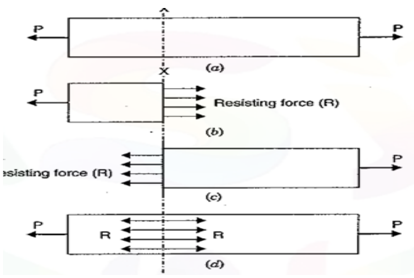
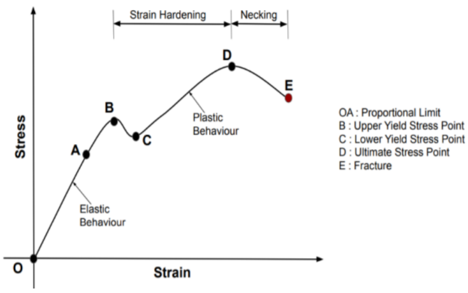
Unit-I.

Theories of failures

Theories of failures:

According to theory of failure a member is said to be fail if the member will not regain its original dimensions soon after removed applied load. (tensile/ compressive)





Principal stress:(σ)

The planes in the xyz system which have no shear stress and only subjected to normal stresses are known as Principal stresses.

Factor of Safety:

$$= \frac{\text{Stress at elastic}}{\text{Principal Shear (or) Max Principal shear}}$$

Types of theories of failures:

a) The Maximum Principal stress (or) Rankine’s Theory
b) The MaximumPrincipal Strain (or) St. Venant's theory
c) The Maximum shear stress theory (or) Guest's theory
d) The Maximum strain energy theory (or)Haigh's theory
e) The maximum shear strain energy theory (or) Mises and menhy theory

a. maximum principal stress theory (or) ranking’s theory

According to this theory the failure will take place if $\sigma_1 \geq \sigma_{t^*}$in Tension

$\sigma_t = \sigma_1$: $|\sigma_3| \geq \sigma_c$in Compression : factor of safety = $\frac{\sigma_t}{\sigma_t}$ (=> $\sigma_1 = \sigma_t$)

σ_1 =PrincipleTensile Stress
σ_2 = Principle Tensile Stress
σ_3 = Principle compressive Stress
σ_t^* = shear at elastic limit

b. The Maximum Principal Strain (or) St. Venant's theory

According to thing they failure will take place if $e_1 \geq \frac{\sigma_t^*}{E}$; $|e_3| \geq \frac{\sigma_e}{E}$ => $E = \frac{\sigma}{e}$

young's modulus = $\frac{\text{stress}}{\text{stain}}$: Poisson ratio (μ) = $\frac{\text{lateral strain}}{\text{longitudinal strain}}$

$e_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$; $e_1 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2))$

$\frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)] \geq \frac{1}{E}\sigma_t$: $\frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)] \geq \frac{1}{E}\sigma_c$

$\sigma_t = \sigma_1 - \mu(\sigma_2 + \sigma_3)$: $\sigma_c = |\sigma_3 - \mu(\sigma_1 + \sigma_2)|$

σ_1 = Principle Tensile Stress
σ_2 = Principle Tensile Stress
σ_3 = Principle compressive Stress
σ_t^* = shear at elastic limit

c. The Maximum shear stress theory (or) Guest's theory

According to this theory failure will take Place if $(\sigma_1 - \sigma_3) \geq \sigma_t$*

maximum Shear stress = ½ (σ1 – 3) in Simple Tension $\sigma_2, \sigma_3 = 0$

so (τ) = (½ σ_t*-0) $= \frac{1}{2}\sigma_t$* $\sigma_t = (\sigma_1 - \sigma_3)$

σ_1 = Principle Tensile Stress
σ_2 = Principle Tensile Stress
σ_3 = Principle compressive Stress
σ_{t*} = shear at elastic limit

d.The Maximum strain energy theory (or)Haigh's theory

According to this failure will take place if $\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 * \sigma_2) \geq \sigma_t^2$

Modules of elasticity (E) $= \frac{\sigma}{e} = \frac{\text{stress}}{\text{strain}}$

Strain (e) = $\frac{\delta L}{L} = \frac{\text{change in length}}{\text{original length}}$; Volume = Area *Length

Stress (σ) $= \frac{P}{A} = \frac{\text{Load}}{\text{Area}}$; Strain energy in a body (u) = Work done by load (p)

$= \frac{1}{2} * p * \delta L$; $= \frac{1}{2} * (\sigma * a) * (l * e)$; $= \frac{1}{2} * \sigma * e * Al$

