Roll No. .... Total Pages : 3

433

DBAQ/A-15

MATHEMATICS

(Analysis)

Paper : BM-301

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *five* questions in all, selecting at least *one* question from each unit.

UNIT–I

1. (a) Give an example to show that every bounded function is not integrable.

   (b) State and prove First Mean Value Theorem on Integral Calculus.

2. (a) By definition, prove that $\int_1^2 x\,dx = \frac{3}{2}$.

   (b) Test the convergence of the improper integral $\int_0^1 \frac{dx}{\sqrt{1-x}}$.

3. (a) State and prove Dirichlet's test for convergence of improper integral.

   (b) Evaluate $\int_0^{\pi} \frac{\log(1+\alpha\cos x)}{\cos x}\,dx$ for $|\alpha| < 1$.

UNIT–II

4. (a) Show that the series $\sum_{n=2}^{\infty} \frac{\sin nx}{n^2 \log n}$ is convergent.

(b) Show that the Cauchy product of two divergent series

$$\sum_{n=0}^{\infty} a_n = 2 + \sum_{n=1}^{\infty} 2^n \text{ and } \sum_{n=0}^{\infty} b_n = -1 + \sum_{n=1}^{\infty} 1^n$$

is convergent.

5. (a) Find the Fourier expansion of the function $f(x) = e^x$ in $-\pi < x < \pi$.

(b) If f_x and f_y are both differentiable at a point (a, b) of the domain of definition of the function f, then $f_{xy} = f_{yx}$.

UNIT–III

6. (a) Prove that an analytic function with constant modulus is constant.

(b) Find the regular function whose imaginary part is $\frac{x-y}{x^2+y^2}$.

7. (a) Prove that $\cosh(\alpha+\beta) - \cosh(\alpha-\beta) = 2\sinh\alpha \sinh\beta$.

(b) Find the Mobius transformation which maps the points $z = -2, 0, 2$ into the points $w = 0, i, -i$ respectively.

UNIT–IV

8. (a) Let $d(x, y) = \min\{2, |x-y|\}$. Show that d is a metric on R.

(b) Prove that, in any metric space (X, d), every open sphere is an open set.

9. State and prove Cantor's Intersection Theorem.

10. (a) Prove that compact space has Bolzano Weierstrass property.

(b) Every totally bounded metric space is bounded, prove it.

Roll No. .... Total Pages : 3

434

DBAQ/A-15

MATHEMATICS

(Abstract Algebra)

Paper–II/BM-302

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks except Section-II (6 marks).

SECTION–I

1. (a) Let Z(G) be the centre of a group G. If G/Z(G) is cyclic, prove that G is abelian.

(b) Find automorphisms of a finite cyclic group G of order *n*.

2. (a) Define Normalizer of an element. If *f* is an automorphism of a group G and $a \in G$ be an element, then show that $f(N(a)) = N(f(a))$.

(b) Show that no group of order 56 is simple.

3. (a) Let P be a prime number such that $P^m | O(G)$, where *m* is a positive integer. Then G has a subgroup of order p^m.

(b) Let G be any cyclic group of order *mn*, where gcd $(m, n) = 1$. Let H and K be its subgroups of order *m* and *n* respectively. Show that $G = H \times K$.

SECTION–II

4. (a) Define the characteristic of a ring. Prove that the characteristic of any integral domain R is either zero or a prime number.

(b) Let R be a commutative ring with unity. Then prove that an ideal M of R is maximal iff R/M is field.

5. (a) The set of Gaussian integers is denoted by $Z[i] = \{a + ib;\ a, b \in Z\}$ and $Z[i]$ is an integral domain w.r.t. addition and multiplication of complex numbers. Show that $Z[i]$ is an Euclidean domain.

(b) State and prove Eisenstein criterion.

SECTION–III

6. (a) Prove that any set of linearly independent vectors of a finitely generated vector space V can be extended to a basis of V.

(b) Show that the set

$$\{(1, i, 0), (2i, 1, 1), (0, 1 + i, 1 - i)\}$$

is a basis for $V_3(\mathbb{C})$ and determine the co-ordinates of the vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) w.r.t. the basis.

7. (a) Define Null space of a Linear transformation. Let $T : U(F) \to V(F)$ is a L.T., then show that N(T) is a subspace of U(F).

