Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Sc.(H)/BT/ SEM-2/BMT-204/2012

2012

BIO-MATHEMATICS – II

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) In Lagrange's mean value theorem $f(x)$ should be continuous in

a) closed interval b) open interval

c) semi-open interval d) none of these.

ii) Rank of $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ is

a) 0 b) 1

c) 2 d) 3.

iii) Degree of the differential equation

$$x\cos x\left(\frac{dy}{dx}\right)^2 + y^3(x\sin x + \cos x) = x^9$$ is

a) 3 b) 1

c) 2 d) 0.

iv) The singularities of the integral $\int_{-1}^{2} \frac{dx}{x(x-1)}$ are

a) 0,1 b) 1, 2

c) –1, 2 d) 0, 2.

v) If for a mapping we have distinct images of distinct objects, then the mapping is

a) injective b) surjective

c) into d) invertible.

vi) If l, m, n are the direction cosines of a straight line in 3D then $l^2 + m^2 + n^2$ is equal to

a) 0 b) 1

c) 2 d) 3.

vii) The value of $\Gamma\left(\frac{7}{2}\right)$ is

a) $\frac{15}{8}\sqrt{\pi}$ b) $\frac{3}{4}\sqrt{\pi}$

c) $\sqrt{\pi}$ d) none of these.

viii) The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

ix) The eigenvalues of the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ are

a) 1, –1 b) $i, -i$

c) 0, 0 d) 1, 1.

x) The sequence $\{u_n\}$, where $u_n = \frac{2n-1}{n+1}$, $n \in N$ is bounded by

a) 2 b) 3

c) 0·5 d) none of these.

xi) If the straight lines $\frac{x+1}{1} = \frac{y-2}{-2} = \frac{z+3}{2}$ and $\frac{x-3}{2} = \frac{y-4}{\lambda} = \frac{z-2}{2}$ are perpendicular then the value of λ is

a) 3 b) –3

c) 2 d) 1.

xii) If $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - 4\hat{j} - 3\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} + 2\hat{k}$, then $|\vec{a} + \vec{b} + \vec{c}|$ is

a) $4\sqrt{2}$ b) $2\sqrt{2}$

c) $5\sqrt{2}$ d) $3\sqrt{2}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the point where line $\frac{x-2}{1} = \frac{y+3}{-1} = \frac{z-1}{6}$ meets the plane $3x + y + z = 7$.

3. Define semigroup. Let $G = \{1, -1, i. - i\}$. Prove that this set forms a group with respect to usual multiplication.

4. Find the rank of the given matrix, $\begin{pmatrix} 2 & 0 & 4 & 2 \\ 3 & 2 & 6 & 5 \\ 5 & 2 & 10 & 7 \\ 0 & 3 & 2 & 5 \end{pmatrix}$.

5. Show that the given mapping is neither injective nor subjective : $f(x) = |x|$, where x is a real number.

6. Show that any square matrix A and its transpose A^T have same eigenvalues.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Let (*G*, *o*) be a group. A non-empty subset *H* of *G* forms a subgroup of (*G*, *o*) iff $a \in H, b \in H \Rightarrow aob^{-1} \in H$.

   b) In a group *G*, if every element is its own inverse, then show that the group *G* is commutative.

   c) The binary operation *o* is defined on the sets of integers *Z* as $aob = a + b - 2$, for all *a*, *b* belong to *Z*. Show that (*Z*, *o*) is a group.

8. a) State Rolle's theorem. Verify the theorem for the following function :

   $f(x) = |x| + |x-1|, x \in [-1, 2]$

   b) State Cauchy's Mean Value Theorem. Then calculate *c* for the following :

   $f(x) = \sin x$ and $g(x) = \cos x$ on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

   c) Let $\{u_n\}$ and $\{v_n\}$ be two convergent sequences that converge to *u* & *v* respectively. Then show that $\lim(u_n \pm v_n) = u \pm v$.

9. a) Show that the straight lines $\frac{x-1}{2}=\frac{y+2}{2}=\frac{z-6}{5}$ and

$2x+y-3z-2=0$
$3x+2y+5z+7=0$ are perpendicular.

b) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and

$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are co-planar.

c) Find the equation of the straight line passing through the point (3, 1, −6) and parallel to the planes $x+y+2z-4=0$, $2x-3y+z+5=0$.

10. a) Verify that the limit of the sequence $\{x_n\}$, where $x_n=\frac{2n+1}{n-1}$, $n\in N$ is 2.

b) Test the convergence of the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$.

c) Determine the interval of convergence of the power series $\sum a_n x^n$, where $a_n=\frac{1}{\log(n+1)}$.

11. a) Evaluate $\int_0^{\infty} e^{-4x} x^{\frac{3}{2}} \, dx$

b) Solve any *two* :

i) $\frac{d^2y}{dx^2} + 4y = 2x + 3$

ii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 2e^{3x}$

iii) $x^2 \frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = x$

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