



Name :
Roll No. :
Invigilator's Signature :

CS/B.Sc.(H)/BT/ SEM-2/BMT-204/2012

2012

BIO-MATHEMATICS – II

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

**GROUP – A
(Multiple Choice Type Questions)**

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) In Lagrange's mean value theorem $f (x)$ should be continuous in

- a) closed interval b) open interval
c) semi-open interval d) none of these.

ii) Rank of $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ is

- a) 0 b) 1
c) 2 d) 3.



viii) The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is

- a) convergent b) divergent
c) oscillatory d) none of these.

ix) The eigenvalues of the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ are

- a) 1, -1 b) $i, -i$
c) 0, 0 d) 1, 1.

x) The sequence $\{u_n\}$, where $u_n = \frac{2n-1}{n+1}$, $n \in N$ is bounded by

- a) 2 b) 3
c) 0.5 d) none of these.

xi) If the straight lines $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z+3}{2}$ and $\frac{x-3}{2} = \frac{y-4}{\lambda} = \frac{z-2}{2}$ are perpendicular then the value of λ is

- a) 3 b) -3
c) 2 d) 1.



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Let (G, o) be a group. A non-empty subset H of G forms a subgroup of (G, o) iff $a \in H, b \in H \Rightarrow aob^{-1} \in H$.
- b) In a group G , if every element is its own inverse, then show that the group G is commutative.
- c) The binary operation o is defined on the sets of integers Z as $aob = a + b - 2$, for all a, b belong to Z . Show that (Z, o) is a group.

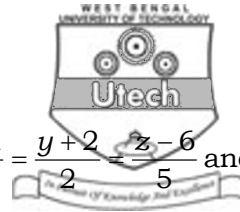
8. a) State Rolle's theorem. Verify the theorem for the following function :

$$f(x) = |x| + |x - 1|, x \in [-1, 2]$$

- b) State Cauchy's Mean Value Theorem. Then calculate c for the following :

$$f(x) = \sin x \text{ and } g(x) = \cos x \text{ on } \left[\frac{\pi}{4}, \frac{3\pi}{4} \right].$$

- c) Let $\{u_n\}$ and $\{v_n\}$ be two convergent sequences that converge to u & v respectively. Then show that $\lim(u_n \pm v_n) = u \pm v$.



9. a) Show that the straight lines $\frac{x-1}{2} = \frac{y+2}{2} = \frac{z-6}{5}$ and

$$\begin{aligned} 2x + y - 3z - 2 &= 0 \\ 3x + 2y + 5z + 7 &= 0 \end{aligned}$$

are perpendicular.

- b) Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

are co-planar.

- c) Find the equation of the straight line passing through the point (3, 1, -6) and parallel to the planes $x + y + 2z - 4 = 0$, $2x - 3y + z + 5 = 0$.

10. a) Verify that the limit of the sequence $\{x_n\}$, where

$$x_n = \frac{2n+1}{n-1}, \quad n \in N \text{ is } 2.$$

- b) Test the convergence of the series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2}$.

- c) Determine the interval of convergence of the power series $\sum a_n x^n$, where $a_n = \frac{1}{\log(n+1)}$.



11. a) Evaluate $\int_0^{\infty} e^{-4x} x^{\frac{3}{2}} dx$

b) Solve any *two* :

i) $\frac{d^2y}{dx^2} + 4y = 2x + 3$

ii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

iii) $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = x$

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