1.Definition of sets

- **Introduction:**
- The set theory was developed by a German mathematician Georg Cantor (1845-1918). Nowadays, set theory is used in almost all branches of mathematics. We also use sets to define relations and functions. The knowledge of sets is required in the study of geometry, sequences, probability, etc. In this unit, we will discuss some basic definitions related to sets.
- **Definition of Sets:**
- A well-defined collection of objects is known as a set. "Well-defined" means in a given set, it must be possible to decide whether the objects belong to the set, and "distinct" implies that the object should not be repeated. Each object of a set is called a member or element of that set. A set is represented by { }.
- Generally, sets are denoted by capital letters X, Y, Z, etc., and their elements are denoted by small letters x, y, z, etc.
- If X is a non-empty set and x is an element of X, then we write x ∈ X and read it as "x is an element of X" or "x belongs to X".
- If x is not an element of X, then we write x ∉ X and read it as "x is not an element of X" or "x does not belong to X".
- Example 1.1.1: Suppose we have a set X defined as the set of all days in a week. In this set, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday are members of the set.
- **Methods of Representation of Sets:**
- Sets can be represented by the following two methods:
- 1.Tabular method or Roster method
- 2.Set builder method
- **Tabular Method or Roster Method:**
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- In this method, elements are listed and put within braces {} and separated by commas.
- Example 1.1.2: Suppose we have a set X defined as the set of even numbers less than 15. [X = {2, 4, 6, 8, 10, 12, 14}]
- **Set Builder Method:**
- In this method, instead of listing all elements of a set, we list the property or properties satisfied by the elements of the set and write it as: [X = {x : P(x)}] It is read as "X is the set of all elements x such that x has the property P(x)." The symbol ":" stands for "such that.
- "Example 1.1.3: Suppose we have a set X defined as the set of even numbers less than 15. [X = {x : x = 2n, n \in \mathbb{N}, 1 \leq n \leq 7}]
- **Types of Sets:**
- 1.Empty (Void/Null) Set: A set that has no element is called an empty set. It is denoted by ϕ or {}.
- Example 1.1.4: Let X be the set of all even prime numbers greater than 3.
- Example 1.1.5: Let Y be the set of all prime numbers less Then
- 2.Singleton Set: A set that has only one element is known as a singleton set.
- Example 1.1.6: Let X = {x : x is an even prime number} and Y = {a}.
- 3.Finite Set: A set that has a finite number of elements is known as a finite set.
- Example 1.1.7: Let X = {x : x is an even number less than 9} and Y = {1, 3, 5, 7, 11, 13, 15}.
- 4.Infinite Set: A set that has an infinite number of elements is known as an infinite set.
- Example 1.1.8: Let X = {x : x is a natural number} and Y = {2, 4, 6, 8, 10, 12, 14, ...}.
- 5.Equivalent Sets: If two finite sets X and Y have the same number of elements, then the sets are known as equivalent sets.
- Example 1.1.9: Let X = {2, 4, 6, 8} and Y = {1, 3, 5, 7}.
- 6.Equal Sets: If X and Y are two non-empty sets and each element of X is an element of Y, and each element of Y is an element of X, then sets X and Y are called equal sets.
- Example 1.1.10: Let X = {x : x = 2n} and Y = {2, 4, 6, 8, 10}.
- 7.Universal Set: If there are some sets under consideration, there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set, and it is denoted by U.
- Example 1.1.11: Suppose we have three sets X = {a, b}, Y = {c, d, e}, and Z = {f, g, h, i, j}. [U = {a, b, c, d, e, f, g, h, i, j}] is a universal set for all given sets.
- 8.Power Set: If X is a non-empty set, then the collection of all possible subsets of set X is known as the power set. It is denoted by P(X).
- Example 1.1.12: Let X = {a, b, c}. [P(X) = {\emptyset, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}}]
- **Subset and Superset:**
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- Let X and Y be two non-empty sets. If each element of set X is an element of set Y, then set X is known as a subset of set Y. If set X is a subset of set Y, then set Y is called the superset of X. Also, if X is a subset of Y, then it is denoted as (X \subseteq Y) and read as "X is a subset of Y."If (x \in X) implies (x \in Y), then (X \subseteq Y).If (x \in X) and (x \notin Y), then X will not be a subset of Y.
- Example 1.1.13: If X = {a, b} and Y = {a, b, c, d}, here each element of X is an element of Y. Thus (X \subseteq Y), i.e., X is a subset of Y and Y is a superset of X.
- **Proper Subset:**
- If each element of X is in set Y, but set Y has at least one element which is not in X, then set X is known as a proper subset of set Y. If X is a proper subset of Y, then it is written as (X \subset Y) and read as "X is a proper subset of Y."
- Example 1.1.14: If (N = {1, 2, 3, 4}) and (W = {0, 1, 2, 3, 4}), then (N \subset W).

Unit 1.2: Venn Diagrams

- **Introduction:**
- In the previous unit, we learned what subsets, supersets, and proper subsets are. In this chapter, we will learn about Venn diagrams. Venn diagrams help us easily solve questions related to everyday life.
- **Venn Diagrams:**
- *Definition:* Venn diagrams represent relationships between sets.
- *Origin:* Named after English logician John Venn.
- *Structure:*Rectangles and closed curves (usually circles) are used.
- A universal set is usually represented by a rectangle, and its subset by a circle.

- Elements of a set are written within their respective sets.
- Example:Universal Set (U): {1, 2, 3, 4, 5, 6, 7}Subset (X): {1, 4, 6, 7}

- U (Universal Set): {1, 2, 3, 4, 5, 6, 7}X (Subset): {1, 4, 6, 7
- }In the Venn diagram:The rectangle represents the universal set (U).The circle within the rectangle represents the subset (X).

