Total No. of Questions—**12**] [Total No. of Printed Pages—**8+2**

[4062]-101

S.E. (Civil) (First Semester) EXAMINATION, 2011

ENGINEERING MATHEMATICS

Paper III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) Attempt Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** or Q. No. **6** from Section I and attempt Q. No. **7** or Q. No. **8**, Q. No. **9** or Q. No. **10**, Q. No. **11** or Q. No. **12** from Section II.

(*ii*) Answer to the sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of logarithmic tables, slide rules, electronic pocket calculator and steam table is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* : [12]

(*i*) $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x)$$

(*ii*) $$\frac{d^2y}{dx^2} - y = x\sin x + e^x(1+x^2)$$

(*iii*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$ (By variation of parameters)

(*iv*) $x^2 \frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 \log x$

(*v*) $\frac{d^2y}{dx^2} + 4y = \sin x \sin 2x$.

(*b*) Solve the following : [5]

$(D-1)x + Dy = 2t + 1$

.

Or

2. (*a*) Solve any *three* : [12]

(*i*)

(*ii*) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$

(*iii*) $\frac{d^2y}{dx^2} + y = x \sin x$ (By variation of parameters)

(*iv*) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$

(*v*) $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = e^{2x}(1+x)$.

(*b*) Solve : [5]

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}.$$

3. (*a*) The differential equation satisfied by a beam, uniformly loaded with one end fixed and second subjected to a tensile force P is given by :

$$\mathrm{EI}\,\frac{d^2y}{dx^2} - \mathrm{P}y = -\frac{\mathrm{W}x^2}{2}.$$

Show that the elastic curve for the beam under conditions :

$y = 0$ and $\dfrac{dy}{dx} = 0$ when $x = 0$ is given by :

$$y = \frac{\mathrm{W}}{2\mathrm{P}}\left[x^2 + \frac{2}{n^2} - \frac{e^{nx}}{n^2} - \frac{e^{-nx}}{n^2}\right]$$

where $\dfrac{\mathrm{P}}{\mathrm{EI}} = n^2$. [8]

$u\,(1, t) = 0°C$

(*b*) The temperature at any point of an insulated metal rod of one meter length is governed by the differential equation :

$$\frac{\partial u}{\partial t} = c^2\,\frac{\partial^2 u}{\partial x^2}$$

Find $u\,(x, t)$ subject to the following conditions :

(*i*)

(*ii*)

(*iii*) . [8]

Or

4. (*a*) It is found experimentally that a weight of 3 kg. Stretches a spring to 15 cm. If the weight is pulled down 10 cm below equilibrium position and then released :

(*i*) find the amplitude, period and frequency of motion

(*ii*) determine the position, velocity and acceleration as a function of time. [8]

(*b*) Solve the equation :

subject to the following conditions :

(*i*) $u(x, \infty) = 0$

(*ii*)

(*iii*)

(*iv*) . [8]

5. (*a*) Solve the following system of equations by Gauss-Seidel iteration method :

$$20x + y - 2z = 17$$

[9]

(*b*) Use Runge-Kutta method of fourth order to solve :

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2} \quad ; \quad y(0) = 1$$

to find y at $x = 0.4$ taking $h = 0.2$. [8]

Or

6. (*a*) Solve the equation :

$$\frac{dy}{dx} = x - y^2 \quad ; \quad y(0) = 1$$

to find y at $x = 0.4$ using modified Euler's method taking $h = 0.2$. [9]

(*b*) Solve the following system of equations by Cholesky's method :

$$3x + 2y + 7z = 4$$

$$2x + 3y + z = 5$$

$$3x + 4y + z = 7.$$

[8]

SECTION II

7. (*a*) The first four moments of a distribution about the value 4 of a variable are –1.5, 17, –30 and 108. Find the moments about the mean. Calculate coefficient of Skewness and Kurtosis. [6]

(*b*) From a group of ten students, marks obtained by each student in papers of Mathematics and Electronics are given as :

