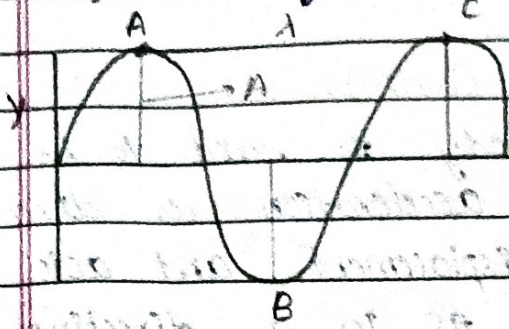


Periodic motion:

Motion repeats with the equal intervals of time is called periodic motion.

Types of periodic motion:

- i) Oscillatory motion
- ii) Vibratory motion



\* Displacement (x): The distance of location of a body from its mean position at a particular instant of time.

\* Amplitude (a): The maximum value of displacement that a body can undergo either side of its mean position during the oscillation.

\* Frequency (ν): The number of oscillations per unit time.  $\nu = \frac{n}{T}$

\* Angular frequency (ω) or Angular velocity: It is the angle covered in unit time by a body moving in circular motion at that instant of time.

$$\omega = 2\pi\nu \Rightarrow \omega = \frac{2\pi}{T}$$

\* Time period (T): It is the time taken by the body to complete one oscillation.  $T =$

Restoring force:

When a body is oscillating in a medium the action of force whose magnitude is proportional to displacement and acting in a direction opposite to displacement w.r.t. equilibrium position.

By Hooke's law,

$$F \propto -x$$

$$F = -kx$$

Simple Harmonic Motion (SHM):

The motion of a body is said to be SHM if the (restoring force) acceleration is directly proportional to the displacement and acts in a direction opposite to that of motion from the equilibrium position.

Restoring force  $\propto y$  (Displacement in opp. direction)

$$F_{\text{restoring}} \propto -y$$

$$y = a \sin \omega t$$

$$x = a \cos \omega t$$

$$\text{velocity} = \dot{y} = a \omega \cos \omega t$$

$$\frac{d(y)}{dt} = a \omega \cos \omega t \Rightarrow \text{velocity}$$

$$\frac{d^2 y}{dt^2} = -a \omega^2 \sin \omega t \Rightarrow \text{acceleration} = \frac{d^2 y}{dt^2} = -\omega^2 a \sin \omega t$$

$$\frac{d^2 y}{dt^2} = -\omega^2 y \quad [ \because a \sin \omega t = y ]$$

$$\begin{aligned} \text{velocity} &= a\omega \cos \omega t \\ &= a\omega \sqrt{1 - \sin^2 \omega t} \\ &= \omega \sqrt{a^2 - a^2 \sin^2 \omega t} \end{aligned}$$

$$[\because \cos \omega t = \sqrt{1 - \sin^2 \omega t}]$$

$$\text{velocity} = \omega \sqrt{a^2 - y^2}$$

Note:

- \* When body at mean position, velocity is maximum, acceleration is zero
- \* When body at extreme position, acceleration is maximum, velocity is zero.

Expression for SHM

By definition of SHM,  
 $F_{\text{restoring}} \propto -y$

$$F_{\text{restoring}} = -ky \quad \text{--- (1)} \quad [\because K \text{ is a force constant / stiffness factor}]$$

$$K = -\frac{F_{\text{restoring}}}{y}$$

[It is the ratio of force exerted on the body to the displacement]

According to Newton's 2<sup>nd</sup> law,

$$F = ma$$

$$F = m \frac{d^2y}{dt^2} \quad \text{--- (2)} \quad [\because a = \frac{d^2y}{dt^2}]$$

Equating eq<sup>n</sup> (1) + (2)

$$-ky = m \frac{d^2y}{dt^2}$$

DATE \_\_\_\_\_ PAGE \_\_\_\_\_

$$m \frac{d^2y}{dt^2} + ky = 0$$

$$\frac{d^2y}{dt^2} + \frac{k}{m} y = 0 \quad [ \div m ]$$

Replacing  $k/m$  by  $\omega^2$

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0}$$

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}} \quad \text{--- (3)}$$

$$\omega = 2\pi f \quad \text{--- (4)}$$

By equating eq<sup>n</sup> (3) + (4)

$$2\pi f = \sqrt{\frac{k}{m}}$$

$$\boxed{f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}}$$

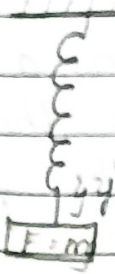
W.K.T.  $T = \frac{1}{f}$

$$\boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

Characteristics of SHM

- \* It is a free and zero motion
- \* It is a periodic motion
- \* Force is proportional to displacement
- \* Acceleration is in the opposite direction of displacement
- \* Restoring force is essential for SHM

# Mechanical Simple Harmonic Oscillator



$$y = a \sin(\omega t + \phi)$$

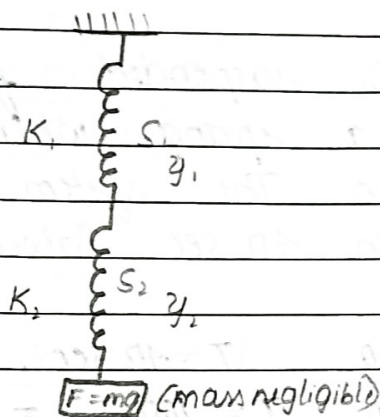
$\phi$  = Phase difference  
 $\omega$  = angular velocity

Consider a mass attached to spring of negligible mass which is suspended from a rigid support as showing fig.

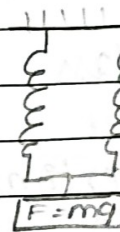
The oscillations in the spring are due to restoring force developing in the spring

## Springs in Series and Parallel

Series :



Parallel :



$$Y = Y_1 + Y_2$$

$$F = -K_y$$

$$y = \frac{-F}{K}$$

$$F_p = F_1 + F_2$$

$$K_y \cong F$$

$$K_p Y = K_1 Y + K_2 Y$$

$$K_p = K_1 + K_2$$

$$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_s = \frac{K_1 K_2}{K_1 + K_2}$$

## Problems

- 1) Man weighing 600N steps on a spring scale machine. The spring in the machine is compressed by 1cm. Find the force constant of the spring.

Given,

$$W = F = mg = 600\text{N}$$

$$x = 1\text{cm} \Rightarrow 1 \times 10^{-2}\text{m}$$

$$K = ?$$

$$F = Kx$$

$$K = \frac{F}{x} \Rightarrow K = \frac{600}{1 \times 10^{-2}}$$

$$K = 6 \times 10^2 \times 10^2 \Rightarrow \boxed{K = 6 \times 10^4 \text{ N/m}}$$

- 2) Mass of 5kg is suspended from the free end of a spring when set for vertical oscillation. The system executes 100 oscillations in 40 sec. Calculate Force constant.

$$m = 5\text{kg}, n = 100, T = 40 \text{ sec}, K = ?$$

$$\text{Time for oscillation } T = \frac{100}{40} = 2.5 \text{ sec}$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$T^2 = 4\pi^2 \frac{m}{K} \quad [\text{squaring on B.S.}]$$

$$K = \frac{4\pi^2 m}{T^2}$$

$$= \frac{4 \times (3.14)^2 \times 5}{(2.5)^2}$$

$$= 4 \times 9.856 \times \frac{5}{6.25}$$

$$= 4 \times 9.856 \times 0.8$$

$$\boxed{K = 31.53 \text{ N/m}}$$

3) Mass 0.5 kg poses an extension 0.03 m in a spring and the system is set for oscillations. Find

- i) Force constant
- ii) Angular frequency
- iii) Time period

$$m = 0.5 \text{ kg}, \quad x = 0.03 \text{ m}, \quad K = ?, \quad \omega = ?, \quad T = ?$$

$$K = F/x$$

$$F = mg$$

$$F = 0.5 \times 9.8$$

$$\boxed{F = 4.9 \text{ N}}$$

$$K = \frac{4.9}{0.03}$$

$$\boxed{K = 163.3 \text{ N/m}}$$

$$\omega = \sqrt{\frac{K}{m}} \Rightarrow \omega = \sqrt{\frac{163.3}{0.5}} = \frac{12.76}{0.70}$$

$$\boxed{\omega = 18.2 \text{ rad/sec}}$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$T = 2(3.14) \sqrt{\frac{0.5}{163}}$$

$$T = 6.28 \left( \frac{0.70}{12.77} \right) \Rightarrow T = \frac{4.396}{12.77} = \underline{\underline{0.34 \text{ sec}}}$$

- 4) An electric motor weighing 50 kg is mounted on four springs each of which has a spring constant  $2 \times 10^3 \text{ N/m}$ . The motor moves only in vertical direction. Find the frequency of system.

$$m = 50 \text{ kg}$$

$$K = 2 \times 10^3 \Rightarrow 2000$$

$$f = \frac{1}{2\pi} \sqrt{\frac{K_{\text{eff}}}{m}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{4 \times 2000}{50}}$$

$$= \frac{1}{6.28} \sqrt{160}$$

$$= \frac{12.64}{6.28}$$

$$f = 2.01 \text{ Hz}$$

- 5) Find the frequency of oscillation of a particle executing SHM of amplitude 0.35 m. If the maximum velocity it can attain is 220 m/s.

$$a = 0.35 \text{ m}, \quad v = 220 \text{ m/s}$$

$$v = \omega a$$

$$\omega = \frac{v}{a} \Rightarrow \omega = \frac{220}{0.35}$$

$$\omega = 628.57 \text{ rad/sec}$$

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{628.57}{6.28}$$

$$f = 100 \text{ Hz}$$



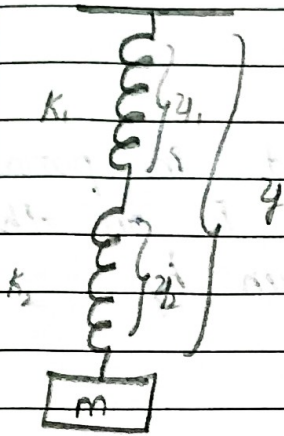
Natural frequency:

When a body exhibits free oscillation the frequency with which oscillation occurs is called natural frequency.

Springs in series and parallel

Consider two springs of negligible masses with force constant  $k_1$  &  $k_2$ ,

Let us calculate effective spring constant when two springs are connected in series to a mass or connected in parallel.



When springs are connected in series to a mass

The net extension  $y$  is given by  $y = y_1 + y_2$

$y$  is the total extension

$y_1$  is the extension in spring with force constant  $k_1$ .

$y_2$  is the extension in spring with force constant  $k_2$ .

If  $k_s$  is the effective force constant of the combination.

If  $F$  is the force applied on the combination.

$$y = y_1 + y_2 \quad \text{--- (1)}$$

$$-\frac{F}{K_s} = -\frac{F}{K_1} - \frac{F}{K_2}$$

$$-\frac{F}{K_s} = -F \left( \frac{1}{K_1} + \frac{1}{K_2} \right)$$

$$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_s = \frac{K_1 K_2}{K_1 + K_2}$$

Effective force constant of series combination

Springs in parallel

When springs are connected in parallel to a mass the net force  $F$  on the system of spring is given by

$$F = F_1 + F_2$$

$F_1$  is the force on the spring with force constant  $K_1$ .

$F_2$  is the force on the spring with force constant  $K_2$ .

If  $K_p$  is the effective force constant of the combination

If  $y$  is the total extension in the combination

Then we get:

$$F_p = F_1 + F_2$$

$$K_p y = K_1 y + K_2 y$$

$$K_p y = y(K_1 + K_2)$$

$$K_p = K_1 + K_2$$

## Application of spring

### Compression spring:

A compression spring is an elastic coil made of spring steel.