Basic Set Operations

- **Union of Sets :**

- Definition: Combines all elements of two sets, including common elements only once.
- Symbol: (\cup)
- Example: If (X = {1, 3, 5, 7}) and (Y = {1, 2, 4, 6, 8}), then (X \cup Y = {1, 2, 3, 4, 5, 6, 7, 8}).
- Properties:
- Commutative Law: (X \cup Y = Y \cup X)
- Associative Law: ((X \cup Y) \cup Z = X \cup (Y \cup Z))
- Identity Law: (X \cup \emptyset = X)
- Idempotent Law: (X \cup X = X)
- Universal Law: (U \cup X = U)
- **Intersection of Sets:**
- Definition: Contains only the elements common to both sets.
- Symbol: (\cap)
- Example: If (X = {1, 3, 5, 7}) and (Y = {1, 2, 3, 4, 6, 8}), then (X \cap Y = {1, 3}).
- Properties:
- Commutative Law: (X \cap Y = Y \cap X)
- Associative Law: ((X \cap Y) \cap Z = X \cap (Y \cap Z))
- Identity Law: (X \cap \emptyset = \emptyset)
- Idempotent Law: (X \cap X = X)
- Universal Law: (U \cap X = X)
- Distributive Law: (X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z))
- **Disjoint sets:**
- Definition: Two sets with no common elements.
- Condition: (X \cap Y = \emptyset)
- Example: If (X = {2, 4, 6}) and (Y = {1, 3, 5}), then (X \cap Y = \emptyset).
- **Difference of Sets:**
- Definition: Elements in one set but not in the other.
- Symbols: (X - Y) and (Y - X)
- Example: If (X = {1, 2, 3, 4, 5, 6}) and (Y = {1, 3, 5}):(X - Y = {2, 4, 6})(Y - X = { \emptyset }) (assuming no additional elements in (Y))
- **Complement of a Set:**
- Definition: All elements in the universal set (U) that are not in set (X).
- Symbol: (X') or (\overline{X})
- Example: If (U = {1, 2, 3, 4, 5, 6, 7, 8}) and (X = {1, 3, 5, 7}), then (X' = {2, 4, 6, 8}).
- **Cartesian Product of Sets:**
- Definition: Set of all ordered pairs from two sets.
- Symbol: (X \times Y)
- Example: If (X = {a, b}) and (Y = {1, 2}), then (X \times Y ={(a, 1), (a, 2), (b, 1), (b, 2)}).

De Morgan's Law

- According to De Morgan's laws:
- The complement of the union of two sets is the intersection of their complements.
- Symbolically: ((X \cup Y)' = X' \cap Y')
- The complement of the intersection of two sets is the union of their complements.
- Symbolically: ((X \cap Y)' = X' \cup Y')
- Example 1.4.1:Given:(U = {1, 2, 3, 4, 5, 6})(A = {2, 3})(B = {3, 4, 5})
- To show: ((A \cup B)' = A' \cap B')
- Steps:(A \cup B = {2, 3, 4, 5})((A \cup B)' = {1, 6})(A' = {1, 4, 5, 6})(B' = {1, 2, 6})(A' \cap B' = {1, 6})
- Hence, ((A \cup B)' = A' \cap B')
- Example
- 1.4.2:Find the value of (A \cap (A \cup B)').
- Steps:(A \cap (A \cup B)' = A \cap (A' \cap B')) [by De Morgan's Law]
- (= (A \cap A') \cap B') [by associative law]
- (= \emptyset \cap B') [since (A \cap A' = \emptyset)](= \emptyset)
- **Distributive Laws:**
- (A \cup (B \cap C) = (A \cup B) \cap (A \cup C))
- (A \cap (B \cup C) = (A \cap B) \cup (A \cap C))
- Example
- 1.4.4:Given non-empty sets (A) and (B), find the value of ((A \cap B) \cup (A - B)).
- Steps:((A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')) [since (A - B = A \cap B')]
- (= A \cap (B \cup B')) [by distributive law]
- (= A \cap U) [since (B \cup B' = U)](= A)
- Example 1.4.5:
- For any two sets (A) and (B), prove (A \cap (A' \cup B) = A \cap B).
- Proof:
- (A \cap (A' \cup B))
- (= (A \cap A') \cup (A \cap B)) [by distributive law]
- (= \emptyset \cup (A \cap B)) [since (A \cap A' = \emptyset)]
- (= A \cap B)
- Summary:
- De Morgan's Law:((X \cup Y)' = X' \cap Y')((X \cap Y)' = X' \cup Y')
- Distributive Laws:(A \cup (B \cap C) = (A \cup B) \cap (A \cup C))(A \cap (B \cup C) = (A \cap B) \cup (A \cap C))

Unit 1.5: Matrix

- **Introduction:**
- In previous units, we learned about set operations and De Morgan's law. In this unit, we will discuss matrices, which are powerful tools in mathematics, used for simplifying

complex calculations, solving systems of linear equations, and various applications in commerce and science.

- **Matrix:**
- A matrix is an arrangement of (m \times n) numbers or functions in (m) horizontal lines (rows) and (n) vertical lines (columns). This arrangement is enclosed in brackets ([]).
- Each number in the matrix is called an element, and its position is determined by two subscripts (row, column). The element in the (i)-th row and (j)-th column is denoted by (a_{ij}).
- Example:
- A (2 \times 3) matrix: [\begin{bmatrix} 2 & 8 & 3 \ 5 & 7 & 4 \end{bmatrix}]
- A (3 \times 2) matrix: [\begin{bmatrix} 2 & 5 \ 8 & 7 \ 3 & 4 \end{bmatrix}]
- **Types of Matrices:**
- Row Matrix:
- A matrix with only one row.
- Example: [[2 , 6 , 3]]
- Column Matrix:
- A matrix with only one column.
- Example: [\begin{bmatrix} 2 \ 5 \end{bmatrix}]
- Null Matrix:
- A matrix where all elements are zero.
- Examples: [\begin{bmatrix} 0 & 0 & 0 \ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \ 0 & 0 & 0 \ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \ 0 & 0 \end{bmatrix}]
- Square Matrix: A matrix with the same number of rows and columns ((m = n)).
- Example: [\begin{bmatrix} 1 & 9 & 25 \ 4 & 12 & 30 \ 8 & 16 & 32 \end{bmatrix}]
- **Diagonal Matrix:**
- A square matrix where all non-diagonal elements are zero.
- Example: [\begin{bmatrix} 1 & 0 & 0 \ 0 & 2 & 0 \ 0 & 0 & 3 \end{bmatrix}]
- 1. Diagonal Matrix
- Definition: A square matrix where all non-diagonal elements are zero.
- Order: (n \times n)
- Example: [\begin{pmatrix} 1 & 0 & 0 \ 0 & 2 & 0 \ 0 & 0 & 3 \ \end{pmatrix}]
- This is a diagonal matrix of order (3 \times 3).
- 2. **Rectangular Matrix:**
- Definition: A matrix where the number of rows and columns are not the same.
- Order: (m \times n) (where (m \neq n))
- Example: [\begin{pmatrix} 1 & 2 & 3 \ 2 & 3 & 4 \ \end{pmatrix}]
- This is a rectangular matrix of order (2 \times 3).
- **3. Scalar Matrix:**
- Definition: A diagonal matrix where all the diagonal elements are equal.Order: (n \times n)
- Example: [\begin{pmatrix} 2 & 0 & 0 \ 0 & 2 & 0 \ 0 & 0 & 2 \ \end{pmatrix}]
- This is a scalar matrix of order (3 \times 3).
- **4. Symmetric Matrix:**
- Definition: A square matrix that is equal to its transpose ((A = A^T)).
- Order: (n \times n)
- Example: [\begin{pmatrix} 1 & 3 & 4 \ 3 & 2 & 5 \ 4 & 5 & 6 \ \end{pmatrix}]
- This is a symmetric matrix of order (3 \times 3).
- **5. Skew-Symmetric Matrix:**
- Definition: A square matrix where the transpose is equal to the negative of the original matrix ((A^T = -A)).
- Order: (n \times n)
- Example: [\begin{pmatrix} 0 & x & y \ -x & 0 & z \ -y & -z & 0 \ \end{pmatrix}]
- This is a skew-symmetric matrix of order (3 \times 3).
- **6. Identity Matrix (Unit Matrix):**
- Definition: A scalar matrix where each diagonal element is 1.
- Order: (n \times n)
- Example: [\begin{pmatrix} 1 & 0 \ 0 & 1 \ \end{pmatrix}]
- This is an identity matrix of order (2 \times 2).
- [\begin{pmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 0 & 0 & 1 \ \end{pmatrix}]
- This is an identity matrix of order (3 \times 3).
- **7. Upper Triangular Matrix:**

