CS/B.Tech(New)/CSE/IT/SEM-4/M-401/2013

2013

MATHEMATICS-III

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)
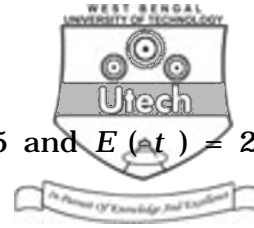
1. Choose the correct alternatives for any *ten* of the following :

    $10 \times 1 = 10$

    i)   The number of generations of an infinite cyclic group is

    a)   1   b)   2

    c)   infinite   d)   none of these.

    ii)   The standard deviation of a sample mean for SRSWR is

    a) σ^2/n b) $\sigma/\sqrt{n}$

    c) σ/n d) n.

4102   [Turn over

iii) If t is a statistic such as $E (t^2) = 5$ and $E (t) = 2$, then the standard error of t is

a) 0

b) 1

c) 2

d) none of these.

iv) If the exponential distribution is given by the probability density function $f (x) = e^{-x}$, $0 < x < \infty$, then the mean of the distribution is

a) 1

b) 3

c) $\dfrac{1}{3}$

d) none of these.

v) The probability of an event A is $\dfrac{1}{3}$, that of $A + B$ is $\dfrac{1}{2}$ and that of AB is $\dfrac{1}{4}$. Then the probability of B is

a) $\dfrac{1}{12}$

b) $\dfrac{5}{12}$

c) $\dfrac{1}{6}$

d) none of these.

vi) Which one of the following sets forms a group under usual multiplication of complex numbers ?

a) $\{ 1, i \}$

b) $\{ 1, \omega, \omega^2 \}$

c) $\{ 1, \omega^2 \}$

d) $\{ 1, \omega \}$.

vii) The distribution for which the mean and variance are equal is

    a) Poisson   b) normal

    c) binomial   d) exponential.

viii) In a Binomial (*n, p*) distribution, if its mean and variance are 2 and 4/3 respectivity, then the values of *n* and *p* are

    a) $8, \frac{1}{4}$ b) $6, \frac{1}{3}$

    c) $4, \frac{1}{2}$ d) none of these.

ix) If *G* is a connected planar graph with *n* vertices, *e* edges and *f* faces, then $n - e + f = 2$. This statement is

    a) True   b) False.

x) The mean of Binomial variate is

    a) *np*   b) $np (1 - p)$

    c) $\sqrt{np}$ d) none of these.

xi) Kuratowski's graph is a

a) planar graph

b) regular graph

c) tree

d) none of these.

xii) The order of the dihedral group D_4 is

a) 4

b) 6

c) 8

d) 64.

xiii) Every finite integral domain is a field. This statement is

a) True

b) False.

xiv) If A and B are two subgroups of a group G, then which of the following is always a subgroup of G ?

a) $A \cup B$

b) $G - A$

c) $G - B$

d) $A \cap B$.

xv) The symmetric group S_3 has

a) 6 elements

b) 8 elements

c) 9 elements

d) none of these.

GROUP – B
### (Short Answer Type Questions)
Answer any *three* of the following.   3 × 5 = 15

2. Let (Q, +) be the additive group of rational numbers and (Q^+, .) be the multiplicative group of positive rational numbers. Are these two groups isomorphic ? Justify your answer.

3. Prove Baye's theorem for repeated trials.

4. Examine whether faction $|x|$ in (– 1, 1) and zero elsewhere is a density function.

5. Show that a connected graph is Eulerian if and only if each of its vertices is of even degree.

6. Show that a field does not contain any zero divisor.

GROUP – C
(Long Answer Type Questions)
Answer any *three* of the following.   3 × 15 = 45

7. a) Prove that a planar graph with n vertices, e number of edges and k number of components determines f number of regions, where $f = e – n + k + 1$.   8

   b) Let $\bar{X}$ be the sample mean of samples of size n drawn at random from a population which is normally distributed with mean μ and variance σ^2. Find the standard error of the statistic $\bar{X}$.   7
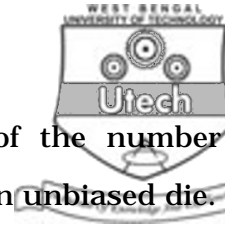
8. a) Find the mathematical expectation of the number of points obtained in a single throw of an unbiased die.   5

   b) Define Poisson distribution and find its mean and variance.   5

   c) Let f be a ring homomorphism from the ring Z of integers into itself such that $f (1) = 1$. Determine the homomorphism f.   5

9. a) Show that any simple connected planar graph with n vertices ($n \geq 3$) has at most ($3n - 6$) edges.   7

   b) Prove that every nontrivial subgroup of the additive group Z of integers is cyclic.   4

   c) Let R and S be two rings and $f : R \rightarrow S$ be a ring homomorphism. Show that kernel of f is a subring of R.   4

10. a) Determine the mean and variance of exponential distribution.   6

    b) Show that every cyclic group is commutative.   4

    c) Let H be a normal subgroup of a group G and G/H be the set of all cosets of H in G. Show that G/H forms a group under the composition

    $$(aH) \cdot (bH) = (ab) H \text{ for all } a, b \in G .$$ 5

11. a) The probability density function of a random variable X is assumed to be of the form $f(x) = cx^{\alpha}$, $0 \leq x \leq 1$ for some number and constant c. If $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n, find the maximum likelihood estimate of α. 5

b) Let S' be the set defined by $S' = \{z \in C : |z| = 1\}$, where C is the set of all complex numbers. Show that S' forms a commutitive group under usual multiplication of complex numbers. 5

c) Let R be the additive group of real numbers and C^* be the multiplicative group of nonzero complex numbers. If $f = R \rightarrow C^*$ is a group homomorphism defined by $f(x) = e^{2\pi i x}$ for all $x \in R$, find the kernel of f. 5

———————