It is a spring characteristic is that it opposes forces or opposes resistance

structure and working.



Compression spring are very effective at building and it has gaps between its coil in a unloaded state.

The distance between the coil is reduced when the spring is loaded and compressed.

### Uses

A compression spring can be used as pure energy accumulator, shock absorber, suspension.

Vibration damper or force generator

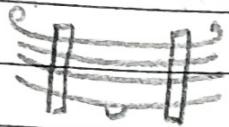
Used to reduce the amount of vibration that you feel when a

It absorbs the vibrations generated by the driving shaft.

It is important that not to be overloaded.

Leaf springs

Structure and working



Uses

The leaf spring is used to control the height at which the vehicle rides and keep the tires aligned on the road.

Leaf springs are most useful for loaded trucks and tanks haulings and heavy loads.

Types of oscillations

i) Free oscillations

ii) Damped oscillation

iii) Forced oscillation

## Theory of Damped Oscillation

A body executing vibrations (oscillations) if the amplitude keeps on decreasing because of frictional resistive force offered by the medium in which the oscillations are carried out.

Due to the resistive force the vibrations die out after some time the motion is said to be damped due to resistive force and hence is called damped oscillations.

$$F_{\text{restoring}} \propto -y \text{ (displacement)}$$

$$F_{\text{resistive}} \propto -v \text{ (velocity)}$$

By Newton's II law

$$F = m \frac{d^2y}{dt^2}$$

$$F_{\text{restoring}} = -ky, \quad F_{\text{resistive}} = -r \frac{dy}{dt}$$

Total force acting on the body

$$F = F_{\text{restoring}} + F_{\text{resistive}}$$

$$m \frac{d^2y}{dt^2} = -ky - r \frac{dy}{dt}$$

$$m \frac{d^2y}{dt^2} + r \frac{dy}{dt} + ky = 0$$

$$\frac{d^2y}{dt^2} + \frac{r}{m} \frac{dy}{dt} + \frac{k}{m} y = 0 \quad \left[ \frac{r}{m} = 2b, \frac{k}{m} = \omega^2 \right]$$

$$\frac{d^2y}{dt^2} + 2b \frac{dy}{dt} + \omega^2 y = 0 \quad \left[ \frac{r}{m} = 2b, \frac{k}{m} = \omega^2 \right]$$

The solution for differential equation

$$y = Ae^{\lambda t} \quad [A \text{ is a arbitrary constant}]$$

$$\frac{dy}{dt} = \lambda Ae^{\lambda t}$$

$$\frac{d^2y}{dt^2} = \lambda^2 Ae^{\lambda t}$$

$$\textcircled{1} \Rightarrow \lambda^2 Ae^{\lambda t} + 2b\lambda Ae^{\lambda t} + Ae^{\lambda t}\omega^2 = 0$$

$$Ae^{\lambda t} [\lambda^2 + 2b\lambda + \omega^2] = 0$$

$$Ae^{\lambda t} \neq 0$$

$$\lambda^2 + 2b\lambda + \omega^2 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-2b \pm \sqrt{(2b)^2 - 4(1)(\omega^2)}}{2(1)}$$

$$= \frac{-2b \pm \sqrt{4b^2 - 4\omega^2}}{2}$$

$$= \frac{-2b \pm 2\sqrt{b^2 - \omega^2}}{2}$$

$$= -b \pm \sqrt{b^2 - \omega^2}$$

The general solution of differential equation

$$y = A_1 \text{ Exponential } [-b + \sqrt{b^2 - \omega^2}] + A_2 \text{ Exponential } [-b - \sqrt{b^2 - \omega^2}]$$

Damped oscillation

Case I -  $b \gg \omega$ , over damping

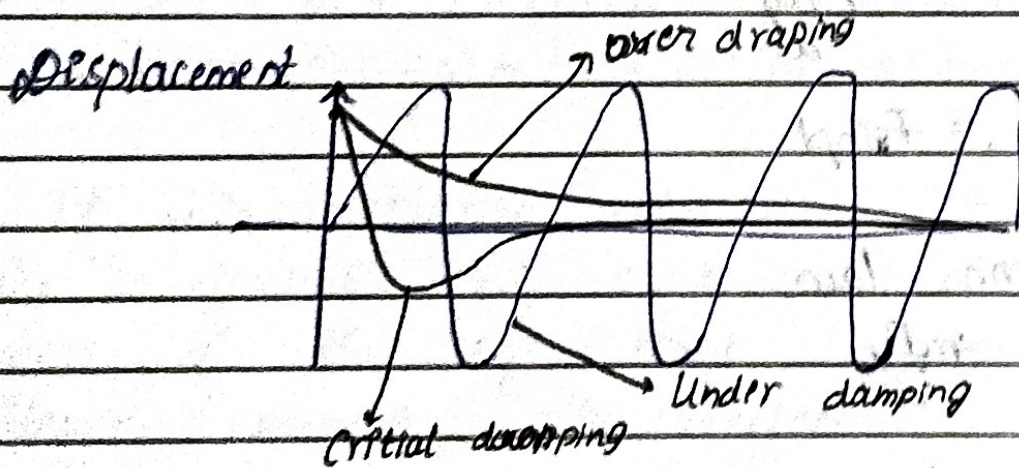
Case II -  $b \ll \omega$  Under damping

Case III -  $b = \omega$  Critical damping

-son for damped oscillation is  $b^2 < 4mk$

Over damping: Oscillations are said to be over damped or heavy damped when the system attains equilibrium state quite slowly without making oscillations. The condition for over damping is  $b^2 > 4mk$ .

Critical damping: When the system approaches equilibrium state quite quickly without making any oscillations is called critical damping. The condition for critical damping is given by  $b = 2\sqrt{mk}$ .



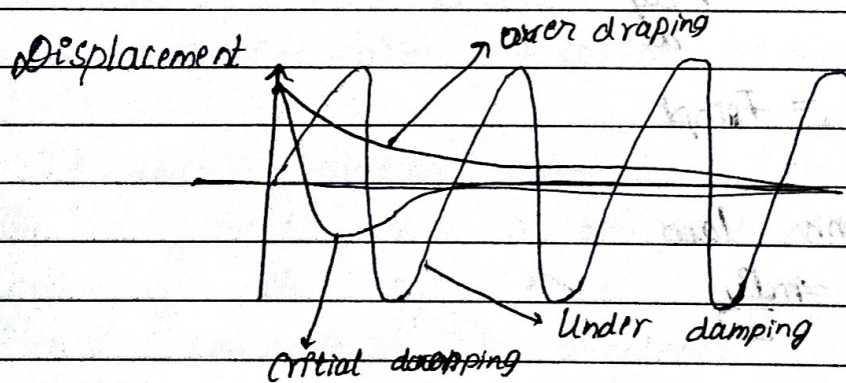
Quality factor: The energy ratio of  $2\pi$  times of energy stored in the system to the energy lost per unit time.

$$Q = 2\pi \left[ \frac{\text{Energy stored in the system}}{\text{Energy lost per unit time}} \right]$$

Under damping: Oscillations are said to be under damped or weakly damped if the retarding force is weaker than the restoring force. The amplitude of oscillations decreases with respect to time. The condition for damped oscillation is  $b^2 < 4mk$ .

Over damping: Oscillations are said to be over damped or heavy damped when the system attains equilibrium state quite slowly without making oscillations. The condition for over damping is  $b^2 > 4mk$ .

Critical damping: When the system approaches equilibrium state quite quickly without making any oscillations is called critical damping. The condition for critical damping is given by  $b^2 = 4mk$ .



Quality factor: The energy ratio of  $2\pi$  times of energy stored in the system to the energy lost per unit time.

$$Q = 2\pi \left[ \frac{\text{Energy stored in the system}}{\text{Energy lost per unit time}} \right]$$



## Theory of Forced Oscillations

When the body is subjected to the external periodic force with its natural frequency under SHM then the kind of SHM is called "Forced Oscillation".

If the body vibrates with the other frequency than its natural frequency then the kind of SHM is called "Forced Oscillations".

$$\frac{d^2y}{dt^2} + \frac{k}{m}y + \frac{r}{m}\frac{dy}{dt} = \frac{F}{m} \sin pt$$

$$\frac{d^2y}{dt^2} + \omega^2 y + 2b\frac{dy}{dt} = f \sin pt$$

$$F_{\text{restoring}} = -ky$$

$$F_{\text{resistive}} = -r\frac{dy}{dt}$$

$$F_{\text{external}} = F \sin pt$$

Newton's law,

$$F = m\frac{d^2y}{dt^2}$$

Total force acting on the body is

$$F = -ky - r\frac{dy}{dt} + F \sin pt$$

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F \sin pt$$

$$\frac{d^2y}{dt^2} + \frac{k}{m} \frac{dy}{dt} + \frac{r}{m} \frac{dy}{dt} = \frac{F}{m} \sin pt$$

$$\frac{d^2y}{dt^2} + \omega^2 y + 2b \frac{dy}{dt} = f \sin pt \quad \text{--- (1)}$$

The gen sol<sup>n</sup> for above differential eq<sup>n</sup>

$$y = A \sin(pt - \theta)$$

$$\frac{dy}{dt} = A \cos(pt - \theta) p$$

$$\frac{d^2y}{dt^2} = -A p^2 \sin(pt - \theta)$$

Substitute in eq<sup>n</sup> (1)

$$-A p^2 \sin(pt - \theta) + \omega^2 A \sin(pt - \theta) + 2b A \cos(pt - \theta) p = f \sin(pt - \theta)$$

$$= A p^2 \sin(pt - \theta) + 2b A \cos(pt - \theta) p + \omega^2 A \sin(pt - \theta) \\ = f (\sin(pt - \theta) \cos \theta + \cos(pt - \theta) \sin \theta)$$

If this relation ~~holds~~ denotes for all values of  $t$   
The co-efficient  $\sin(pt - \theta)$  &  $\cos(pt - \theta)$  on both  
sides of the eq<sup>n</sup> must be equal  
Hence comparing the co-efficient of  $\sin(pt - \theta)$   
&  $\cos(pt - \theta)$  on both side

$$A(\omega^2 - b^2) \sin(pt - \theta) + 2b A \cos(pt - \theta) p = f \sin(pt - \theta) \cos \theta + f \cos(pt - \theta) \sin \theta$$

$$A(\omega^2 - p^2) = f \cos \theta \quad \text{--- (2)}$$

$$2b p A = f \sin \theta \quad \text{--- (3)}$$

Squaring on BS of eq<sup>n</sup> (2) & (3)

$$A^2(\omega^2 - p^2)^2 = f^2 \cos^2 \theta \quad \text{--- (4)}$$

$$4b^2 p^2 A^2 = f^2 \sin^2 \theta \quad \text{--- (5)}$$

Adding eq<sup>n</sup> (4) & (5)

$$A^2(\omega^2 - p^2)^2 + 4b^2 p^2 A^2 = f^2 (\sin^2 \theta + \cos^2 \theta)$$

$$A^2(\omega^2 - p^2)^2 + 4b^2 p^2 A^2 = f^2$$

$$A^2 [(\omega^2 - p^2)^2 + 4b^2 p^2] = f^2$$

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}}$$

$$A = \frac{f}{\sqrt{\omega^2 - p^2 + 4b^2 p^2}}$$

Divide eq<sup>n</sup> (5) by (4)

$$\frac{f \sin \theta}{f \cos \theta} = \frac{2b p A}{A(\omega^2 - p^2)}$$

$$\tan \theta = \frac{2bp}{\omega^2 - p^2}$$

$$\theta = \tan^{-1} \left( \frac{2bp}{\omega^2 - p^2} \right)$$

Case 1: When driving frequency is low  
 $p \ll \omega$      $p \approx 0$

Case 2: The amplitude of the vibration is given by  $p \approx \omega$

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}}$$

$$A = \frac{f}{\omega^2}$$

$$\theta = \tan^{-1} \left( \frac{2bp}{\omega^2 - p^2} \right)$$

$$\theta = \tan^{-1}(0) = 0$$

This shows that the amplitude of the vibration is independent of force the amplitude depends on magnitude of the applied force & the force constant. The force & the displacement are always in phase.