- Definition: A square matrix where all elements below the principal diagonal are zero.
- Order: (n \times n)
- Example: [\begin{pmatrix} 4 & 2 & 6 \ 0 & 5 & 3 \ 0 & 0 & 6 \ \end{pmatrix}]
- This is an upper triangular matrix of order (3 \times 3).
- **Lower Triangular Matrix:**
- Definition:
- A square matrix in which each element above the principal diagonal is 0.
- Example: [\begin{bmatrix} 4 & 0 & 0 \ 2 & 5 & 0 \ 3 & 4 & 6 \end{bmatrix}] This is a lower triangular matrix of order (3 \times
- **Singular Matrix:**
- Definition: A square matrix is called singular if its determinant is 0.
- Example: [A = \begin{bmatrix} 1 & 2 \ 4 & 2 \end{bmatrix}] Determinant, (|A| = 1 \times 2 - 2 \times 4 = 2 - 8 = -6) Since (|A| = 0), (A) is a singular matrix.
- **Non-singular Matrix:**
- Definition: A square matrix is called non-singular if its determinant is not equal to 0.
- Example: [A = \begin{bmatrix} 1 & 2 \ 3 & 4 \end{bmatrix}] Determinant, (|A| = 1 \times 4 - 2 \times 3 = 4 - 6 = -2) Since (|A| \neq 0), (A) is a non-singular matrix.
- **Equal Matrices:**
- Definition: Two matrices (A) and (B) are said to be equal, written as (A = B), if they are of the same type and their corresponding elements are equal.
- Example: [A = \begin{bmatrix} a & c & e \ b & d & f \end{bmatrix}] and [B = \begin{bmatrix} a & c & e \ b & d & f \end{bmatrix}] are equal matrices. However, [C = \begin{bmatrix} a & c & e \ g & h & i \end{bmatrix}] is not equal to (A) or (B) because the corresponding elements are not all equal.

Basic Concepts of Mathematical Logic

- **Introduction:**
- Statements: Constructed with words; ambiguity is possible due to multiple meanings.
- Symbolic Language: Used to express mathematical statements, important in discrete mathematics and areas of computer science like AI and digital logic.
- **Proposition or Statement:**
- Definition: A declarative sentence that is either true or false, but not both.
- Examples:"One plus two equals five" (False)"One plus two equals three" (True)
- Truth Value: The truth or falsity of a statement.
- Two-Valued Logic: Only admits two truth values, true (T or 1) and false (F or 0).
- Non-Propositions: Questions, exclamations, and commands.
- Propositional Variables: Represented by p, q, r, etc., can be true (T) or false (F).
- **Compound Proposition:**
- Atomic Proposition: A single propositional variable or constant, cannot be subdivided.
- Composite/Compound Proposition: Formed by combining two or more propositions using logical operators or connectives.
- **Connectives:**
- Definition: Words, phrases, or symbols used to form compound propositions.
- Basic Connectives:
- Negation, Conjunction, Disjunction, Conditional, Biconditional.
- **Symbols and Their Connectives:**
- Negation (~, , N): "Not", negation of p (~p), makes a statement false if it is true and vice versa.
- Conjunction (): "And", combines p and q (p q).
- Disjunction (): "Or", combines p and q (p q).
- Conditional (): "If... then", conditional statement (p q).
- Biconditional (): "If and only if", biconditional statement (p q).
- Examples of Connectives:
- Negation:
- Statement: p: Dehradun is in Uttarakhand.
- Negation: ~p: Dehradun is not in Uttarakhand.
- Negation of Quantified Statement:
- Statement: q: No student is intelligent.
- Negation: ~q: Some students are intelligent.
- **Logic and Compound Statements**
- Conjunction
- Definition: If (p) and (q) are statements, the conjunction (p \land q) is true if both (p) and (q) are true.
- Symbol: (p \land q)
- Truth Table: | p | q | (p \land q) | |-----|-----|----------------| | T | T | T | | T | F | F | | F | T | F | | F | F | F |
- **Disjunction**
- Definition: If (p) and (q) are statements, the disjunction (p \lor q) is true if at least one of (p) or (q) is true.
- Symbol: (p \lor q)
- Truth Table: | p | q | (p \lor q) | |-----|-----|---------------| | T | T | T | | T | F | T | | F | T | T | | F | F | F |
- **Conditional (Implication)**
- Definition: If (p) and (q) are statements, (p \rightarrow q) is false only if (p) is true and (q) is false.
- Symbol: (p \rightarrow q)
- Truth Table: | p | q | (p \rightarrow q) | |-----|-----|----------------------| | T | T | T | | T | F | F | | F | T | T | | F | F | T |
- **Biconditional**
- Definition: If (p) and (q) are statements, (p \leftrightarrow q) is true only if (p) and (q) have the same truth value.
- Symbol: (p \leftrightarrow q)
- Truth Table: | p | q | (p \leftrightarrow q) | |-----|-----|--------------------------| | T | T | T | | T | F | F | | F | T | F | | F | F | T |
- **Truth Tables for Compound Statements**
- Negation
- Symbol: (\neg p)
- Truth Table: | p | (\neg p) | |-----|-------------| | T | F | | F | T |
- Negation of Conjunction
- Symbol: (\neg (p \land q))
- Truth Table: | p | q | (p \land q) | (\neg (p \land q)) | |-----|-----|----------------|------------------------| | T | T | T | F | | T | F | F | T | | F | T | F | T | | F | F | F | T |
- Negation of Disjunction
- Symbol: (\neg (p \lor q))
- Truth Table: | p | q | (p \lor q) | (\neg (p \lor q)) | |-----|-----|---------------|-----------------------| | T | T | T | F | | T | F | T | F | | F | T | T | F | | F | F | F | T |
- Negation of Conditional
- Symbol: (\neg (p \rightarrow q))
- Truth Table: | p | q | (p \rightarrow q) | (\neg (p \rightarrow q)) | |-----|-----|----------------------|------------------------------| | T | T | T | F | | T | F | F | T | | F | T | T | F | | F | F | T | F |
- Negation of Biconditional
- Symbol: (\neg (p \leftrightarrow q))
- Truth Table: | p | q | (p \leftrightarrow q) | (\neg (p \leftrightarrow q)) | |-----|-----|--------------------------|----------------------------------| | T | T | T | F | | T | F | F | T | | F | T | F | T | | F | F | T | F |
- **Related Conditional Propositions**