(b) Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be a Linear transformation such that

$$T(e_1) = (1, 1, 1),\ T(e_2) = (1, -1, 1)$$
$$T(e_3) = (1, 0, 0),\ T(e_4) = (1, 0, 1).$$

Then verify that $\rho(T) + \mu(T) = \dim R^4 = 4$.

8. (a) Show that the quadratic form

$$4x^2 + 9y^2 + 2z^2 + 8yz + 6zx + 6xy$$

is not positive definite.

(b) Let T be a Linear operator on R^2 defined by $T(x, y) = (4x - 2y, 2x + y)$.

Find the matrix of T w.r.t. the basis B = {(1, 1), (–1, 0)}. Also verify that $[T : B]\,[u : B] = [T(u), B]$.

SECTION–IV

9. (a) Let $\{u_1, u_2, \ldots\ldots\ldots\ldots u_n\}$ be an orthonormal basis of an inner product space V(F), then prove that

$$\sum_{i=1}^{n} |\langle u, u_i \rangle|^2 \le \| u \|^2 \quad \forall\, u \in V.$$

(b) Define orthogonal complement of a subspace. Let W be a non-empty subset of an inner product space V(F). Then prove that orthogonal complement of W is a subspace of V(F).

10. (a) Prove that a Linear operator T on a unitary space V is Hermitian iff $\langle T(\alpha), \alpha \rangle$ is real for every α.

(b) Prove that kernel of an R-homomorphism of an R-module M into R-module N is R-submodule of M.

Roll No. .... Total Pages : 3

DBAQ/A-15 **435**

MATHEMATICS

(Programming in C and Numerical Analysis)

Paper : BM-303

Time : Three Hours] [Maximum Marks : 20

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION–I

1. (a) What is an algorithm ? What are its characteristics ?

(b) Draw a flow chart to check a number to be a prime number.

2. (a) Describe arithmetic, logical and assignment operators with the help of examples.

(b) Differentiate between break and continue statements.

3. (a) Distinguish between Local and Global variables.

(b) What are Files ? What is File formatting ? Illustrate its importance through suitable examples.

SECTION–II

4. (a) Find the real roots of $x^3 = 2x + 5$ by Regula-Falsi method correct to three places of decimal.

(b) Describe Lagrange's Interpolation formula for unequal intervals.

5. (a) The table given ahead (page 2) reveals the velocity v of a body during the time t specified. Find its acceleration at $t = 1.1$.

t:	1.0	1.1	1.2	1.3	1.4	1.5
v:	43.1	47.7	52.1	56.4	60.8	65.6

(b) The table below gives the velocity v of a moving particle at time t seconds. Find the distance covered by the particle in 12 seconds.

t:	0	2	4	6	8	10	12
v:	4	6	16	34	60	94	136

6. (a) Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by the Cholesky method.

(b) Transform the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to tridiagonal form by Given's method. Find its eigen vector corresponding to the largest eigen value.

SECTION–III

7. (a) Find the solution at $x = 0.8$ for

$$\frac{dy}{dx} = \sqrt{x + y}, \ y(0.4) = 0.41,$$

using Runge-Kutta formula of fourth order.

(b) Find difference approximations of the solution $y(x)$ of the boundary value problem

$$y'' + 8(\sin^2 \pi x)y = 0, \ 0 \le x \le 1,$$

$$y(0) = y(1) = 1,$$

taking step length 0.5. Also find $y'(0)$.

8. (a) Prove that Chebyshev polynomial $T_n(x)$ satisfies the differential equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + n^2y = 0$.

(b) Fit a curve of the form $y = ae^{bx}$ by the method of Least square for

x:	0	5	8	12	20
y:	3.0	1.5	1.0	0.55	0.18

SECTION–IV

9. (a) Generate 15 successive random numbers using congruential generators

$X_{i+1} = aX_i \pmod{m}$ with $a = 3$, $m = 100$, $X_0 = 4$.

(b) Using Composition method, generate a random variate from $f(x) = \frac{5}{12}(1 + (x - 1)^4)$, where $0 \le x \le 2$.

10. (a) What is Monte-Carlo method ? Write its applications.

(b) Illustrate approximate integration by Monte-Carlo method of the integral $\int_0^1 x^2 \, dx$, using the random numbers 0.01, 0.13, 0.69, 0.97, 0.61.