When  $P = \omega$  i.e. the frequency of force is equal to the frequency of body the amplitude of vibration is given by

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}} = \frac{f}{2b\omega} = \frac{f}{\frac{r}{m}\omega}$$

$$= \frac{f/m}{r\omega}$$

$$\theta = \tan^{-1} \left( \frac{2bp}{\omega^2 - p^2} \right) = \tan^{-1} \left( \frac{2bp}{0} \right) = \tan^{-1}(\infty)$$

$$\theta = \pi/2$$

Thus, the amplitude of the vibration is covered by damped & for small damping force the amplitude of vibration is quite large hence the displacement lags behind the force  $= \pi/2$ .

Case 3:  $P > \omega$  The frequency of the force is greater than the natural frequency  $\omega$  of the body.

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}}$$

$$A = \frac{f}{(-p^2)^2 + 4b^2 p^2} = \frac{f}{-p^2 + 2bP}$$

$$= \frac{-f}{p^2}$$

$$\theta = \tan^{-1} \frac{2bP}{(\omega^2 - p^2)}$$

$$\therefore = \tan^{-1} \left( \frac{2bP}{\omega^2 - p^2} \right)$$

$$= -\tan^{-1} \left( \frac{2bP}{-p^2} \right) = \frac{0}{p^2}$$

$$\theta = \tan^{-1} (-0)$$

$$\theta = \pi$$

Thus, This case the amplitude a goes on decreasing & phase difference tends towards  $\pi$

- Resonance, Sharpness of resonance

Sharpness of resonance: The ratio of change in the amplitude to the change in the frequency.

Sharpness of Resonance =  $\frac{\text{change in amplitude}}{\text{change in frequency}}$

$$= \frac{A_{\max} - A_{\min}}{(\omega - \omega_0)} \quad \text{--- (1)}$$

$$A_{\max} = \frac{F/m}{2b\omega} \quad \text{--- (1)} = \frac{F/m}{\gamma/\omega} \Rightarrow \frac{F}{\gamma\omega}$$

$$A = \frac{f}{\sqrt{(\omega^2 - p^2)^2 + 4b^2p^2}}$$

$$= \frac{F/m}{\sqrt{(\omega^2 - p^2)^2 + 4b^2 p^2}} \quad [(\omega^2 - p^2)^2 = 0]$$

$$= \frac{F/m}{\sqrt{4b^2 p^2}}$$

$$= \frac{F/m}{2bp}$$

$$= \frac{F/m}{2b\omega} \quad \text{--- (3)} \quad [p = \omega]$$

Substitute (2) & (3) in (1)

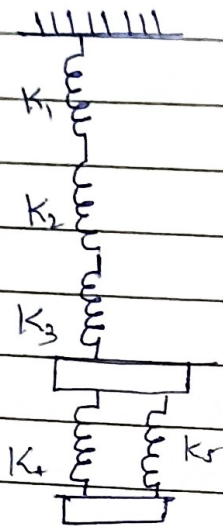
$$= \frac{\frac{F/m}{2b\omega} - \frac{F/m}{2bp}}{(p - \omega)}$$

$$= \frac{\frac{F/m}{2b} \left[ \frac{1}{\omega} - \frac{1}{p} \right]}{(p - \omega)}$$

$$= \frac{\frac{F/m}{2b} \left[ \frac{p - \omega}{\omega p} \right]}{(p - \omega)}$$

$$= \frac{F/m}{2b\omega p}$$

1) In the 2 mass spring system shown in the figure  $K_1 = 2000 \text{ N/m}$ ,  $K_2 = 1500 \text{ N/m}$ ,  $K_4$  &  $K_5$  are equal to  $500 \text{ N/m}$ . Find mass such that the system has a natural frequency of  $10 \text{ Hz}$   $K_3 = 3000 \text{ N/m}$ .



$$\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$$

$$= \frac{1}{2000} + \frac{1}{1500} + \frac{1}{3000}$$

$$\frac{1}{K_s} = 5 \times 10^{-4} + 6.66 \times 10^{-4} + 3.33 \times 10^{-4}$$

$$= 14.99 \times 10^{-4} \approx 15 \times 10^{-4}$$

$$= \cancel{0.06666} \times 10^4 = \underline{\underline{6666 \text{ N/m}}}$$

$$= \cancel{6666} \text{ N/m}$$

$$K_p = 500 + 500$$

$$= \underline{\underline{1000 \text{ N/m}}}$$

$$\frac{1}{K_H} = \frac{1}{667} + \frac{1}{1000}$$

$$= 1.499 + 0.001$$

$$\frac{1}{K_{eff}} = 1.5 \Rightarrow \underline{\underline{0.66 \text{ N/m}}}$$

$$= 0.0014 + 0.001$$

$$K_{eff} =$$

2) Three particles executing SHM in a straight line with a period of 25 sec, 5 sec after it has crossed the equilibrium point the velocity is found to be 0.7 m/s. Find the displacement at the end of 10 sec also find the amplitude of oscillation.

$$t = 5 \text{ sec} \quad T = 25 \text{ sec} \quad v = 0.7 \text{ m/s}$$

$$y = a \sin \omega t$$

$$\omega = 2\pi f$$

$$= 2 \times 3.14 \times$$

$$= 0.25$$

$$\Rightarrow$$

$$a = \frac{v}{\omega}$$

$$a = \frac{v}{\omega \cos \omega t}$$

$$a = \frac{v}{\omega \cos \omega t}$$

$$= \frac{0.7}{0.25 \cos(0.25)(5)}$$

$$= \frac{0.7}{0.25 \cos(1.25)}$$

$$= \frac{0.7}{0.25(0.99)}$$

$$= \frac{0.7}{0.25}$$

$$= 2.8 \text{ m}$$

$$a \Rightarrow 2.86 \text{ m}$$

$$y = a \sin \omega t$$

$$= 2.8 \sin(0.25)(10)$$

$$= 2.8 \sin(2.5) \Rightarrow 0.112 \text{ m}$$

$$y = 0.112 \text{ m}$$



3. A sonometer wire under tension is plucked and left free for vibrations. Find its frequency of vibrations if the mid point on the string attains a maximum velocity of 1.57 m/s when its amplitude of oscillation is 5 mm. Treat the vibration as SHM.

$$v = 1.57$$

$$v = \omega a \quad \omega = ?$$

$$a = 5 \text{ mm}$$

$$2\pi f =$$

$$v = \omega a$$

$$\omega = \frac{v}{a}$$

$$2\pi f = \frac{1.57}{5 \times 10^{-3}}$$

$$= \frac{1.57}{6.28 \times 5 \times 10^{-3}} = \frac{1.57}{31.4 \times 10^{-3}}$$

$$= 0.05 \times 10^3$$

$$\boxed{f = 50 \text{ Hz}}$$

4. A body of mass 500 g is attached to a spring and the system is driven by an external periodic force of amplitude 15 N of frequency 0.796 Hz. The spring extends by a length of 88 mm under the given load. Calculate the amplitude of oscillation if the resistance co-efficient of the medium is 5.05 kg/s.

$$m = 500 \times 10^{-3} = 0.5 \text{ kg}$$

$$F = 15 \text{ N}$$

$$f = 0.796 \text{ Hz}$$

$$y = 88 \times 10^{-3} \text{ m} = 0.88 \text{ m}$$

$$b = 5.05 \text{ kg/s}$$

$$F = r \frac{dy}{dt} \Rightarrow 15 = 0.05 \times \frac{dy}{dt} \quad w = 2\pi f$$

$$\frac{dy}{dt} = \frac{15}{0.05} = 300 \text{ m/s}$$

$$= 8.28$$

$$w = 4.99 \text{ m}$$

$$\frac{dy}{dt} = 300 \text{ m/s}$$

$$K = \frac{F/m}{b} \quad w = \sqrt{\frac{K}{m}}$$

$$K = \frac{F}{2} = \frac{15}{0.88} = 17.04 \text{ N/m} = 55.6 \text{ N/m}$$

$$w = \sqrt{\frac{17.04}{0.5}} \Rightarrow \sqrt{34.08} \Rightarrow 5.83$$

$$w = \sqrt{\frac{55.6}{0.5}} = 10.54$$

$$b = 5.05$$

$$w = \sqrt{\frac{55.6}{0.5}} = 10.54$$

$$b = \frac{5.05}{2} \Rightarrow 0.25$$

$$A = \frac{F/m}{\sqrt{4b^2p^2 + (w^2 - p^2)^2}}$$

$$= \frac{15/0.5}{\sqrt{4(0.25)^2(4.99)^2 + ((10.5)^2 - (4.99)^2)^2}}$$

$$= \frac{30}{\sqrt{4(0.0625)(24.90) + (24.90 - 0.25)^2}}$$

$$= \frac{30}{\sqrt{6.225 + (24.65)^2}} \Rightarrow \frac{30}{85.28} = 0.3$$

$$\sqrt{6.225 + (24.65)^2}$$

$$\frac{30}{85.28} = 0.3$$

## Shock waves

Ans.

Acoustic waves These are the normal waves  
(Sound speed 332 m/s)

Frequency of sound  $\rightarrow 20-20\text{KHz}$

Types of waves

- i) Subsonic  $\rightarrow M < 1$
- ii) Transonic  $\rightarrow 0.8 < M < 1.2$
- iii) Supersonic  $\rightarrow M > 1$
- iv) Hypersonic  $\rightarrow M > 5$

Mach number,

$$M = \frac{\text{speed of object}}{\text{speed of sound}}$$

Shock waves:

A shock wave is narrow surface that manifests as a discontinuity in a fluid medium in which it is propagating with supersonic speed. The disturbance is characterized by sudden increase in pressure, temperature and density of the gas through which it is propagating.