- Converse
- Definition: The converse of $p \rightarrow q$ is $q \rightarrow p$.
- Contrapositive
- Definition: The contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.
- Inverse
- Definition: The inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$.
- Truth Table for Conditional, Converse, Inverse, and Contrapositive
- pq($p \rightarrow q$)($q \rightarrow p$)($\neg p \rightarrow \neg q$)($\neg q \rightarrow \neg p$)TTTTTTTTFFTTFFTTFFTFFTTTT
- **Algebra of Propositions**
- Laws
- Idempotent Law:
- $p \lor p \equiv p$
- $p \land p \equiv p$
- Associative Law:
- $(p \lor q) \lor r \equiv p \lor (q \lor r)$
- $(p \land q) \land r \equiv p \land (q \land r)$
- Commutative Law:
- $p \lor q \equiv q \lor p$
- $p \land q \equiv q \land p$
- Distributive Law:
- $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
- $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
- Identity Law:
- $p \lor T \equiv T$
- $p \lor F \equiv p$
- $p \land T \equiv p$
- $p \land F \equiv F$
- Complement Law:
- $p \lor \neg p \equiv T$
- $p \land \neg p \equiv F$
- Involution Law:
- $\neg (\neg p) \equiv p$
- De Morgan's Laws:
- $\neg (p \lor q) \equiv \neg p \land \neg q$
- $\neg (p \land q) \equiv \neg p \lor \neg q$
- **Logical Equivalence**
- Definition: Two propositions $P(p, q, \ldots)$ and $Q(p, q, \ldots)$ are logically equivalent if they have the same truth value in every possible case, denoted by $P \equiv Q$.
- Tautology, Contradiction, and Contingency
- Tautology: A statement that is always true.
- Contradiction: A statement that is always false.
- Contingency: A statement that is neither always true nor always false.

Unit 2.2: Switching Circuits

- **Switching Circuits**
- A switch is a two-state device used to control the flow of current in a circuit. Switches are denoted by letters such as S, S1, S2, etc.
- **Switch Circuit Example**
- Consider a circuit containing a lamp controlled by a switch S. When the switch is closed (on or 1), current flows in the circuit, and the lamp glows. When the switch is open (off or 0), no current flows, and the lamp does not glow.
- **Symbolic Logic Representation**
- The theory of symbolic logic can represent a circuit by a statement pattern. Conversely, we can construct a circuit for given statement patterns. Switches in the same state are represented by the same letter (equivalent switches), and switches in opposite states are represented by complementary letters.
- Example
- Switch S1 corresponds to statement letter p)(p): switch S1($\sim p$): switch ($S'1$)
- Switch S2 corresponds to statement letter q)(q): switch S2($\sim q$): switch ($S'2$)
- **Input Output Table**
- We consider all possible combinations of states of all switches in the circuit and prepare a table, called the "Input Output table," which is similar to the truth table of the corresponding statement pattern.
- **Two Switches in Series**
- Two switches S1 and S2 are connected in series to control an electric lamp 'L'.
- Let p: switch S1
- Let q: switch S2
- L: the lamp L
- Input Output Table for $p \land q$
- (p)(q)($p \land q$)111100010000
- **Two Switches in Parallel**
- Two switches S1 and S2 are connected in parallel to control an electric lamp 'L'.
- Let p: switch S1
- Let q: switch S2
- L: the lamp L
- Input Output Table for ($p \lor q$)(p)(q)($p \lor q$)111101011000

- **GROUP**
- **Introduction:**
- Group theory is an important part of mathematics that started in the 19th century with the solution of algebraic equations. It generalizes to the concept of an abstract group and applies in various fields like crystallography, quantum mechanics, geometry, topology, analysis, algebra, and biology.
- **Binary Operations:**
- A binary operation (o) on a set (G) is a mapping from (G \times G) to (G). If (o(a, b) = aob) for (a, b \in G), then (o) is a binary operation.
- **Types of Binary Operations:**
-
- Commutative Operation:A binary operation (o) on a set (G) is commutative if (aob = boa) for all (a, b \in G).
- Examples:

- Addition and multiplication of natural numbers are commutative.
- Subtraction and division are not commutative.
- **Associative Operation:**
- A binary operation (o) on a set (G) is associative if ((aob)oc = ao(boc)) for all (a, b, c \in G).
- **Distributive Operation:**
- An operation (o') is left distributive with respect to (o) if (ao'(boc) = (ao'b)o(ao'c)) for all (a, b, c \in G).
- It is right distributive if ((boc)o'a = (bo'a)o(co'a)) for all (a, b, c \in G).
- **Identity**:
- An element (e) in (G) is an identity element with respect to (o) if (aoe = a = eoa) for all (a \in G)
- **Inverse**:
- An element (a \in G) has an inverse (b \in G) with respect to (o) if (aob = e = boa), where (e) is the identity element.
- **Algebraic Structure:**
- An algebraic structure is a non-empty set (G) together with at least one binary operation defined on it.
- Examples:

- ((N, +)), ((I, +)), ((R, +, .))
- Addition and multiplication on the set of real numbers are binary operations. Thus, ((R, +, .)) is an algebraic structure with two operations.
- **Group:**
- An algebraic structure ((G, o)) is a group if it satisfies the following axioms:
- Closure (G1): (G) is closed under (o), i.e., (aob \in G) for all (a, b \in G).
- Associativity (G2): (o) is associative, i.e., ((aob)oc = ao(boc)) for all (a, b, c \in G).
- Identity (G3): There exists an identity element (e \in G) such that (eoa = aoe = a) for all (a \in G).
- Inverse (G4): Each element (a \in G) has an inverse (b \in G) such that (aob = e = boa).
- **Types of Groups**:
- 1.Groupoid (Quasi-group):
- If ((G, o)) satisfies only the closure axiom (G1), it is called a groupoid.
- Example: The set of odd integers is not closed under addition, so it is not a groupoid.
- 2.Semi-group:
- If ((G, o)) satisfies the closure (G1) and associativity (G2) axioms, it is a semi-group.Examples: ((I, +)), ((I, .)), ((N, .)), and ((R, +)) are semi-groups.
- 3.Monoid:
- A semi-group that also satisfies the identity axiom (G3) is called a monoid.A monoid is an associative groupoid with an identity element.
- 4.Quaternion Group:
- The set (A = {\pm 1, \pm i, \pm j, \pm k}) with the operation defined by (i^2 = j^2 = k^2 = -1) and (ij = -ji = k), (jk = -kj = i), (ki = -ik = j) forms a quaternion group.
- 5.Abelian Group:
- A group ((G, o)) is abelian if (o) is commutative, i.e., (aob = boa) for all (a, b \in G).
- Examples: ((Z, +)), ((Q, +)), ((R, +)), ((C, +)), ((Q_0, \times)), ((R_+, \times)).
- **Properties and Theorems:**
- 1.Unique Identity Element:
- The identity element of a group is unique.
- Proof: Suppose (e) and (e') are identity elements, then (eoe' = e) and (eoe' = e') implies (e = e').
- 2.Unique Inverse Element:
- The inverse of each element in a group is unique.
-
- Proof: Suppose (a^{-1}) and (a') are inverses of (a), then (a^{-1}oa = e = aoa^{-1}) and (a'oa = e = aoa'). By associativity, (a^{-1} = a').Inverse of the Inverse:If (a^{-1}) is the inverse of (a), then the inverse of (a^{-1}) is (a), i.e., ((a^{-1})^{-1} = a).
- 3.Inverse of a Product:
- The inverse of the product of two elements is the product of the inverses taken in reverse order, i.e., ((aob)^{-1} = b^{-1}oa^{-1}).
- 4.Cancellation Laws:
- If (aob = aoc), then (b = c) (left cancellation law).
- If (boa = coa), then (b = c) (right cancellation law).
- 5.Solutions to Linear Equations:
- In a group, the equations (aox = b) and (yoa = b) have unique solutions, which are (x = a^{-1}ob) and (y = boa^{-1}) respectively.
- 6.Identity Element Property:
- If an element satisfies (a + 0a = a) or (0a + a = a), then (0a = a)
- **Examples and Problems:**
- Example 3.1.5: Commutative and Associative OperationFor (Q) (set of rational numbers) with operation (aob = a + b - ab):
- Commutative: (aob = boa)

