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06MAT31

Third Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions choosing at least TWO from each part.***

Part A

1 a. Find the Fourier series for the function $f(x)=x+x^2$ from $x=-\pi$ to $x=\pi$ and deduce that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dots\dots\dots$ **(07 Marks)**

b. Obtain the cosine half-range Fourier series for $f(x)=Kx, \quad \text{in } 0\le x\le l/2$; $=K(l-x) \quad \text{in } l/2\le x\le l$. **(07 Marks)**

c. The following table gives the variating of periodic current over a period:

t (sec)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (Amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a direct current part of 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(06 Marks)**

2 a. Obtain the finite Fourier Cosine transform of the function $f(x)=e^{ax}$ in (0, l). **(07 Marks)**

b. Find the Fourier sine and cosine transforms of

$$f(x)=\begin{cases} x, & 0<x<2 \\ 0, & \text{otherwise} \end{cases}$$ **(07 Marks)**

c. Solve the integral equation,

$$\int_0^\infty f(x)\cos(\alpha x)dx=\begin{cases} 1-\alpha, & 0\le\alpha\le 1 \\ 0, & \alpha>1 \end{cases}.$$

Hence evaluate $\displaystyle\int_0^\infty \frac{1-\cos x}{x^2}dx$. **(06 Marks)**

3 a. Form the P.D.E by eliminating the arbitrary function from $z=y^2+2f\left(\dfrac{1}{x}+\log y\right)$. **(07 Marks)**

b. Solve $x^2\dfrac{\partial u}{\partial x}+y^2\dfrac{\partial u}{\partial y}=0$ by the method of separation of variables. **(07 Marks)**

c. Solve $(y^2+z^2)p+x(yq-z)=0$. **(06 Marks)**

4 a. Derive the one dimensional heat equation. **(07 Marks)**

b. Solve the wave equation $\dfrac{\partial^2 u}{\partial t^2}=C^2\dfrac{\partial^2 u}{\partial x^2}$ given $u(0,t)=0$; $u(l,t)=0$; $\dfrac{\partial u}{\partial t}=0$ when $t=0$ and $u(x,0)=u_0\sin\dfrac{\pi x}{l}$. **(07 Marks)**

c. Obtain the various possible solutions of the Laplace's equation $u_{xx}+u_{yy}=0$ by the method of separation of variables. **(06 Marks)**

Part B

5 a. Find the real root of the equation $3x = \cos x + 1$ correct to four decimal places using Newton's method. (07 Marks)

b. Solve the system of equations,

$2x + y + z = 10$

$3x + 2y + 3z = 18$

$x + 4y + 9z = 16$

by Gauss-Jordan method. (07 Marks)

c. Find the largest eigen value and the corresponding eigen vector of the following matrix by using power method.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

Taking $[1 \quad 0 \quad 0]^T$ as the initial eigen vector. Carry out four iterations. (06 Marks)

6 a. Given f(0) = 1, f(1) = 3, f(2) = 7, f(3) = 13. Find f(0.1) and f(2.9) using Newton Interpolation formula. (07 Marks)

b. Using Newton's divided difference formula evaluate f(8) and f(15), given that (07 Marks)

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

c. Evaluate $\int_4^{5.2} \log_e x\,dx$ by using Weddle's rule, taking 7 ordinates. (06 Marks)

7 a. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. (07 Marks)

b. Find the extremal of the functional $\int_0^{\pi/2} \left[y^2 - (y')^2 - 2y\sin x\right]dx$ under the conditions $y(0) = y\left(\pi/2\right) = 0$. (07 Marks)

c. Find the geodesics on a surface, given that the arc length on the surface is $s = \int_{x_1}^{x_2} \sqrt{x\left[1 + (y')^2\right]}\,dx$. (06 Marks)

8 a. Find the z-transforms of i) $(n+1)^2$ ii) sin(3n+5). (07 Marks)

b. Obtain the inverse Z transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (07 Marks)

c. Solve the difference equation, $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$ using Z transforms. (06 Marks)

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