Mach number

It is the ratio of speed of an object to the speed of the sound in the surrounding medium.

$$M = \frac{\text{Speed of object}}{\text{Speed of sound}} \Rightarrow M = \frac{V_o}{V_s}$$

## Types of waves

1) Subsonic

2) Transonic

3) Supersonic

The waves which are travels with, in the speed less than the speed of sound in a medium are called subsonic waves ( $M < 1$ )

4) Transonic

The speed overlapping b/w subsonic and supersonic speed is the transonic waves ( $0.8 < M < 1.2$ )

5) Supersonic

The waves which travel with speed greater than the speed of sound in a medium are called supersonic waves ( $M > 1$ )

6) Hypersonic

If the mach number of the moving object is greater than 5 such waves are called hypersonic waves ( $M > 5$ )

## Properties of shock waves

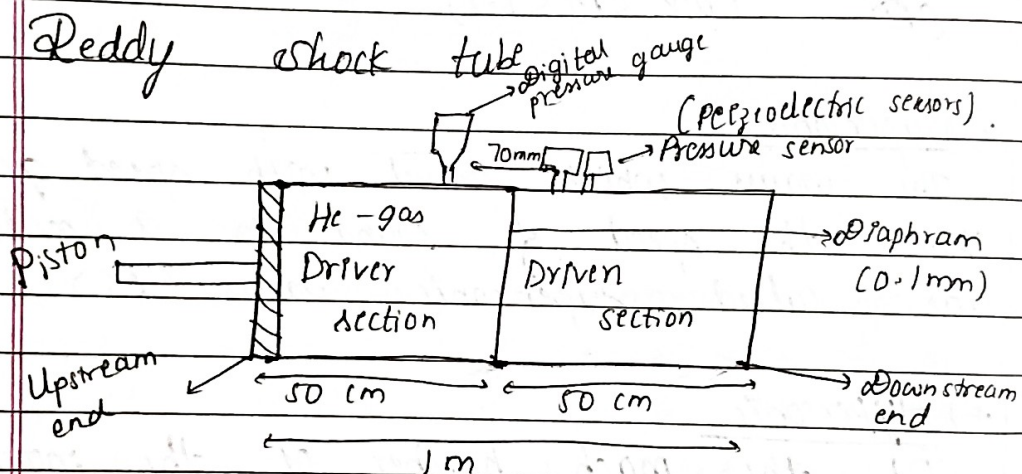
- \* Shock waves travel in a medium with the mach number greater than 1
- \* Shock waves obey the law of fluid dynamics

- \* When shock waves pass through a medium, the entropy of the system increases
- \* When shock waves pass through a medium the changes are adiabatic
- \* General wave properties cannot be associated with shock waves

### Mach angle

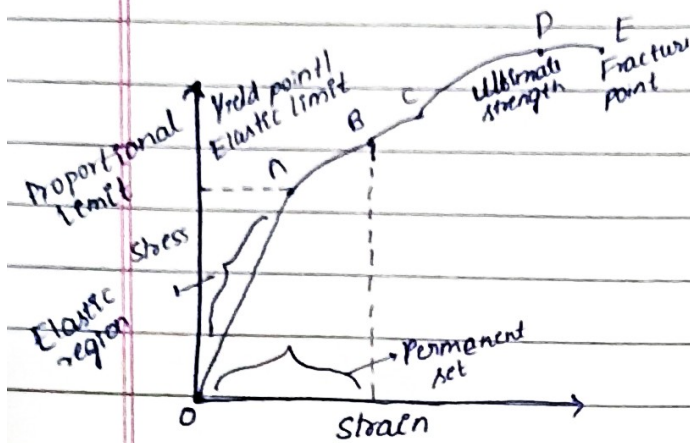
Mach angle is half of the vertex angle of a mach cone whose sign is the ratio of speed of the sound to the speed of the moving body.

$$\theta = \sin^{-1} \left( \frac{1}{M} \right)$$



It has is divided into two section as driver section & driven section

## Stress - Strain Curve



- 1) OA  $\rightarrow$  Elastic region  $\rightarrow$  Hooke's law is applied
- 2) AB  $\rightarrow$  Elastic limit  $\rightarrow$  There is a loss of proportionality or Yield point - try b/w stress and strain
- 3) BC  $\rightarrow$  Permanent set  $\rightarrow$
- 4) CD  $\rightarrow$  Ultimate strength
- 5) DE  $\rightarrow$  E pt  $\rightarrow$  Fracture point  $\rightarrow$  Where the deforms

1) Consider a bar of uniform cross section fixed at one end and loaded at other the stress - strain relation could be understand by plotting the stress-strain curve

- 1) The region OA is the elastic region in which Hooke's law is observed with the proportionality of stress v/s strain
- 2) Between the points A and B the material ~~accelerates~~ exhibits elastic material but lost the proportionality b/w stress and strain the point B is called Elastic limit or yield point.
- 3) Between the points B and C: The material posses permanent strain and plastic behaviour is observed ~~at~~ ~~it~~ ~~is~~ thus the material is said to be in permanent set.

- 4) The point D corresponds to the maximum stress that the material can withstand and is called point of ultimate strain
- 5) At the point 'E' material breaks and the body no more exists in single piece. Hence, point 'E' is called fracture point
- 6) In the region DE strain hardening and strain softening is observed

### Elastic Moduli:

- 1) Young's Modulus, longitudinal stress  $\rightarrow$  linear strain
- 2) Bulk Modulus, compressive stress  $\rightarrow$  volume strain
- 3) Rigidity Modulus, shearing stress  $\rightarrow$  shearing strain

Limiting values for  $\sigma$

$$Y = 2\eta(1+\sigma)$$

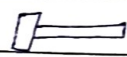
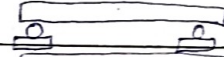


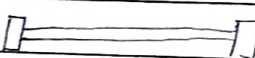

$$Y = 3K(1-2\sigma)$$

$$2\eta(1+\sigma) = 3K(1-2\sigma)$$

Bending of Beams:

- 1) Weight than stress applied
- 2) Cross sectional area
- 3) No shearing forces

Types of Beam

- 1) Cantilever 
- 2) Simply supported 
- 3) Over hanging 
- 4) Continuous Beam 
- 5) Fixed ended 
- 6) Cantilever simply supported 

Bending moment & Expression for bending moment

The moment of applied couple due to which the beam bends longitudinally is called bending moment

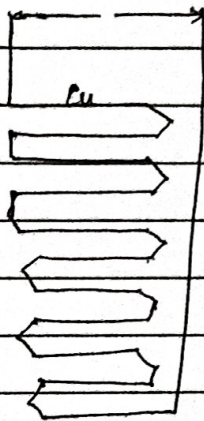
Consider a Beam. Let AB, CD and EF be three layers before applying the load. Hence,  $AB = CD = EF$ . The beam bends by applying load. Hence, AB extends to A'B'. Similarly, EF contracts to E'F' but the layer CD remains same even after the action of load. Hence, it is called neutral surface.



Disadvantages:

- The output voltage reduced is low
- The stray magnetic field can introduce errors in output voltage Hence, the accuracy is low

Thermo-pile: Thermo-pile is an electronic device that converts ~~electr~~ thermal energy to electrical energy. This composed of several thermo-couples connected usually in series or less commonly in parallel

Construction:

The output voltage of single thermo-couple is extremely small so, a number of couples are connected in series or parallel to get a large output. The arrangement of this number of thermo-couple is called thermo-pile

Working: These thermo-couples are arranged to out the cold and hot areas and the hot junctions are isolated thermally from the cold junction. Due to the temp<sup>r</sup> variations the thermo EMF across the two materials

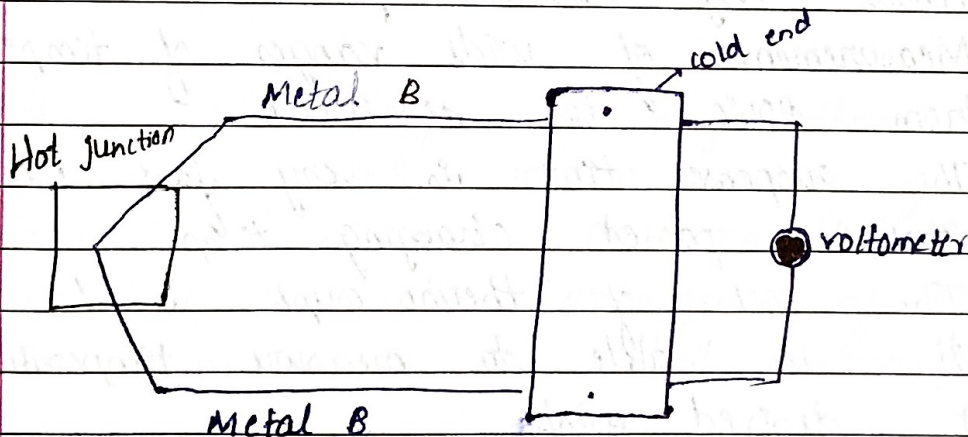
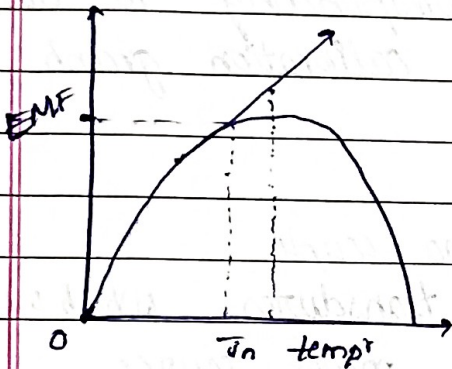
The beam forms an arc of a circle of radius  $R$  and subtends an angle  $\theta$ . The layer  $A'B'$  is concentric to  $CD$  and  $EF$  the radius  $R+r$

Thermo-couple & thermopile

Thermo-couple:

principle: It is based on the principle of Seebeck effect

Def<sup>n</sup>: A thermo couple is a transducer that converts thermal energy into electrical energy



Consider dissimilar metals A & B which are joined at two ends. The thermo-couple is connected or joined at two ends.

## Measurement of temperature

The thermo couple is calibrated by studying by variation of thermo EMF as a function of the temperature of hot junction keeping the temperature in temperature of cold junction ( $0^{\circ}\text{C}$ )

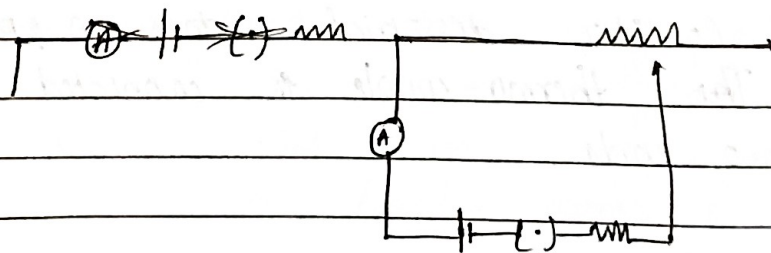
The graph of thermo EMF vs  $T_{\text{hot}}$  junction is plotted this graph is known as calibration graph

The hot junction is placed in a bath of unknown temperature and the EMF developed in the thermo couple is noted using voltmeter.

The temperature corresponding to this EMF is found from the calibration graph

## Advantages of thermo-couple

- \* This is a active transducers which works without any external power source
- \* Measurement of wide ranges of temperature from  $0^{\circ}\text{C}$  to  $2800^{\circ}\text{C}$ .
- \* The response time is very fast which can measure ~~is~~ passed changing temp<sup>s</sup>
- \* The cost of thermo-couple is low
- \* It is able to measure temperatures at desired points

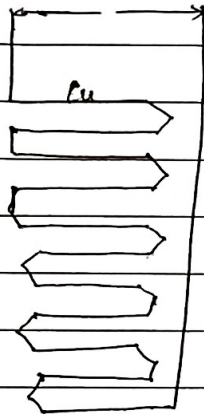


Disadvantages:

- > The output voltage reduced is low.
- x The strain magnetic field can introduce errors in output voltage Hence, The accuracy is low.

Thermo-pile: Thermo-pile is an electronic device that converts ~~electr~~ thermal energy to electrical energy. This composed of several thermo-couples connected usually in series or less commonly in parallel.

Construction:



The output voltage of single thermo-couple is extremely small so, a number of couples are connected in series or parallel to get a large output. The arrangement of this number of thermo-couple is called thermo-pile.