- .Associative: $a o (b o c) = (a o b) o c$.
- Example 3.1.6: Closure under Union and Intersection
- For $S = \{A, B, C, D\}$ where $A = \emptyset, B = \{a\}, C = \{a, b\}, D = \{a, b, c\}$:Union ($\cup$) and intersection ($\cap$) are binary operations on S.
- Example 3.1.9: Abelian Group of Even IntegersThe set of all even integers (including zero) with additive property forms an abelian group.
- Example 3.1.10: Group of Non-zero Rational NumbersThe set of all non-zero rational numbers with multiplication forms a group.
- **Finite and Infinite Groups:**
- A group with a finite number of elements is called a finite group; otherwise, it is an infinite group.
- **Order of a Group:**
- The number of elements in a finite group is called the order of the group. An infinite group has an infinite order.
- **Unit 3.2: Subgroups and Other Groups**
- **Binary Operations**
- Definition:
- A binary operation on a set is a rule for combining any two elements of the set to form a third element.
- Types:
- Commutative: $a * b = b * a$
- Associative: $(a * b) * c = a * (b * c)$
- **Group Theory Basics**
- Group:
- A set G with a binary operation satisfying closure, associativity, identity, and invertibility.
- Groupoid: A set with a binary operation that is not necessarily associative.Semi-group: A set with an associative binary operation.
- Monoid: A semi-group with an identity element.
- Quaternian Group: A specific group with properties and elements resembling quaternion multiplication.
- Abelian Group: A group where the binary operation is commutative.
- Non-Abelian Group: A group where the binary operation is not commutative.
- Finite Group: A group with a finite number of elements.
- Infinite Group: A group with an infinite number of elements.
- Order of a Group: The number of elements in the group.
- Unit Outcome
- **Subgroups:**
- Define and identify subgroups within a larger group.
- Understand properties and significance of subgroups.
- **Order of an Element:**
- Calculate the order of an element within a group.
- **Cyclic Groups:**
- Define cyclic groups and their properties.
- Understand the significance of generators in cyclic groups.
- **Introduction to Subgroups**
- Subgroups are smaller groups derived from a given group G.
- **Importance**: Subgroups retain the characteristic properties of the parent group and act as true representatives.
- **Subgroup Definition and Properties**
- Definition:
-
- A non-empty subset H of a group G is a subgroup if:
- The composition in G induces a composition in H.
- H forms a group under this induced composition.
- **Trivial Subgroups:**
- Identity element alone.
- The group G itself.
- **Proper Subgroup**:
- Any subgroup other than the trivial subgroups.
- **Complex:**
- Any subset of a group, regardless of whether it is a subgroup.
- **Theorems and Proofs**
- **Intersection of Subgroups:**
- Theorem: $H_1 \cap H_2$ is a subgroup of G.
- Proof: If $a, b \in H_1 \cap H_2$, then $a \cdot b^{-1} \in H_1 \cap H_2$.
- **Union of Subgroups:**

- Theorem: The union of two subgroups is not necessarily a subgroup.
- Example:
- $H_1 = \{0, \pm2, \pm4, \pm6, \ldots\}$
- $H_2 = \{0, \pm3, \pm6, \pm9, \ldots\}$
- $H_1 \cup H_2 = \{0, \pm2, \pm3, \pm4, \pm6, \pm9, \ldots\}$
- Not closed under addition.
- **Condition for Union:**
- Theorem: The union of two subgroups is a subgroup if and only if one is contained within the other.
- Proof: If $H_1 \cup H_2$ is a subgroup, then either $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$.
- **Finite Subset Subgroup:**
- Theorem: A finite subset H of G is a subgroup if $a, b \in H$ implies $ab \in H$.
- Proof:Closure: Given by $a, b \in H \Rightarrow ab \in H$.
- Associativity: Inherited from G.
- Identity: Derived from the finite nature and closure.
- Inverses: Derived from the finite nature and closure.
- **Order of an Element**
- Definition: The order of an element a in a group G is the smallest positive integer n such that $a^n = e$ (identity element).

- Infinite Order: If no such n exists, a is of infinite or zero order.
- Notation: $o(a)$.
- Example:Multiplicative group $G = \{1, -1, i, -i\}$:
- Order of 1: 1
- Order of -1: 2
- Order of i: 4
- Order of $-i$: 4
- **Cyclic Group**
- Definition: A group G is cyclic if for some $a \in G$, every element $x \in G$ is of the form a^n for some integer n.
- Generator: The element a that generates the group.
- Examples:Multiplicative group $\{1, \omega, \omega^2\}$ with generators ω and ω^2.Multiplicative group of nth roots of unity, with generator $e^{2\pi i/n}$.
- **Properties of Cyclic Groups**
- 1.Abelian Property:
- Theorem: Every cyclic group is abelian.
- Proof: For any $a^r, a^s \in G$, $a^r \cdot a^s = a^{r+s} = a^{s+r} = a^s \cdot a^r$.
- 2.Order:
- Theorem: The order of a cyclic group is the same as the order of its generator.Proof: If a has order n, the cyclic group generated by a has n
- elements: $\{e, a, a^2, \ldots, a^{n-1}\}$.
- 3.Inverse Generator:
- Theorem: If a is a generator of a cyclic group G, then a^{-1} is also a generator.
- Proof: For any $a^r \in G$, $(a^{-1})^{-r} = a^r$. Every element can be generated by a^{-1}.
- 4.Relatively Prime Powers:
- Example: Find generators of the cyclic group $\{a, a^2, a^3, a^4, e\}$ with order 5:$\gcd(1, 5) = 1$, $\gcd(2, 5) = 1$, $\gcd(3, 5) = 1$, $\gcd(4, 5) = 1$.
- Generators are a, a^2, a^3, a^4.

