SE | IT - III - 15 may 2012
Applied Mathematics-III

V-L-1st-Hf-Ex-12-BB-99

Con. 3568-12. **GN-5393**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory.**
(2) Solve any **four** out of remaining **six** questions.
(3) Answers to **subquestions** should be answered **together.**

1. (a) If $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 5 & 3 & 4 \end{bmatrix}$, find adj A, A^{-1}. Also find B such that $AB = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 6 & 1 \\ 5 & 6 & 4 \end{bmatrix}$. **5**

(b) Find $L\left\{\frac{\cosh 2t \sin 3t}{t}\right\}$. **5**

(c) A regular function of constant magnitude is cosntant. **5**

(d) Find the Fourier series for $f(x) = 1 - x^2$ in $(-1, 1)$. **5**

2. (a) Expand $f(x) = \begin{cases} \pi x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$ with period 2, into a Fourier series. **6**

(b) Find the orthogonal trajectories of the family of curves $e^{-x}(x \sin y - y \cos y) = c$. **7**

(c) Using convolution theorem, prove that, $L^{-1}\left\{\frac{1}{s}\tan^{-1}\frac{a}{s}\right\} = \int_0^t \frac{1}{u} \sin au\,du$. **7**

3. (a) Show that every square matrix A can be uniquely expressed as P + iQ. Where P and Q are Hermitian matrices. **6**

(b) Using Cauchy's residue theorem, evaluate, $\oint_C \frac{12z-7}{(z-1)^2(2z+3)}dz$ where C is the circle (i) $|z| = \frac{1}{2}$ (ii) $|z + i| = 3$. **7**

(c) Solve the following equation by using Laplace transform, $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ given that $y(0) = 1$. **7**

4\. (a) State Laplace's equation in polar form and verify it for $u = r^2 \cos 2\theta$ and also find V and f(z). **6**

(b) Find Fourier series for $f(x) = \sqrt{1 - \cos x}$ $0 < x < 2\pi$ and hence show that **7**

$$\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} = \frac{1}{2}.$$

(c) Evaluate $\int_0^{\infty} t\sqrt{1 + \sin t}\, dt$. **7**

[TURN OVER

5. (a) Using Residue theorem, Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$. 6

(b) Reduce the following matrix to normal form and find its rank. 7

$$\begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

(c) (i) Express the function as Heaviside's unit step function and find their Laplace transforms. 4

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ t^2 & 1 < t < 3 \\ 0 & t > 3. \end{cases}$$

(ii) Find L { f(t) } where $f(t) = t \quad 0 < t < 1$ 3

$= 0 \quad 1 < t < 2$

and f(t) is a periodic function with period 2.

6. (a) Investigate for what values of λ and μ the equations— 6

$x + 2y + 3z = 4$

$x + 3y + 4z = 5$

$x + 3y + \lambda z = \mu$

have (i) no solution (ii) a unique solution **(iii)** an infinite number of solution.

(b) Show that the set of functions $\sin(2n + 1)x$, $n = 0, 1, 2, \cdots$ is orthogonal over $[0, \pi/2]$. Hence construct orthogonal set of functions. 7

(c) Find all Laurent's expansions of the function $f(z) = \frac{2 - z^2}{z(1-z)(2-z)}$. 7

7. (a) Find L { cost cos 2t cos 3t }. 6

(b) Show that the vectors [1, 0, 2, 1], [3, 1, 2, 1], [4, 6, 2, – 4], [– 6, 0, – 3, – 4] are linearly dependent and find the relation between them. 7

(c) Obtain half range sine series for f(x) where $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$ 7

Hence find the sum of $\sum_{2n-1}^{\infty} \frac{1}{n^4}$.

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

———S———