Working: These thermo-couples are arranged to out the cold and hot areas and the hot junctions are isolated thermally from the cold junction. Due to the temp<sup>r</sup> variations The thermo EMF across the two materials

Peltier - Co-efficient: The heat absorbed  $Q$  in unit time is directly proportional to ~~area~~ current

$$\frac{H}{t} \propto I \quad \text{or} \quad H = \pi_{ab} I t$$

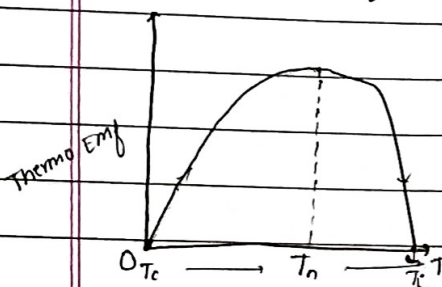
$$\frac{H}{t} = \pi_{ab} I \Rightarrow H = \pi_{ab} I t \Rightarrow H = \pi_{ab} q \quad [I t = q]$$

$$\pi_{ab} = \frac{H}{q} \Rightarrow \pi_{ab} = H / I t$$

The amount of heat absorbed or evolved at the junction of two dissimilar metals when 1A of current flows through it for one second

Thermo - Electric materials

Variation of thermo - Emf with the temperature



$T_n \rightarrow$  Neutral temperature

$T_i \rightarrow$  Inversion temperature

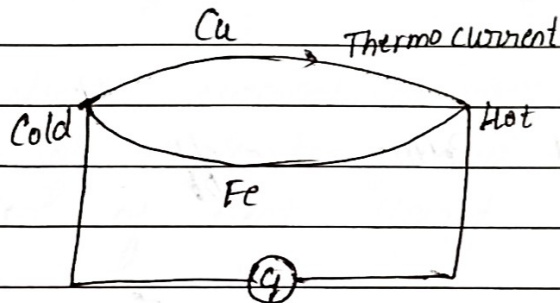
Let us take Cu-Fe thermocouple with Galvanometer  $G$  in the circuit. Let the temperature of the cold end B as  $0^\circ\text{C}$ . Now, the temperature of hot junction is gradually increased and the corresponding thermo - EMF is noted

A graph is plotted between the temperature of the hot junction and thermo EMF

• When both the junctions are at the same temperature ( $0^\circ\text{C}$ ) then thermo EMF is also zero

• As a temperature of the hot junction increases thermo EMF also increases till it becomes maximum

- The temperature at which thermo EMF becomes maximum is called neutral temperature. It is represented by  $T_n$ .
- When the temperature of the hot junction is increased beyond neutral temperature, the thermo-EMF starts decreasing instead of increasing.
- At another particular temperature of the hot junction the thermo EMF becomes zero on heating slightly further.
- The direction of the thermo EMF is reversed and is called inversion temperature.



### Relation between Neutral temp<sup>r</sup> and Inversion temp<sup>r</sup>

$$e = at + \frac{1}{2} bt^2 \quad \text{--- (1)}$$

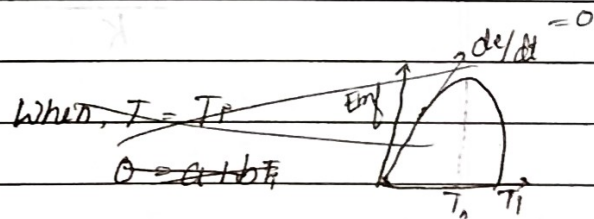
$$\frac{de}{dt} = a + bt \quad \text{--- (2)}$$

where (a) and (b) are the Seebeck co-efficient  
 $t = T_h - T_c$

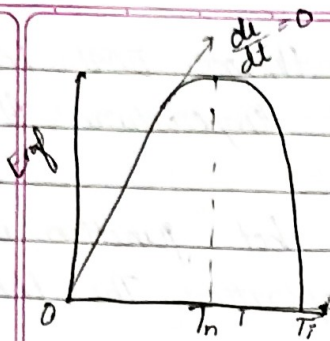
When  $T = T_n$

$$0 = a + bT_n$$

$$T_n = -a/b$$



$$0 =$$



When  $T = T_r$

$$0 = aT_r + \frac{1}{2} bT_r^2$$

$$0 = T_r(a + \frac{1}{2} bT_r)$$

$$0 = a + \frac{1}{2} bT_r$$

$$T_r = -\frac{2a}{b}$$

$$T_i = 2T_n$$

### Thermo - Electric Power

The rate of change of EMF with respect to temperature is called Thermo - Electric power

$$\frac{dE}{dT} = P$$

In terms of Peltier co-eff

$$\pi = PT$$

Figure of merit: Thermo - Electric system is defined by device efficiency and it depends on electrical conductivity, thermal conductivity and Seebeck co-efficient all of these changes on temp.

The maximum efficiency of the energy conversion process at a given temp. in the material is determined by the figure of merit which is indicated as

$$ZT = \frac{\alpha^2 \sigma T}{K}$$

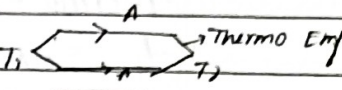
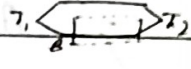
$\alpha \rightarrow$  Seebeck co-efficient

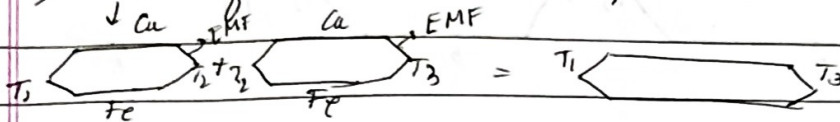
$\sigma \rightarrow$  Electrical conductivity

$K \rightarrow$  Thermal conductivity

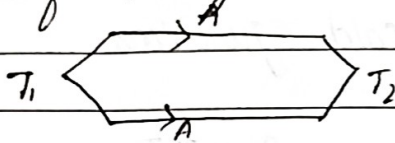
$T \rightarrow$  temp.

## Laws of thermo - Elasticity Electricity:

- 1) law of homogeneous metals  $\rightarrow T_1$  
- 2) law of intermediate metals  $\rightarrow T_1$  
- 3) law of intermediate temperatures  $\rightarrow$



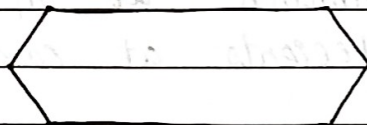
- 1) law of homogeneous metal



Thermo-electric current cannot be sustained in a circuit of single homogeneous material by the application of heat allowed

The thermal EMF produced by two thermocouples at  $T_1$  and  $T_2$  is independent of the temperature variation

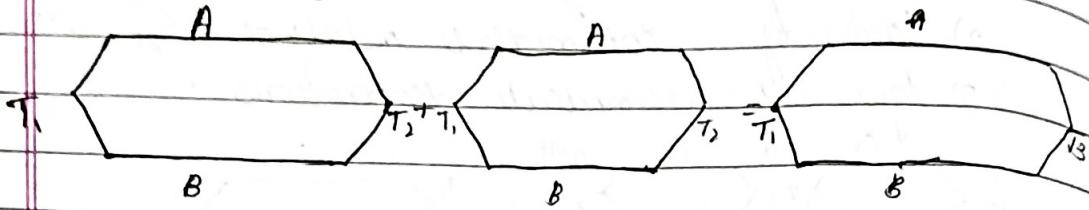
- 2) law of intermediate metals: A third metal may be inserted into a thermo-couple system without affecting the EMF generated if and only if the junctions with the third metal are kept at the same temperature



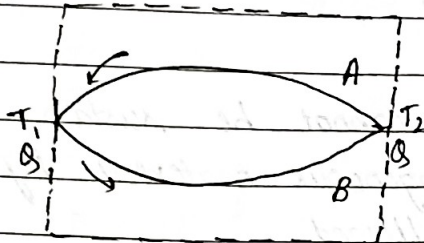
- 3) law of intermediate temperatures: The sum of EMFs developed by a thermo couple with its junctions at temperatures  $t_1$  and  $t_2$  and with its junction at temperature  $t_2$  and  $t_3$  will be



the same as the EMF developed if the thermo-couple junctions are at temperatures  $t_1$  and  $t_3$



Expression for thermo-EMF in terms of temp<sup>s</sup> of hot and cold junctions



$$Q_s = \pi_1 T_1$$

$$Q' = \pi_2 T_2$$

Consider circuit of two dissimilar metals A and B with two junctions. Let  $T_1$  &  $T_2$  be the temperatures of cold and hot junctions. The electric current flows through the circuit due to Seebeck effect. Due to the flow of electric current through hot and cold junction, the Peltier effect will be in action which results in the absorption of heat by hot junction and loss of heat at cold junction. Let  $\pi_1$  &  $\pi_2$  be the Peltier co-efficients at cold & hot junctions.

$$Q = \pi_1 q \quad \text{at temp } T_1$$

$$Q' = \pi_2 q \quad \text{at temp } T_2$$

$$e = (\pi_2 - \pi_1) q$$

Considering the process as similar to Carnot's Engine, hot junction is represented as  $T_1$  and cold junction is represented as  $T_2$ .

$$\text{By def}^n \quad \frac{\pi_1 Q}{T_1} = \frac{\pi_2 Q}{T_2}$$

$$\frac{\pi_1}{T_1} = \frac{\pi_2}{T_2}$$

$$\frac{\pi_1}{\pi_2} = \frac{T_1}{T_2}$$

$$\frac{\pi_2}{\pi_1} = \frac{T_2}{T_1}$$

$$\frac{\pi_2}{\pi_1} - 1 = \frac{T_2}{T_1} - 1$$

$$\frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1}$$

$$\pi_2 - \pi_1 = \frac{\pi_1 (T_2 - T_1)}{T_1}$$

$$e = \pi_2 - \pi_1$$

$$e = \frac{\pi_1 (T_2 - T_1)}{T_1}$$

$$e \propto T_2 - T_1$$

EMF  $\propto$  difference in temp<sup>r</sup>

## Thermo-pile

### Advantages and Disadvantages of Thermo-pile

- \* Does not need any external power supply.
- \* It gives a stable response to radiation which is gone from temperature measuring goals.
- \* It has stable response characteristics and available in small sizes and it is less costly.
- \* Thermo-pile is a non-contact temperature detecting device that uses IR radiation to transfer heat.
- \* It generates large output voltage due to the usage several thermo-couple devices.

### Disadvantages

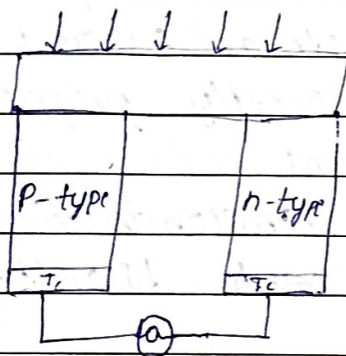
- \* The thermo-pile that are not within conductive material to depend them from static discharges and fields.
- \* These can be damaged due to stress and reverse in the polarity of the power supply.
- \* These should not be directly exposed to moisture or sunlight because this may cause the deposition of corrosion on the device.
- \* This device should not be operated with dirty or oily fingers because the dust will affect the device performance. For better performance

It is needed to be cleaned with cotton swabs

## Thermo - Electric Generators:

Thermo - Electric generators are the devices that convert the temperature differences that is generated between the two sections into the electrical form of energy when a load is properly connected.

### Construction and Working



**Construction:** The simplest thermo - Electric generator consists thermo - couple comprising two semiconductor thermo - element which are connected electrically in series and thermally in parallel.

The p - type & n - type semiconductor are interconnected through a metal. The load is connected to free end of p & n - type semiconductor.

Semiconductors are used due to the high electrical conductivity and low thermal conductivity.

Heat is pumped into one-side of the couple and rejected from the opposite end. The electrons present at the hot end would be at higher energy level as compared to the electrons present at the cold end. Hence, the hot electrons tends to move towards the cold end due to the temperature gain. When temperature gradient is produced b/w two ends the electrons start flowing from one end to another end. and create a potential difference at the electrical current is produced which is directly proportional to temperature gradient.