Unit 4.1: Graph Theory

- **Introduction**
- Graph theory is an applied branch of mathematics with applications in operations research, genetics, physical, biological and social sciences, engineering, and computer science. It involves modeling problems as graphs and solving these graph-theoretic problems to interpret solutions in the original context.
- **Definitions and Types of Graphs**
- *1.Graph:*
- A graph ($G = (V, E)$) consists of a finite set (V) of vertices and a collection (E) of unordered pairs ((u, v)) of distinct elements from (V).
- Vertices (V): Nodes or points in the graph.
- Edges (E): Links or lines connecting pairs of vertices.
- Order: The number of vertices ($|V|$).
- Size: The number of edges ($|E|$).
- Example: A graph with 6 vertices and 10 edges.
- *2.Simple Graph:*
- A graph without self-loops (edges that connect a vertex to itself) and parallel edges (multiple edges between the same pair of vertices).
- Example: Fig. 4.1.2.
- *3.Multi-Graph:*
- A multi-graph (M) consists of a set (V) of vertices and a set (E) of edges where two vertices can be joined by multiple edges, and an edge can join a vertex to itself (loop).
- Parallel edges: Two or more edges joining the same pair of vertices.
- Loop: An edge joining a vertex to itself.
- Example: Fig. 4.1.3.
- *4.Complete Graph:*
- A simple graph where there is an edge between every pair of vertices.
- Denoted by (K_n), where (n) is the number of vertices.
- Example: Complete graphs with 3 and 4 vertices (Fig. 4.1.4).
- *5.Bi-Graph:*
- Consists of two orthogonal structures: a place graph (describes nesting of entities) and a link graph (provides non-local hyperlinks between entities).
- Bi-graphs are compositional, meaning larger bi-graphs can be built by placing regions in sites and connecting faces with like-names.
- Example: Fig. 4.1.5.
- **Degree of a Vertex**
- Degree: The degree of a vertex (v) in a graph (G) is the number of edges incident to (v).
- Self-loop counts twice.
- Degree notation: ($\deg(v)$).
- Minimum degree: ($\delta(G)$).
- Maximum degree: ($\Delta(G)$).
- Example: Fig. 4.1.6 (Graph with 6 vertices and edges).
- **Special Types of Graphs**
- ***1.Isomorphic Graphs:***
- Two graphs ($G_1 = (V_1, E_1)$) and ($G_2 = (V_2, E_2)$) are isomorphic if there exists a bijective function ($f: V_1 \rightarrow V_2$) that preserves adjacency.
- Bijective function: One-to-one and onto mapping.Adjacency preservation: If ($\{a, b\} \in E_1$), then ($\{f(a), f(b)\} \in E_2$).
- Example: Fig. 4.1.7 and Fig. 4.1.8.
- ***2.Euler Graph:***
- A graph (G) containing a closed walk that includes all the edges of (G) (Euler cycle).
- Euler path: An open walk containing all edges of (G).
- *Theorem 1:*
- A connected graph (G) is an Euler graph if all its vertices have an even degree.
- Proof:If (G) is an Euler graph, every vertex must have an even degree as the Euler line enters and exits each vertex.
- Conversely, if all vertices have an even degree, an Euler line can be constructed by starting at any vertex and tracing edges without repetition until all edges are included.
- *Theorem 2*:
- A connected graph (G) is an Euler graph if it can be decomposed into edge-disjoint circuits.
- Proof:If (G) can be decomposed into edge-disjoint circuits, each vertex in the circuit has an even degree, making (G) an Euler graph.
- Conversely, if (G) is an Euler graph, it can be decomposed into edge-disjoint circuits by tracing circuits from each vertex and removing them until no edges are left.
- Example: Fig. 4.1.9 and Euler graph Fig. 4.1.11.
- ***3.Hamiltonian Graph:***
- A graph (G) containing a circuit that includes each vertex exactly once (Hamiltonian cycle).
- *Hamiltonian path:*
- A simple path that includes all vertices of (G) with distinct endpoints.Example: Fig. 4.1.12.
- ***4.Bipartite Graph:***
- A graph ($G = (V, E)$) where the vertex set (V) can be partitioned into two disjoint subsets (V_1) and (V_2) such that each edge ($e \in E$) has one endpoint in (V_1) and the other in (V_2).
- Bipartition: ($V = V_1 \cup V_2$).
- Example: Fig. 4.1.13.
- ***5.Complete Bipartite Graph:***
- A bipartite graph where each vertex in (V_1) is connected to each vertex in (V_2).
- Denoted by ($K_{m, n}$), where (m) and (n) are the number of vertices in (V_1) and (V_2) respectively.
- Star graph: ($K_{1, n}$).
- Example: Fig. 4.1.14 (Star graph ($K_{1,6}$)).
- **Detailed Examples and Notes**

- *1.Graph Example:*
- Consider a graph (G) with vertices (V(G) = {v1, v2, v3, v4, v5, v6}) and edges (E(G) = {e1, e2, e3, e4, e5, e6, e7, e8, e9, e10}).
- *2.Simple Graph Example:*
- A graph without self-loops and parallel edges. E.g., Fig. 4.1.2 shows a simple graph.
- *3.Multi-Graph Example:*
- A graph with parallel edges and possibly loops. E.g., Fig. 4.1.3 shows a multi-graph with parallel edges and a loop.
- *4.Complete Graph Example:*
- A graph where every pair of vertices is connected by an edge. E.g., Fig. 4.1.4 shows complete graphs for 3 and 4 vertices.
- *5.Bi-Graph Example:*
- A compositional structure with place and link graphs. E.g., Fig. 4.1.5 shows a bi-graph with entities connected regardless of location.
- *6.Degree of Vertex Example:*
- In a graph (G) with vertices ({v1, v2, v3, v4, v5, v6}) and edges ({e1, e2, e3, e4, e5, e6, e7, e8, e9, e10}), the degree of each vertex can be computed by counting incident edges.
- *7.Isomorphic Graph Example:*
- Graphs (G1) and (G2) where there is a one-to-one correspondence between vertices and edges such that adjacency is preserved. E.g., Fig. 4.1.7 and Fig. 4.1.8.
- *8.Euler Graph Example:*
- A graph with a closed walk containing all edges. E.g., Fig. 4.1.11 shows an Euler graph with an Euler line.
- *9.Hamiltonian Graph Example:*
- A graph with a Hamiltonian cycle. E.g., Fig. 4.1.12 shows a Hamiltonian cycle where each vertex is visited exactly once.
- *10.Bipartite Graph Example:*
- A graph where vertices can be divided into two disjoint sets (V1) and (V2) with edges only between these sets. E.g., Fig. 4.1.13.
- *11.Complete Bipartite Graph Example:*
- A bipartite graph where every vertex in (V1) is connected to every vertex in (V2). E