Marks in Mathematics (x)	**Marks in Electronics (y)**
23	25
28	22
42	38
17	21
26	27
35	39
29	24
37	32
16	18
46	44

Calculate coefficient of correlation. [6]

(*c*) Probability of man now aged 60 years will live upto 70 years of age is 0.65. Find the probability of out of 10 men sixty years old, 8 or more will live upto the age of 70 years. [5]

Or

8. (*a*) For the following distribution find first four moments about the mean : [6]

x	f
2	5
2.5	38
3	65
3.5	92
4	70
4.5	40
5	10

(*b*) The regression equations are $8x - 10y + 66 = 0$ and $40x - 18y = 214$. The value of variance of x is 9.

Find :

(*i*) The mean value of x and y.

(*ii*) The correlation coefficient between x and y.

(*iii*) The standard deviation of y. [6]

(*c*) A random sample of 200 screws is drawn from a population which represents the size of screws. If a sample is distributed normally with a mean 3.15 cm and standard deviation 0.025 cm, find expected number of screws whose size falls between 3.12 cm and 3.2 cm. [5]

Given :

z = 1.2 , 2.0

Area = 0.3849 , 0.4772

9. (*a*) The position vector of a particle at time t is :

.

Find the condition imposed on m by requiring that at time $t = 1$, the acceleration is perpendicular to the position vector. [5]

(*b*) Find the directional derivative of :

$$\phi = 4xz^3 - 3x^2y^2z \quad \text{at} \quad (2, -1, 2)$$

along tangent to the curve

$$x = e^t \cos t, \quad y = e^t \sin t, \quad z = e^t \quad \text{at} \quad t = 0. \qquad [5]$$

(*c*) Show that :

is irrotational. Find scalar potential f such that :

$$\bar{F} = \nabla\phi. \qquad [6]$$

Or

10. (*a*) If a particle P moves along the curve $r = a e^{\theta}$ with constant angular velocity w, then show that the radial and transverse components of its velocity are equal and its acceleration is always perpendicular to radius vector and is equal to $2rw^2$. [5]

(*b*) Find the function $f(r)$ so that $f(r)\bar{r}$ is solenoidal. [5]

(*c*) Establish any *two* : [6]

(*i*) $$\nabla \cdot \left[r \, \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$$

(*ii*) $$\nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^n} \right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$$

(*iii*) $$\nabla \cdot \left(\frac{\bar{a} \times \bar{r}}{r} \right) = 0.$$

11. (*a*) Verify Green's theorem for the field :

$$\bar{F} = x^2 \, \hat{i} + xy \, \hat{j}$$

over the region R enclosed by $y = x^2$ and the line $y = x$. [6]

(*b*) Evaluate :

$$\iint_s (x^3 \, \hat{i} + y^3 \, \hat{j} + z^3 \, \hat{k}) \cdot d\bar{s}$$

where s is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [6]

(*c*) Evaluate :

$$\iint_s (\nabla \times \bar{F}) \cdot d\bar{s}$$

where $$\bar{F} = (x^3 - y^3) \, \hat{i} - xyz \, \hat{j} + y^3 \, \hat{k}$$

and s is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. [5]

Or

12. (*a*) Evaluate :

$$\int_c \overline{\mathrm{F}}\,.\,d\overline{r}$$

where $\overline{\mathrm{F}} = (2xy + 3z^2)\,\hat{i} + (x^2 + 4yz)\,\hat{j} + (2y^2 + 6xz)\,\hat{k}$

and c is the curve $x = t$, $y = t^2$, $z = t^3$ from $t = 0$ to $t = 1$. [6]

(*b*) Show that the velocity potential :

$$\phi = \frac{1}{2}a\,(x^2 + y^2 - 2z^2)$$

satisfies the Laplace's equation. Also determine the stream lines. [6]

(*c*) Show that :

$$\iiint_v \frac{dv}{r^2} = \iint_s \frac{\overline{r}\,.\,\hat{n}}{r^2}\,ds\,.$$ [5]