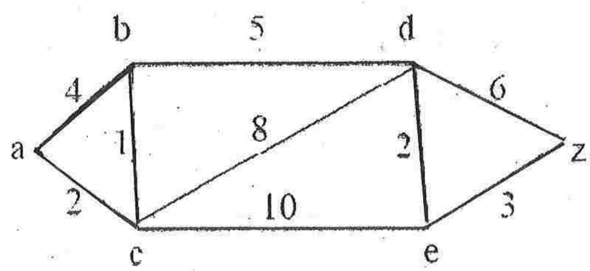
iii) two kings

b) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 0$, $a_1 = -9$ and $a_2 = 15$. **(5+5)**

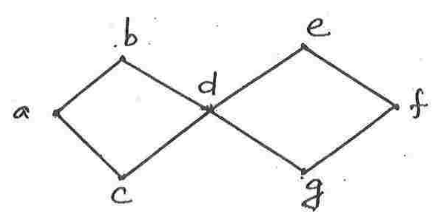
14. a) If G is an Euler graph, show that all the vertices of G are of even degree. **5**

b) Prove that a tree with n vertices has (n – 1) edges. **5**

15. a) Find the length of a shortest path between a to z in the weighted graph.



b) Find the spanning tree of graph G.



(5+5)

PG – 783

I Semester M.C.A. Degree Examination, January 2015
(CBCS)
COMPUTER SCIENCE
MCA - 104 T : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **any five** questions from Part – **A** and **four** questions from Part – **B**.*

PART – A **(5×6=30)**

1. a) Prove that "null set is a subset of every set". **(3+3)**

   b) For any three sets, A, B, C prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2. a) Define :
   i) Reflexive relation
   ii) Irreflexive relation
   iii) Symmetric relation.
   iv) Antisymmetric relation and
   v) Transitive relation with an example. **6**

3. Given p and q as statements, explain the following terms.
   i) Conjunction
   ii) Disjunction
   iii) Implication
   iv) Logically equivalence
   v) Tautology
   vi) Contradiction

4. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers prove that $\sum_{i=0}^{n} F_i^2 = F_n \times F_{n+1}$ for all positive integers n. **6**

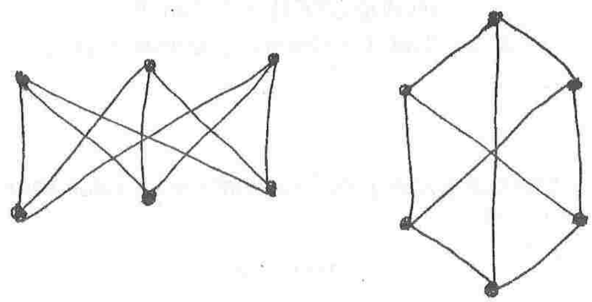
5. An integer is selected at random from 3 through 15 inclusive. If A is the event that a number divisible by 3 is chosen and B is the event that the number exceeds 10, determine $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$ and $P_r(A \cup B)$. **6**

6. Prove that the open interval (0, 1) is not a countable set. **6**

7. Prove that every set of 37 positive integers contain at least two integers that leave the same remainders upon division by 36. **6**

P.T.O.

8. Define Isomorphism of graphs. Verify that the two graphs shown below are isomorphic. 6



PART – B

Answer **any four** questions. **(4×10=40)**

9. a) Using Venn diagram prove that $A\Delta(B\Delta C) = (A\Delta B)\Delta C$.

   b) A survey of 500 televisions viewers of sports channel produced the following information : 285 watch cricket, 195 watch hockey, 115 watch football, 45 watch cricket and football, 70 watch cricket and hockey, 50 watch hockey and football and 50 do not watch any of the three games.

      i) How many viewers in the survey watch all three kinds of games ?

      ii) How many viewers watch exactly one of the sports ? **(5+5)**

10. a) Draw the Hasse diagram representing the partial ordering { (a, b) / a divides b}.

    b) Let A and B be two non-empty sets. Define :

       i) A function from A to B

       ii) One-to-one function

       iii) On-to function

       iv) Bijective function. If $|A| = 4$ and $|B| = 7$, find the number of functions from A to B and one to one functions from A to B. **(5+5)**

11. a) Prove that, for any propositions p, q and r the compound proposition $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a Tautology.

    b) If a band could not play rock music or the refreshments were not delivered on time, then the new year party would have been cancelled and Alicia would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made. Therefore, the band could play rock music. Establish the validity of the argument by using the rules of inferences. **(5+5)**

12. a) For all positive integers n, prove that if $n \geq 24$, then n can be written as a sum of 5's and / or 7's.

b) State and prove the extended pigeon hole principle. **(5+5)**

13. a) Three coins are tossed in succession. Find out the probabilities of occurrence of

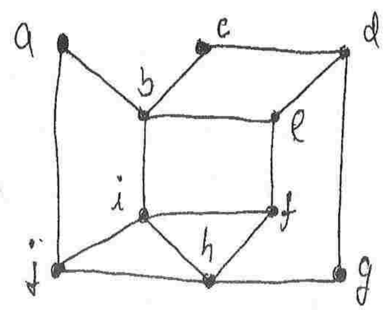
i) Two consecutive heads

ii) Two heads and

iii) Two heads in the following order, head, tail and head.

b) Let $b_0, b_1, b_2, \ldots$ be defined by the formula $b_n = 4^n$, for all integers $n \geq 0$. Show that this sequence satisfies the recurrence relation $b_k = 4b_{k-1}$ for all integers ≥ 1. **(5+5)**

14. a) Prove that the graph G shown below does not have a Hamiltonian circuit. **(5+5)**



b) Find the length of a shortest path between a and z in the weighted graph. **(5+5)**