=1 2 * σ *e*volume

So, Strain energy Per unit Volume

$= \frac{1}{2} * \sigma * e = \frac{1}{2}$*stress*strain

So total strain energy per unit volume U $=\frac{1}{2}\sigma_1 * e_1 * + \frac{1}{2}\sigma_2 * e_2 + \frac{1}{2} * \sigma_3 * e_3$

$e_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2+\sigma_3)]$; $e_2 = \frac{1}{E}[\sigma_2 - \mu(\sigma_1+\sigma_2)]$; $e_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma 1+\sigma_2)]$;

submitted in above equation

strain energy at elastic limit

$=\frac{1}{2} * (\sigma_t *) * (e_t *)$

$=\frac{1}{2E} (\sigma_{t*})^{2} \because [E = \frac{\sigma}{e}]$

$\sigma_t^2 = \sigma_1^2 + \sigma^2 - 2\mu (\sigma_1 * \sigma_2)$

$U = \frac{1}{2E} [\sigma_1^2 + \sigma_1^3 + \sigma_1^3 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]$

e. maximum strain energy theory

According to this failure will take place if $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1) \geq (2\sigma_t *)^2$

total shear strainenergy $=\frac{1}{2c}(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)$

In tension $\sigma_2, \sigma_3 = 0$

$=\frac{1}{2c}[2\sigma_t *^2]$

σ_1 = Principle Tensile Stress
σ_2 = Principle Tensile Stress
σ_3 = Principle compressive Stress
σ_{t*} = shear at elastic limit

<u>Problems on bolt design:</u>

Given data:

Axial load (P) =9 KN =9000N

Shear force (f)=4.5 KN =4500N

Shear at elastic limit in tension

$\sigma_t^* = 225\ N/mm^2$

Factor of safety =3

Poisson ratio =0.3

Data: -

Diameter of bolts =(d) mm

- Factor of safety
  $=\frac{\sigma t*}{\sigma t}$
  $\sigma_t = \frac{\sigma t*}{Fs}$
  $=\frac{225}{3}$
  $=75 N/mm^2$
- tensile stress (σ) $=\frac{P}{A}$
  $=\frac{9000}{\frac{\pi}{2}(d^2)} = \frac{9000}{(d^2)}$ **N/mm^2**
- shear stress (τ) $=\frac{f}{\frac{\pi}{2}(d^2)}$

$\frac{4500}{\frac{\pi}{2}(d^2)} = \frac{4500}{(d^2)}$ **N/mm^2**

- max/min principal stresses $=>\frac{\sigma}{2} \pm \sqrt{((\left(\frac{\sigma}{2}\right) d^2 + \tau^2)}$
  $=>\frac{11459}{2(d^2)} \pm \sqrt{(\frac{11459}{2d^2})^2 + (\frac{5729.5}{d^2})^2}$

By solving

$\sigma_1(+) = \frac{1382.5}{d^2} = N/mm^2$

$\sigma_2(-) = -\frac{2373.5}{d^2} N/mm^2$

(Principal stress $\sigma_3 = 0$)

A. **By using principal stress theory:**

$\sigma_1 = \sigma_t$

$= \frac{1382.5}{d^2} = 7$

D=13.6

B. By using principal strain theory

$e = \frac{1}{E}[\sigma 1 - \mu (\sigma_2 + \sigma_3)]$

$$e = \frac{1}{E}\left[\frac{13832.51}{d^2} - 0.3\left(-\frac{2378.5}{d^2}\right) + 0\right)]$$

$=> \frac{1}{E}\left(\frac{14554.65}{d2}\right)$

$=> e = \frac{\sigma t}{E} => \frac{75}{E}$

$=> \frac{14544.55}{d^2 E}$

=>d=14mm

c. By using maximum shear Theory

$\tau = ½ (\sigma_1 - \sigma_2)$

$\tau = \frac{1}{2}\left[\frac{13832.5}{d^2} - \left(-\frac{2373.5}{d^2}\right)\right]$

$=> \frac{8103}{d^2}$ N/mm^2

$\frac{1}{2}(\sigma_t - 0) = \frac{\sigma t}{2} = \frac{75}{2} = 37.5$

Equating $37.5 = \frac{8103}{d^2}$

D =15 mm

d. By using shear strain energy theory

$\sigma_1^2 + \sigma_2^2 - 2\mu(\sigma_1 * \sigma_2) = \sigma_t^2$

$\left[\frac{13832.5}{d^2}\right]^2 + \left[\frac{-2373.5}{d^2}\right]^2 - 2(0.3)(\sigma_1 * \sigma_2) = 75^2$

By solving d= 14.01 mm

e. By maximum shear strain energy theory

$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_t^2$

D=14.213 mm