- ## Unit 5.1: Data and Their Representation
- **Introduction:**
- Statistics is concerned with the scientific method for collecting, organizing, summarizing, presenting, and analyzing data (datum), as well as drawing valid conclusions and making reasonable decisions based on such analysis. In this unit, we will discuss data, statistical data, the arrangement of raw data, frequency distribution, and graphical representation of the data.
- **Data and Statistical Data**
- **Definition:**
- *Data:*
- A collection of information in raw form. When processed, it becomes information. Processed, meaningful, and logical data is called information.
- *Statistical Data:*
- Refers to numerical data of any phenomenon placed in relation to each other, e.g., population, production, price levels, national income, crimes, literacy, unemployment, etc.
- *Variable*
- Definition: A variable is a quantity that can vary or change from one individual to another. The values a variable takes are called observations.
- *Types of Variables:*
- Discrete Variables:
- These take only finite or countable distinct values (e.g., number of students in a class).
- Continuous Variables:
- These can theoretically assume all values within certain intervals (e.g., height, temperature, weight).
- *Arrangement of Raw Data*
- Raw Data: Data in its original form.
- Seriation: The process of arranging data in a logical or systematic order.
- Statistical Series: Data arranged in a series.Array: Data arranged in ascending or descending order.
- Example: Heights of students: 150, 151, 149, 175, 180, 164, 151, 145, 170, 160.
- Arranged: 145, 149, 150, 151, 151, 160, 164, 170, 175, 180.
- Frequency: Number of times each value occurs.
- *Frequency Distribution*
- Definition:
- Frequency Distribution: Arrangement of data according to the frequency of values.
- Types of Frequency Distributions:
- Univariate Frequency Distribution: Frequency of different values of a single variable.
- Bivariate Frequency Distribution: Based on two variables.
- Discrete Frequency Distribution: Formed by distinct values of a discrete variable.
- Grouped Frequency Distribution: Dividing the entire range into groups and distributing frequencies over these groups.
- Example: Marks of 24 students:Marks: 18, 17, 16, 24, 25, 19, 41, 22, 32, 42, 44, 21, 43, 26, 28, 40, 29, 30, 37, 27, 49, 27, 34, 31.
- Frequency Table:MarksFrequency10-20420-30930-40540-506
- *Cumulative Frequency and Table*
- Definition:
- Cumulative Frequency: Sum of frequencies up to a class interval.
- Example:Marks
- FrequencyCumulative Frequency10-204420-3091330-4051840-50624
- Less Than Cumulative Frequency Table:
-
- MarksCumulative FrequencyLess than 100Less than 204Less than 3013Less than 4018Less than 5024
- *Graphical Representation of Frequency Distribution*
- **Types of Graphs:**
- **Histogram**
- Rectangles with class-intervals as bases and heights proportional to frequencies.
- **Frequency Polygon:**
- Points plotted with central values of class intervals as x-coordinates and frequencies as Curvy-coordinates, joined by straight lines.
- **Frequency Curve:**
- A smoothed frequency polygon.Cumulative Frequency Curve (Ogive): Plotted cumulative frequencies joined by straight lines or a smooth curve.
- **Summary:**Data becomes information when processed.Variables can be discrete or continuous.Frequency distribution can be univariate, bivariate, discrete, or grouped.Graphs like histograms, frequency polygons, and cumulative frequency curves help in data visualization.
- ## Unit-5.2: Measure of the Central Tendency
- **Introduction**
- In the previous unit, we studied about data and statistical data, variable, arrangement of raw data, frequency distribution, graphical representation of data, and the types of graph. When two or more different series of the same type are compared, it is not enough to classify and to tabulate the observations. To make the data more comprehensive, it is often desirable to define quantitatively the characteristics of frequency distribution. There are four fundamental characteristics in which similar frequency distribution may differ. One of the characteristics is central tendency. In this unit, we will discuss about arithmetic mean or average, properties of arithmetic mean, median, positional measure, mode and the relationship between mean, mode, and median.
- **Arithmetic Mean or Average**
- Arithmetic mean of a group of observations is the quotient obtained by dividing the sum of all the observations by their number. Thus, arithmetic mean denoted by x̄ is:
- [x bar= \frac{\text{Sum of all the observations}}{\text{Number of Observations}}]
- Formula:Individual Series: If (X_1, X_2, X_3, ..., X_n) are the n values of a variate x, then the arithmetic mean (A.M.) is given by:

- $\text{A.M.} = \frac{X_1 + X_2 + ... + X_n}{n} = \frac{\sum{x}}{n}$
- **Discrete Frequency Distribution:**
- If the value (x_1) occurs (f_1) times, the value (x_2) occurs (f_2) times, and so on, then: $\text{A.M.} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n} = \frac{\sum{f_ix_i}}{N}$ Where ($N = f_1 + f_2 + ... + f_n = \text{Total frequency}$).
- **Methods of Calculating Arithmetic Mean:**
- Direct Method: $\text{A.M.} = \frac{\sum{f x}}{N}$
- Short-Cut Method: $\bar{x} = A + \frac{\sum{f (x - A)}}{\sum{f}}$
- Step-Deviation Method: $\bar{x} = A + h \cdot \frac{\sum{f u}}{\sum{f}}$
- Where:
- (A) = Assumed mean
- (h) = Class size
- ($u = \frac{x - A}{h}$)($\sum{f} = \text{Sum of the frequencies given, can be denoted by N}$)
- **Properties of Arithmetic Mean:**
- The algebraic sum of the deviations of all the variate values from their mean is zero.
- If every value of the variable is increased by the same constant (a), then arithmetic mean is also increased by (a).
- Arithmetic mean is not independent of the change of origin and scale.
- The sum of the squares of the deviations of all the values taken about their mean is minimum.
- **Geometric Mean:**
- The nth root of the product of the values is called geometric mean.
- Geometric Mean for Ungrouped Data:
- If ($X_1, X_2, X_3, ..., X_n$) be n observations, then geometric mean is given by: $G = \sqrt[n]{X_1 \cdot X_2 \cdot .... \cdot X_n}$ $\log{G} = \frac{1}{n} \sum_{i=1}^{n} \log{X_i}$ $G = \text{antilog} \left(\frac{1}{n} \sum_{i=1}^{n} \log{X_i} \right)$
- Geometric Mean for Grouped Data:
- If ($X_1, X_2, X_3, ..., X_n$) be n observations whose corresponding frequencies are ($f_1, f_2, f_3, ..., f_n$) then geometric mean is given by: $G = \sqrt[N]{X_1^{f_1} \cdot X_2^{f_2} \cdot .... \cdot X_n^{f_n}}$ $G = \text{antilog} \left(\frac{1}{N} \sum_{i=1}^{n} f_i \log{X_i} \right)$
- Example 5.2.3: Find the geometric mean of the numbers ($3, 3^2, 3^3, ..., 3^n$).Solution: $GM = \sqrt[n]{3 \cdot 3^2 \cdot 3^3 \cdot .... \cdot 3^n}$ $= 3^{\frac{1+2+....+n}{n}} = 3^{\frac{n(n+1)/2}{n}} = 3^{(n+1)/2}$
- **Median:**
- When the observations are arranged in ascending or descending order of magnitude, then the middle value is called the median of these observations. Median is that value of the variable which divides a given series into two parts so that one-half or more of the items are equal to or less than it.
- **Formula to Compute Median of the Given Data:**
- Individual Series: Let ($X_1, X_2, X_3, ..., X_n$) be n values of a variable written in ascending order of magnitude. Then median denoted by Me or M is given by:
- When n is odd, then: $\text{Median} = \left(\frac{n+1}{2} \right)^\text{th term}$
- When n is even, then: $\text{Median} = \frac{\left(\frac{n}{2} \right)^\text{th term} + \left(\frac{n}{2} + 1 \right)^\text{th term}}{2}$
- <u>Formula for Grouped Data:</u> $\text{Median} = l + \left(\frac{\frac{N}{2} - C.F.}{f} \right) \cdot h$
- Where:(l) = Lower limit of the median class
- (f) = Frequency of the median class
- (h) = Width of the median class
- (C.F.) = Cumulative frequency(N) = Total frequency(\frac{N
- **Positional Measure:**
- **Median**
- When the observations are arranged in ascending or descending order of magnitude, the middle value is called the median. The median divides the data into two equal parts, such that one-half of the data values are below the median and one-half are above.
- **Median for Ungrouped Data**
- <u>For ungrouped data:</u>
- If the number of observations (n) is odd, the median is the ($(n+1)/2$)-th value.
- If the number of observations (n) is even, the median is the average of the ($(n/2)$)-th and ($(n/2 + 1)$)-th values.
- <u>Median for Grouped Data</u>
- The formula for the median in a grouped data set is:
- $\text{Median} = l + \left(\frac{\frac{N}{2} - CF}{f} \right) \cdot h$
- Where:(l) = lower boundary of the median class
- (N) = total number of observations
- (CF) = cumulative frequency of the class preceding the median class
- (f) = frequency of the median class
- (h) = class width
- **Mode**
- The mode is the value that appears most frequently in a data set. It is the value at the point of maximum frequency in a frequency distribution.
- <u>Mode for Ungrouped Data</u>
- For ungrouped data, the mode is simply the value that occurs most frequently.
- <u>Mode for Grouped Data</u>
- For grouped data, the mode can be estimated using the following formula:
- $\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \cdot h$
- Where:
- (l) = lower boundary of the modal class
- (f_1) = frequency of the modal class
- (f_0) = frequency of the class preceding the modal class
- (f_2) = frequency of the class succeeding the modal class
- (h) = class width
- **Relationship Between Mean, Median, and Mode**
- In a symmetrical distribution, the mean, median, and mode are all equal. However, in a skewed distribution, they differ:
- For a positively skewed distribution: Mode < Median < Mean

- For a negatively skewed distribution: Mean < Median < Mode
- An empirical relationship between the three measures in a moderately skewed distribution is given by: $\text{Mean} - \text{Mode} \approx 3(\text{Mean} - \text{Median})$
- Summary
- This unit covers the measures of central tendency, including the arithmetic mean, geometric mean, median, and mode. We also discussed the properties of these measures and their applications in different types of data. Understanding these measures is crucial for analyzing and interpreting data effectively.

Unit-5.3: Measure of Dispersion

- **Introduction**
- Dispersion in statistics refers to the scatteredness of values of a variable due to variations among them or deviations from a measure of central tendency. This unit discusses the measure of dispersion, variance, properties of standard deviation, and the coefficient of variation.
- **Measure of Dispersion**
- Common measures of dispersion include:
- Range
- Quartile Deviation or Semi-Interquartile Range
- Mean Deviation
- Standard Deviation
- **Range:**
- Range is the difference between the greatest (L) and the smallest (S) values of the variable. $\text{Range} = L - S$
- Quartile Deviation or Semi-Interquartile Range
- The difference between the upper (Q3) and lower (Q1) quartiles is the interquartile range. Half of this range is the quartile deviation (QD). $QD = \frac{1}{2} (Q3 - Q1)$
- **Mean Deviation**
- Mean deviation of a distribution is the arithmetic mean of the absolute deviation of the terms from the statistical mean (A.M., median, or mode).
- For ungrouped or individual series: $MD = \frac{\sum |x_i - A|}{n}$ For grouped data: $MD = \frac{\sum f_i |x_i - A|}{\sum f_i}$
- **Standard Deviation**
- Standard deviation (σ) is the square root of the arithmetic mean of the squares of all deviations of the values from the arithmetic mean.
- For individual series: $\sigma = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n}}$
- For frequency distribution: $\sigma = \sqrt{\frac{\sum f_i (x_i - \overline{x})^2}{N}}$
- For continuous series: $\sigma = \sqrt{\frac{\sum f_i d_i^2}{N}} \times h$ where h is the class interval.
- **Variance**
- Variance (σ^2) is the arithmetic mean of the squares of deviations from the arithmetic mean. $\sigma^2 = \frac{\sum (x_i - \overline{x})^2}{n}$ For grouped series: $\sigma^2 = \frac{\sum f_i (x_i - \overline{x})^2}{N}$
- **Properties of Standard Deviation**
- It is independent of origin.
- It is dependent on the change of scale.
- It is not less than the mean deviation from the mean.
- **Coefficient of Variation (C.V.)**
- The coefficient of variation is the ratio of the standard deviation to the arithmetic mean, useful for comparing variabilities of data sets measured in different units. $\text{C.V.} = \frac{\sigma}{\overline{x}} \times 100$

Unit-5.4: Skewness and Kurtosis

- **Introduction**
- Skewness denotes the lack of symmetry in a frequency distribution. A symmetrical series has identical mode, median, and arithmetic mean. Skewness can be positive (Mean > Median > Mode) or negative (Mode > Median > Mean).
- **Measures of Skewness**
- Bowley's Coefficient of Skewness: $\text{Coeff. of Skewness} = \frac{(Q3 + Q1 - 2 \text{Median})}{Q3 - Q1}$
- Karl Pearson's Coefficient of Skewness: $\text{Coeff. of Skewness} = \frac{\text{Mean} - \text{Mode}}{\sigma}$
- **Kurtosis**
- Kurtosis measures the degree of peakedness or flatness of a frequency distribution curve.
- Mesokurtic (normal curve): $\beta_2 = 3$ and $\gamma_2 = 0$
- Leptokurtic (more peaked): $\beta_2 > 3$ and $\gamma_2 > 0$
- Platykurtic (flatter): $\beta_2 < 3$ and $\gamma_2 < 0$
- **Summary**
- Skewness indicates lack of symmetry; it can be positive or negative.Kurtosis measures the peakedness or flatness of a distribution curve.Bowley's and Karl Pearson's coefficients measure skewness.Mesokurtic, leptokurtic, and platykurtic describe types of kurtosis.