BTS – III – 11.13 – 1162

B. Tech. Degree III Semester Examination November 2013

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 1301 ENGINEERING MATHEMATICS II

(2012 Scheme)

Time : 3 Hours | Maximum Marks : 100

PART A

(Answer *ALL* questions)

(8 x 5 = 40)

I. (a) Find the rank of the matrix.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

(b) Check whether the vectors $X_1 = (1,1,2)$, $X_2 = (1,2,5)$ and $X_3 = (5,3,4)$ are linearly dependent or not.

(c) Find the Laplace transform of $t^2\, u(t-3)$

(d) Evaluate $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}\, dt$

(e) Find the Fourier sine and cosine integrals of $f(x) = e^{-kx}$, for $x > 0, k > 0$

(f) Express $f(x) = x$ as a Fourier cosine series in $0 < x < 2$

(g) Find the work done by the force $\overline{F} = 3xy\,\overline{i} - y^2\,\overline{j}$ when it moves a particle along the curve $y = 2x^2$ in the xy plane

(h) Find (i) $\nabla^2\left(\frac{1}{r}\right)$ where $r = |\overline{r}|$ and (ii) $\nabla\left(\frac{1}{r}\,\overline{r}\right)$

PART B

(4 x 15 = 60)

II. (a) Test for consistency of the following system of equations and solve them if consistent: (8)

$$x_1 + x_2 - x_3 = 0$$
$$2x_1 - x_2 + x_3 = 3$$
$$4x_1 + 2x_2 - 2x_3 = 2$$

(b) Verify Cayley Hamilton theorem and hence find A^4 (7)

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

OR

(P.T.O.)

III. (a) For what values of k the equations $x+y+z=1, 2x+y+4z=k$, $4x+y+10z=k^2$ have a solution and solve them completely in each case. (10)

(b) Check whether $W=\{(a,b,o): a=b^2, a,b,\in R\}$ is a subspace or not (5)

IV. Find the inverse Laplace transform of

(i) $\dfrac{5S+3}{(S-1)(S^2+2S+5)}$ (5)

(ii) $\tan^{-1}\left(\dfrac{2}{S}\right)$ (5)

(iii) $\log\left(\dfrac{1+S}{S}\right)$ (5)

OR

V. (a) Solve the equation : $y^{11}-3y^1+2y=4t+e^{3t}$ when $y(0)=1$, $y^1(0)=-1$ (8)

(b) Apply convolution theorem to evaluate $L^{-1}\left\{\dfrac{1}{S(S^2+4)}\right\}$ (7)

VI. (a) Find the Fourier transform of e^{-x^2} (8)

(b) Solve the integral equation:

$$\int_0^\infty F(x) Cos\, px\, dx = \begin{Bmatrix} 1-p & 0 \le p \le | \\ 0 & p > | \end{Bmatrix}$$ (7)

OR

VII. (a) Obtain the Fourier series for the function $f(x)=x^2, -\pi < x < \pi$. Hence show that (10)

(i) $1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+........=\dfrac{\pi^2}{6}$

(ii) $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+......=\dfrac{\pi^2}{12}$

(iii) $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$

(b) Find the finite Fourier Sine transform of $f(x)=2x$ *in* $0<x<4$ (5)

VIII. (a) Verify divergence theorem for $\bar{F}=x^2\bar{i}+z\bar{j}+yz\bar{k}$ over the cube formed by $x=\pm1, y=\pm1, z=\pm1$ (9)

(b) Prove that $\nabla.(\nabla x \bar{A})=0$ for any vector function $\bar{A}$ (6)

OR

IX. (a) Verify Stoke's theorem for $\bar{F}=(2x-y)\bar{i}-yz^2\bar{j}-y^2z\bar{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is the circular boundary in the XY plane. (8)

(b) Show that $\bar{F}=(y^2+2xz^2)i+(2xy-z)\bar{j}+(2x^2z-y+2z)\bar{k}$ is irrotational and hence find its scalar potential. (7)

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