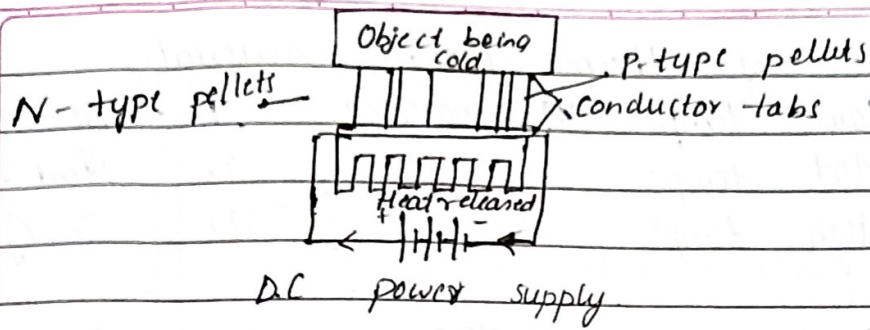
The commonly used semiconductors in TEG are ~~thin~~ bismuth-telluride and silicon-germanium.

### Applications:

TEG's enhances the fuel performance of cars by converting heat liberated into electricity.

Seeback power generation is utilized to provide power for the spacecraft.

Thermo-electric generators are implemented ~~are~~ to provide the power for remote stations such as weather systems, relay networks and others.



Thermo-Electric coolers are solid state heat pump used in applications where temperature stabilization cycling or cooling below ambient temperature

It is based on the peltier-effect that is when electric current is passed in a circuit consisting of two dissimilar metals heat is evolved at one end and absorbed at other end

Construction:

Thermo-electric cooler consists of an array of bismuth-telluride semiconductor that have been doped so that the conduction of electrons will be higher and these two pellets are connected electrically in series but thermally in parallel the conducting tabs connect the semiconductors to the hot end and the cold end

The metalized ceramic substrate provide the platform for the pellets

The size of the thermo-electric power varies 0.2-2mm Thermo-electric cooler can function singularly or in groups in series connection

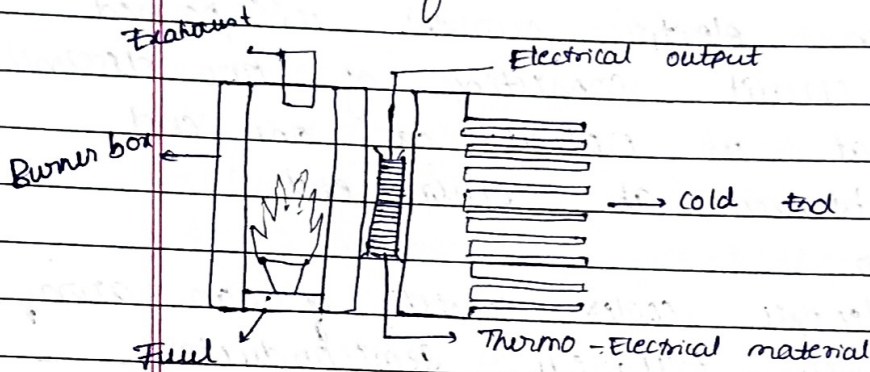
Types of thermo-Electric materials

- 1) low temp<sup>r</sup> TE material.  $Pb - Bi - Te$
- 2) Med temp<sup>r</sup>  $Pb - Hg - Te$
- 3) High temp<sup>r</sup>  $Pb - Se - Ge$

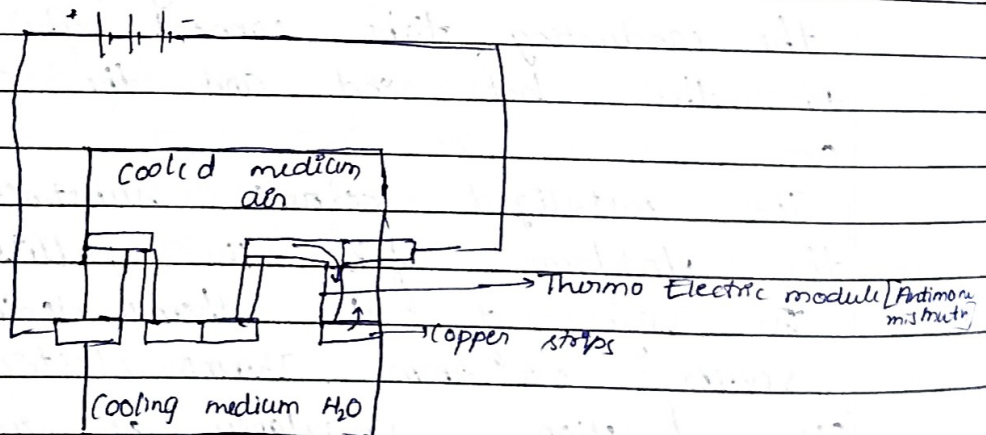
Applications on TEG:

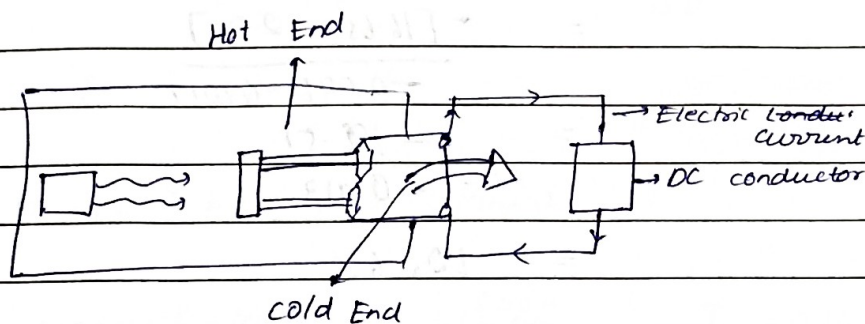
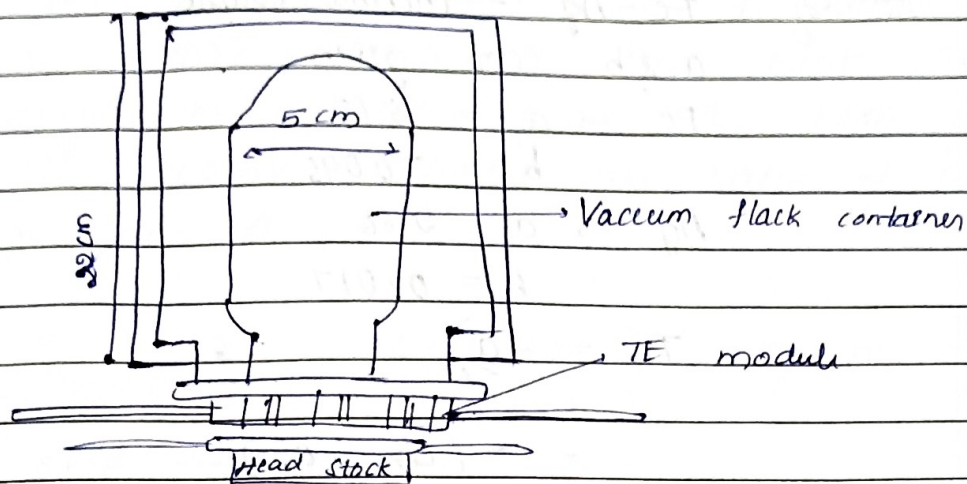
- Exhaust of automobiles
- Refrigerators
- Space programming devices

Exhaust of automobile:



Thermo - Electric Refrigeration





- 1) State and Explain Seebeck Effect and Hence Define Seebeck Co-efficient.
- 2) Discuss the variation of thermo-EMF with respect to temp<sup>r</sup>
- 3) Define neutral temp<sup>r</sup>, inversion temp<sup>r</sup> and Hence deduce relation b/w ~~temp~~  $T_e$  &  $T_n$ .
- 4) Explain the principal construction & working of thermo couple with neat diagram.
- 5) Describe radio isotope TEG and automobile TEG.

2) Calculate the Neutral temp<sup>r</sup> for iron silver thermo couple the values of  $A+B$  are  $16.685$  ~~17.7~~ and  $-0.096$  for iron and  $2.86$  and  $0.017$  for silver respectively.



$T_n = \frac{a}{b}$  Fe-Ag - thermo couple

a & b are given

$$\text{Fe} \rightarrow a = 16.65$$

$$b = -0.096$$

$$\text{Ag} \rightarrow a = 2.86$$

$$b = 0.017$$

$$T_n = \frac{-a}{b}$$

$$= - \frac{[a_{\text{Fe}} - a_{\text{Ag}}]}{b_{\text{Fe}} - b_{\text{Ag}}}$$

$$= - \frac{[16.65 - 2.86]}{-0.096 - 0.017}$$

$$= \frac{-13.79}{-0.113}$$

$$= 122.6^\circ\text{C}$$

2. Calculate the Iron cadmium thermo-couple the value of a & b are 17.5 & -0.1 for iron and for cadmium these values are 3 & 0.08

$$\text{Fe} \rightarrow a = 17.5$$

$$b = -0.1$$

$$\text{Cd} \rightarrow a = 3$$

$$b = 0.08$$

$$T_n = \frac{-a}{b}$$

$$= - \frac{[a_{\text{Fe}} - a_{\text{Cd}}]}{b_{\text{Fe}} - b_{\text{Cd}}}$$

$$= - \frac{[17.5 - 3]}{-0.1 - 0.08}$$

$$= \frac{-14.5}{-0.18}$$

$$= 80.5^\circ\text{C}$$

3. The neutral temp<sup>r</sup> of a thermo-couple is 300°C when it is kept at temp<sup>r</sup> differences 100°C & 0°C. The EMF generated is 1300 mV. Calculate the value of co-efficients  $a$  &  $b$ .

$$T_n = 300^\circ\text{C} \quad e = 1300 \times 10^{-3} \text{ V} \quad T_c = 0^\circ \quad T = 100^\circ\text{C}$$

$$t = 100^\circ\text{C}$$

$$EMF = -a + \frac{1}{2} bT^2$$

$$1300 \times 10^{-6} = -a + \frac{1}{2} b(100)^2 \quad \frac{1300}{150} \times 10^{-6} = -a + b$$

$$E = at + \frac{1}{2} bt^2$$

$$1300 \times 10^{-3} = a(100) + \frac{1}{2} b(100)^2$$

$$1300 \times 10^{-3} = 100a + \frac{1}{2} b(10000) \quad T_n = -a/b$$

$$1300 \times 10^{-3} = 100a + 5000b \quad \text{--- (1)} \quad 300 = -a/b$$

$$1300 \times 10^{-3} = 100[a + 50b] \quad \text{--- (2)} \quad a = -300b$$

$$\frac{1300 \times 10^{-3}}{100} = a + 50b$$

$$13 \times 10^{-3} = -250b$$

$$b = \frac{13 \times 10^{-3}}{250} = \frac{13 \times 10^{-4}}{25}$$

$$b = -\frac{13 \times 10^{-4}}{25} = -5.2 \times 10^{-5}$$

$$\text{(1)} \rightarrow 1300 \times 10^{-3} = 100a + 5000(-5.2 \times 10^{-5})$$

$$1300 \times 10^{-3} = 100a - 26000 \times 10^{-5}$$

$$2700 \times 10^{-3} = 100a$$

$$a = 27$$

$$T_n = -\frac{a}{b}$$

$$300 = -\frac{a}{-5.2 \times 10^{-5}}$$

$$1.56 \times 10^{-4} = -a \Rightarrow a = -0.156$$

1. The EMF of thermo couple is  $1200 \mu\text{V}$  when working b/w  $100^\circ\text{C}$  &  $0^\circ\text{C}$  Pt's neutral temp is  $300^\circ\text{C}$  find the values of  $a$  &  $b$ .

