

## B. Tech I Year Examinations, May/June -2012

## MATHEMATICS-I

(Common to all Branches)

Time: 3 hours

Max. Marks: 75

Answer any five questions  
All questions carry equal marks

1. a) Find whether the series  $\sum (-1)^n \frac{\sin\left(\frac{1}{\sqrt{n}}\right)}{n-1}$  is absolute convergent or conditional convergent.
- b) Test for convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{2^n + 3^n}$ .
- c) Test the convergence of the series  $\sum \frac{n^3 - 5n^2 + 7}{n^5 + 4n^4 - n}$ . [6+6+3]
2. a) Prove using mean value theorem  $|\sin u - \sin v| \leq |u - v|$ .
- b) If the sum of the three numbers is a constant, then prove that their product is maximum when they are equal.
- c) Prove that the functions  $u = xy + yz + zx$ ,  $v = x^2 + y^2 + z^2$ ,  $w = x + y + z$  are functionally dependent and find the relation between them. [6+5+4]
3. a) Show that the evolute of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$  is another cycloid.
- b) Trace the curve  $x = a \cos^3 \theta$ ,  $y = b \sin^3 \theta$ . [8+7]
4. a) Find the volume of the solid generated by the revolution of the Cissoid  $y^2 = \frac{x^2}{(2a-x)}$  about its asymptote.
- b) By changing the order of integration evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx$ . [7+8]
5. a) Solve  $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$ .
- b) Obtain the orthogonal trajectories of the family of curves  $r(1 + \cos \theta) = 2a$ . [7+8]

6. a) By using variation of parameters solve  $\frac{d^2 y}{dx^2} + 4y = \tan 2x$ .

b) Solve  $\left( (2x-1)^3 \frac{d^3 y}{dx^3} + (2x-1) \frac{dy}{dx} - 2y = x \right)$ . [8+7]

7. a) Find the Laplace transform of  $te^{2t} \sin 3t$ .

b) Use Laplace Transforms, to solve  $(D^2 + 1)x = t \cos 2t$  given  $x(0) = x'(0) = 0$ . [7+8]

8. a) Verify divergence theorem for  $2x^2 y i - y^2 j + 4xz^2 k$  taken over the region of first octant of the cylinder  $y^2 + z^2 = 9$  and  $x=2$ .

b) Find the directional derivative of  $\nabla \cdot \nabla \phi$  at the point  $(1, -2, 1)$  in the direction of the normal to the surface  $xy^2z = 3x + z^2$  where  $\phi = 2x^3 y^2 z^4$ . [8+7]

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