$$\text{EMF} = 1200 \times 10^{-6} \text{ V}$$

$$e = at + \frac{1}{2}bt^2$$

$$1200 \times 10^{-6} = a(100) + \frac{1}{2}b(100)^2$$

$$1200 \times 10^{-6} = 100[a + 50b]$$

$$T_n = -\frac{a}{b}$$

$$300 = -\frac{a}{b} \Rightarrow a = -300b$$

$$1200 \times 10^{-6} = 100(-300b + 50b)$$

$$b = \frac{12 \times 10^{-6}}{-250} \Rightarrow b = \frac{12 \times 10^{-6}}{25 \times 10^1}$$

$$b = -0.48 \times 10^{-7}$$

$$T_n = -\frac{a}{b}$$

$$b =$$

$$300 = \frac{-a}{-0.48 \times 10^{-7}}$$

$$144 \times 10^{-7} = a \Rightarrow a = 1.44 \times 10^{-4}$$

2. The EMF of a thermo couple is given by  $e = 1600t$  Find neutral temp & peltier coefficient

$$e = 1600t = 4t^2$$

$$e = at + \frac{1}{2}bt^2$$

$$a = 1600 \quad b = 4$$

$$\frac{de}{dt} = a + 2bt$$

$$\frac{de}{dt} = a + bt$$

$$\pi = t \frac{de}{dt}$$

$$\pi = t[a + bt]$$

$$\pi = 1600t + 4t^2$$

The neutral temperature is  $270^\circ\text{C}$  & the temp<sup>r</sup> inversion  $525^\circ\text{C}$  calculate the temp<sup>r</sup> of cold junction.

$$T_h - T_c = T_i - T_h$$

7. The temp<sup>r</sup> of cold junction and neutral temp<sup>r</sup> of thermo couple are  $15^\circ$  &  $280^\circ\text{C}$

$$T_i =$$

$$T_c + T_i = 2T_h \Rightarrow$$

$$\frac{15 + 280}{2} = T_h$$

$$T_i = 2T_h - T_c$$

$$= 2(280) - 15$$

$$= 560 - 15$$

$$T_i = 545^\circ\text{C}$$

8. For thermo-couple the neutral temp<sup>r</sup> is  $270^\circ\text{C}$  where in its cold junction is at  $20^\circ\text{C}$ . What will be the neutral temp<sup>r</sup> and temp<sup>r</sup> of inversion when temp<sup>r</sup> of cold junction is increased to  $40^\circ\text{C}$

$$T_i = 2T_h - T_c \Rightarrow T_i = 2(270) - 20 \Rightarrow 540 - 20 = 520^\circ\text{C}$$

$$T_i = 2T_h - T_c \Rightarrow 2T_h = \frac{T_i + T_c}{2} = \frac{520 + 40}{2} = 5$$

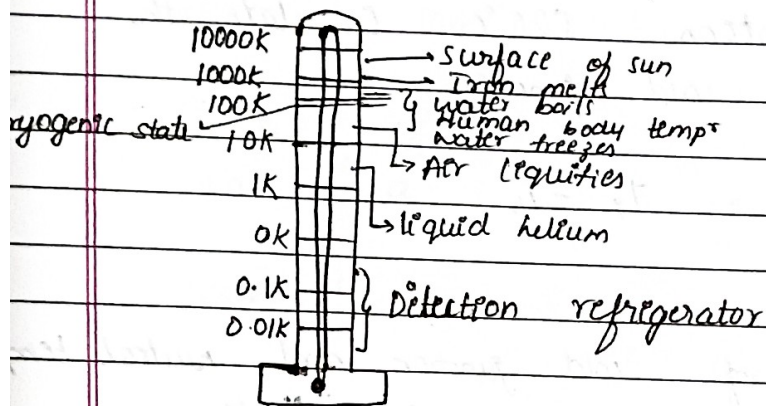
## Module - IV

## Cryogenics

study of production of low temp<sup>r</sup> → cryogenics

Less than 123 K

Cryogenics is the branch of physics which deals with the study of the production of low temp<sup>r</sup> and their effect on the matter



## Joule - Thomson's Effect

If a gas initially having at constant high pressure is allowed to suffer throttle expansion through the porous plug of silk, cotton which has a number of fine to a region of constant low pressure adiabatically. The change in the temp<sup>r</sup> of gas is observed. This effect is called Joule - Thomson's or Joule - Kelvin effect

adiabatic → No exchange of energy or temp<sup>r</sup>

### Joule - Thomson co-efficient

One mole of gas is allowed to expand through a porous plug from a pressure  $P_1$  and volume  $V_1$  to  $P_2$  and  $V_2$ , let the temperature changes from  $T_1$  to  $T_2$  due to Joule - Thomson effect

Net external work done by the gas =  $P_2 V_2 - P_1 V_1$   
 Now, the work done by overcoming by the

forces is the work done is given by  
 $P = \frac{a}{v^2}$  where  $a$  is constant

Total internal work done by the gas when it expands from  $V_1$  to  $V_2$

$$\int_{V_1}^{V_2} P \cdot dv = \int_{V_1}^{V_2} \frac{a}{v^2} dv$$

$$= \left[ -\frac{a}{v} \right]_{V_1}^{V_2} \Rightarrow -\left[ \frac{a}{V_2} - \frac{a}{V_1} \right] \Rightarrow \frac{a}{V_1} - \frac{a}{V_2} \quad \text{--- (1)}$$

Total work done by the gas is given by

$$W = [\text{Ext} + \text{Int}] \text{ work done}$$

$$W = [P_2 V_2 - P_1 V_1] + \frac{a}{V_1} - \frac{a}{V_2} \quad \text{--- (2)}$$

By Vander wall eq<sup>n</sup>

$$\left( P + \frac{a}{v^2} \right) (v - b) = RT$$

$$Pv = RT + Pb - \frac{a}{v} + \frac{ab}{v^2}$$

$$Pv = RT + Pb - \frac{a}{v} \quad \text{[neglecting } \frac{ab}{v^2}]$$

$$P_1 V_1 = RT_1 + P_1 b - \frac{a}{V_1}$$

$$P_2 V_2 = RT_2 + P_2 b - \frac{a}{V_2}$$

$$W = \left[ RT_2 + P_2 b - \frac{a}{V_2} - \left[ RT_1 + P_1 b - \frac{a}{V_1} \right] \right] + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = RT_2 + P_2 b - \frac{a}{V_2} - RT_1 - P_1 b + \frac{a}{V_1} + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = RT_2 + P_2 b - RT_1 - P_1 b$$

$$W = RT_2 + P_2 b - \frac{a}{V_2} - RT_1 - P_1 b + \frac{a}{V_1} + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = RT_2 - RT_1 + P_2 b - P_1 b + \frac{2a}{V_1} - \frac{2a}{V_2}$$

$$W = R[T_2 - T_1] - b[P_1 - P_2] + 2a \left[ \frac{1}{V_1} - \frac{1}{V_2} \right] \quad \text{--- (4)}$$

$$PV = RT \Rightarrow V = \frac{RT}{P}$$

$$V_1 = \frac{RT_1}{P_1}, \quad V_2 = \frac{RT_2}{P_2}$$

Use the temp<sup>r</sup> changes in a small amount

$$T_1 = T_2 = T$$

$$V_1 = \frac{RT}{P_1}, \quad V_2 = \frac{RT}{P_2}$$

Substituting  $V_1$  &  $V_2$  for eq<sup>n</sup> (4)

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2aP_1}{RT} - \frac{2aP_2}{RT}$$

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2a}{RT} [P_1 - P_2]$$

$$W = -R \Delta T - b(P_1 - P_2) + \frac{2a}{RT} [P_1 - P_2]$$

$$W = \left[ \frac{2a}{RT} - b \right] (P_1 - P_2) - R \Delta T \quad \text{--- (5)}$$

Since, The gas is thermally insulated  
The energy necessary for doing this  
work is drawn from the kinetic  
energy of molecule. Hence the kinetic  
energy decreases resulting in fall of  
temp<sup>r</sup> by  $\Delta T$

$$W = \left[ RT_2 + P_2 b - \frac{a}{V_2} - \left( RT_1 + P_1 b - \frac{a}{V_1} \right) \right] + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = RT_2 + P_2 b - \frac{a}{V_2} - RT_1 - P_1 b + \frac{a}{V_1} + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = RT_2 + P_2 b - RT_1 - P_1 b$$

$$W = RT_2 + P_2 b - \frac{a}{V_2} - RT_1 - P_1 b + \frac{a}{V_1} + \frac{a}{V_1} - \frac{a}{V_2}$$

$$W = RT_2 - RT_1 + P_2 b - P_1 b + \frac{2a}{V_1} - \frac{2a}{V_2}$$

$$W = R[T_2 - T_1] + b[P_2 - P_1] + \frac{2a}{V_1} - \frac{2a}{V_2} \quad \text{--- (4)}$$

$$P_1 V_1 = RT_1 \Rightarrow V_1 = \frac{RT_1}{P_1}$$

$$V_1 = \frac{RT_1}{P_1}, \quad V_2 = \frac{RT_2}{P_2}$$

Use the temp<sup>r</sup> changes in a small amount

$$T_1 = T_2 = T$$

$$V_1 = \frac{RT}{P_1}, \quad V_2 = \frac{RT}{P_2}$$

Substituting  $V_1$  &  $V_2$  for eq<sup>n</sup> (4)

$$W = R(T_2 - T_1) - b(P_2 - P_1) + \frac{2aP_1}{RT} - \frac{2aP_2}{RT}$$

$$W = R(T_2 - T_1) - b(P_2 - P_1) + \frac{2a}{RT} [P_1 - P_2]$$

$$W = -R \Delta T - b(P_2 - P_1) + \frac{2a}{RT} [P_1 - P_2]$$

$$W = \left[ \frac{2a}{RT} - b \right] (P_1 - P_2) - R \Delta T \quad \text{--- (5)}$$

Since, The gas is thermally insulated  
The energy necessary for doing this  
work is drawn from the kinetic  
energy of molecule Hence the kinetic  
energy decreases resulting in fall of  
temp<sup>r</sup> by  $\Delta T$



The heat loss is given by

$$C_p \Delta T = (P_1 - P_2) \left( \frac{2a}{RT} - b \right) - R \Delta T$$

$$[C_p + R] \Delta T = \frac{(2a - b)}{RT} (P_1 - P_2)$$

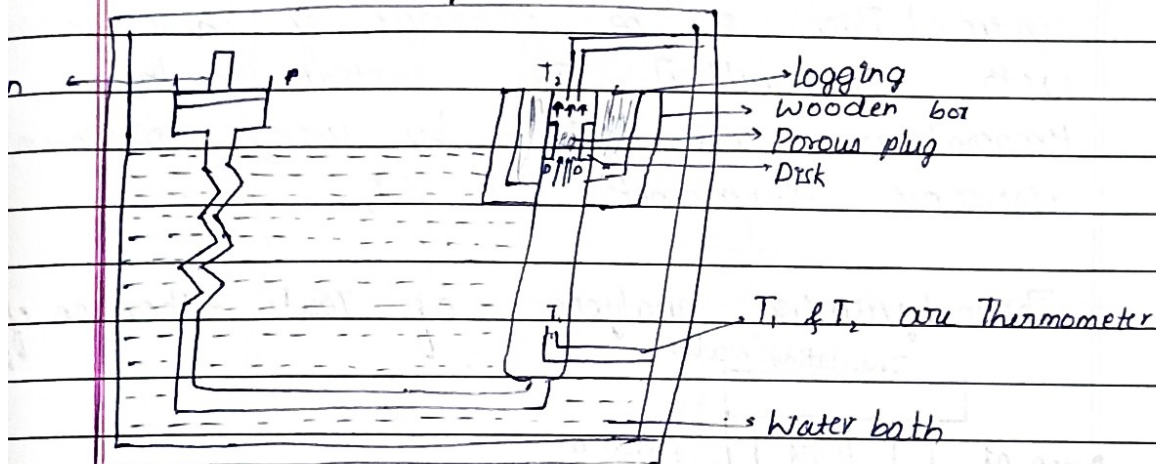
$$C_p \Delta T = \left( \frac{2a - b}{RT} \right) (P_1 - P_2)$$

$$\Delta T = \frac{P_1 - P_2}{C_p} \left( \frac{2a}{RT} - b \right)$$

Joule Thomson constant coefficient

$$U_{JT} = \frac{\partial T}{\partial P} = -\frac{1}{C_p} \left[ \frac{2a}{RT} - b \right]$$

Porous plug Experiment



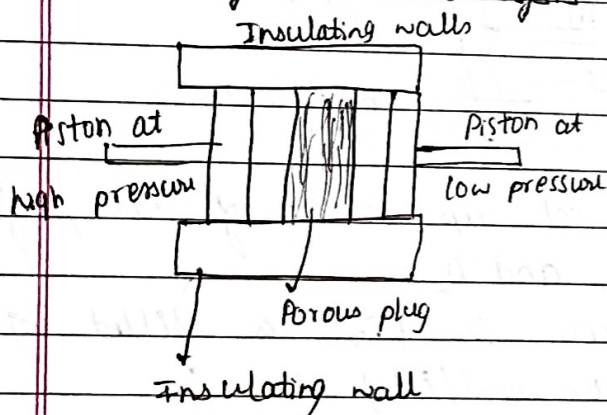
1. The experiment set up consists of two perforated brass-discs  $D_1$  and  $D_2$ .
2. The space between  $D_1$  &  $D_2$  is filled with porous plug [cotton wool/silk].
3. The porous plug arrangement is fitted in a wooden box which is surrounded by cotton to avoid the exchange of heat energy with the surrounding.

4.  $T_1$  &  $T_2$  are the platinum thermometers
5. The piston is fixed at the top to apply pressure
6. The whole arrangement is kept in water bath

### Procedure:

The experimental gas passed slowly and uniformly to the porous plug keeping the high pressure which can be read using pressure gauge. During the passage to the porous plug the gas is throttled. The separation b/w molecules increases due to which the volume of the gas increases against the atmospheric pressure as the experimental set up kept in an adiabatic system [There is no exchange of heat energy with surrounding]. The initial & final temperatures are noted by using platinum resistance thermometer  $T_1$  &  $T_2$ .

### Thermodynamic analysis of Joule-Thomson effect



Work done by the piston 1 on the gas =  $P_1 A_1 dx = P_1 V_1$

Work done by the piston 2 on the gas =  $P_2 A_2 dx = P_2 V_2$

Net external workdone =  $(P_2 V_2 - P_1 V_1)'$

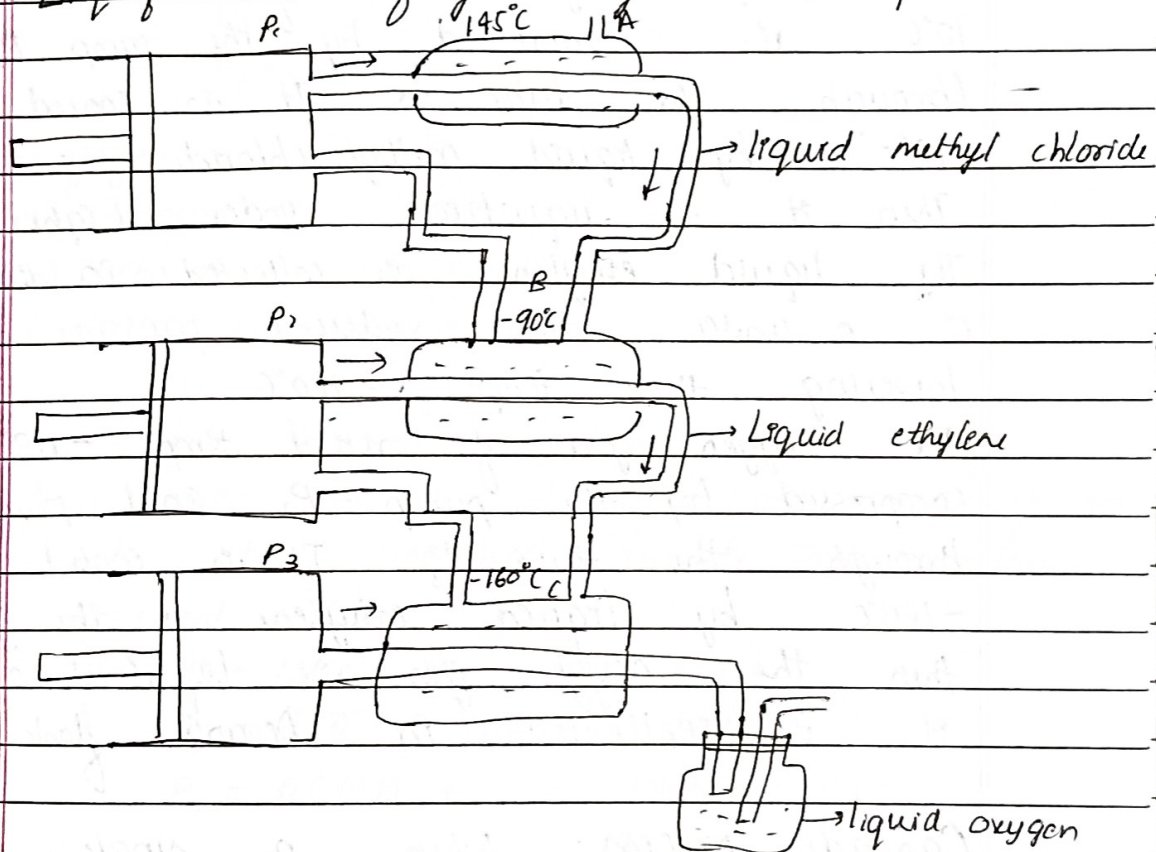
Let  $W$  be the work done by gas in overcoming intermolecular force of attraction

i) Below Boyle's temp<sup>r</sup>: If  $P_2 V_2 > P_1 V_1$ ,  
 $P_2 V_2 - P_1 V_1 = +ve$  cooling effect

ii) When  $P_2 V_2 = P_1 V_1 = +ve$  workdone cooling effect

iii) When  $P_1 V_1 > P_2 V_2 \Rightarrow -ve$  workdone heating effect

Liquification of gas by cascade process



Construction

The apparatus consists of compression pumps  $P_1$ ,  $P_2$  and  $P_3$ ,  $T_1$ ,  $T_2$  &  $T_3$  are the tubes which are surrounded by the outer jackets AB and C. The Dewar's flask is used to collect

## Liquid oxygen

Working:

The methyl chloride gas of critical temp<sup>r</sup>  $145^{\circ}\text{C}$  is compressed by the pump  $P_1$  through the tube  $T_1$ . It is cooled by the water filled in the jacket A.

Here the methyl chloride reaches temp<sup>r</sup> lower than its critical temp<sup>r</sup>. Then it is liquified under high pressure. The liquid methyl chloride is collected in the jacket B and Evaporates under the reduced pressure by lowering the temperature to  $-90^{\circ}\text{C}$ .

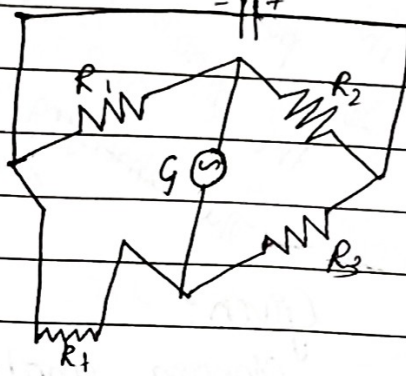
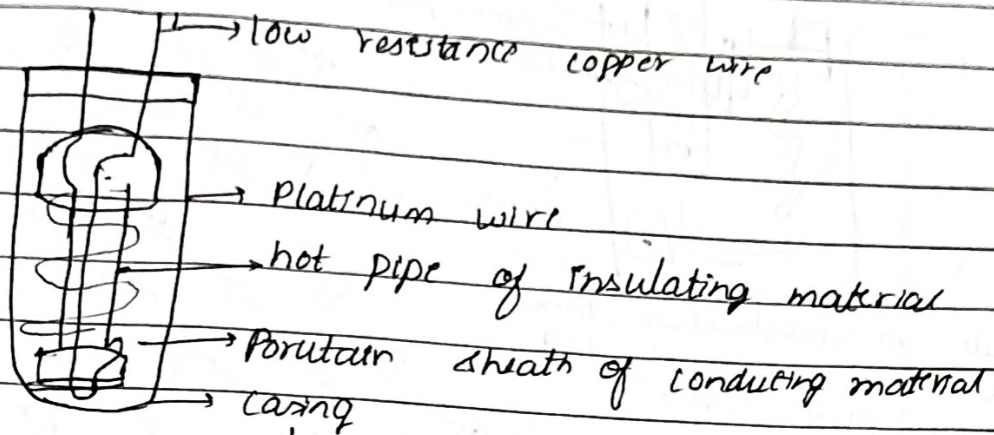
The Ethylene gas is of critical temp<sup>r</sup>  $10^{\circ}\text{C}$  is compressed by the pump  $P_2$  through the tube  $T_2$ . It is cooled to  $-90^{\circ}\text{C}$  by liquid methyl chloride.

Then it is liquified under higher pressure. The liquid ethylene is collected in the jacket C and under the reduced pressure and lowering the temp<sup>r</sup>  $-160^{\circ}\text{C}$ .

The Oxygen gas of critical temp<sup>r</sup>  $-119^{\circ}\text{C}$  is compressed by the pump  $P_3$  and passed through the tube  $T_3$ . It is cooled to  $-160^{\circ}\text{C}$  by liquid ethylene in the jacket C. Then the oxygen gas is liquified and it is collected in Dewar's flask.

**Cascade process:** When a single stage is not enough to produce the desired results the process taking place in a number of stages in a sequence is used for the purpose of liquification of gas.

# Platinum resistance thermometer



## Cryogenics:

- 1) Aerospace
- 2) Tribology
- 3) Food processing

Q. In Joule-Thomson experiment temp<sup>r</sup> changes from  $100^{\circ}\text{C}$  to  $150^{\circ}\text{C}$  for pressure change of  $20\text{MPa}$  to  $170\text{MPa}$ . Calculate Joule Thomson Co-efficient

$$T_1 = 100^{\circ}\text{C} = 100 + 273 = 373\text{K}$$

$$T_2 = 150^{\circ}\text{C} = 150 + 273 = 423\text{K}$$

$$P_1 = 20\text{MPa} \quad P_2 = 170\text{MPa}$$

$$\frac{\partial T}{\partial P} \text{ Joule Thomson Coefficient} = \mu_J = \frac{\partial T}{\partial P}$$

$$= \frac{T_2 - T_1}{P_2 - P_1} = \frac{50\text{K}}{150\text{MPa}} = \frac{1}{3} \text{K/MPa}$$