Reg. No. [ ]

B.E./ B. Tech (Full Time) DEGREE END SEMESTER EXAMINATION, April / May - 2014

THIRD SEMESTER

MA8357 - TRANSFORMS TECHNIQUES AND PARTIAL DIFFERENTIAL EQUATIONS

(Regulation: 2012)

COMMON TO AGRI/GEO/CIVIL/PRINT/EEE/ECE/BIO

Time: 3 hours **Maximum: 100 Marks**

Answer ALL Questions

Part – A ($10 \times 2 = 20$ marks)

1. Obtain the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 - b)$.
2. Find the complete solution of $\sqrt{p} + \sqrt{q} = 1$.
3. If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$, in $0 < x < 2\pi$, then deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
4. Define the complex form Fourier series of $f(x)$, in $(c, c+2l)$.
5. What is meant by steady state?
6. What are the possible solutions of the one dimensional heat flow equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.
7. If the Fourier transform of $f(x)$ is $F(s)$, then find the Fourier transform $e^{iax} f(x)$.
8. Solve for $f(x)$ if $\int_0^{\infty} f(x) \cos \alpha x \, dx = e^{-\alpha}$.
9. Find the Z-transform of n^2.
10. State initial value theorem for Z-transform.

Part – B ($5 \times 16 = 80$ marks)

11. i) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & if\ |x| < 1 \\ 0, & otherwise \end{cases}$.

Hence deduce that $\int_0^{\infty} \left(\frac{\sin s - s \cos s}{s^3} \right) ds = -\frac{\pi}{4}$. (8)

ii) Find the Fourier sine transform of e^{-ax} ($a > 0$), hence evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2 + 16)^2}$. (8)

12. (a) i) Find the singular solution of $z = px + qy + \sqrt{1+p^2+q^2}$. (8)

ii) Find the general solution of $(D^2 - 2DD' + D'^2)z = \cos(x-3y) + e^{-2x}$. (8)

(OR)

(b) i) Find the general solution of $z(x-y) = px^2 - qy^2$. (8)

ii) Find the complete solution of $p^2 + q^2 = x^2 + y^2$. (8)

13. (a) i) Find the Fourier series expansion of $f(x) = x^2$ in $(-\pi, \pi)$ of periodicity 2π.

Hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ...to\ \infty = \frac{\pi^2}{12}$. (8)

ii) Obtain half range Fourier cosine series expansion of $f(x) = (x-1)^2$ in $0 < x < 1$. (8)

(OR)

(b) i) Obtain the Fourier series expansion of $f(x) = \begin{cases} -\pi, & if\ -\pi < x < 0 \\ x, & if\ \ 0 < x < \pi \end{cases}$. (8)

ii) Obtain half range Fourier sine series expansion of $f(x) = x$ in $0 < x < l$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. (8)

14. (a) If the string of length l is initially at rest in its equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3 \frac{\pi x}{l}$, $0 < x < l$, determine the transverse displacement $y(x,t)$ at any time at any distance x. (16)

(OR)

(b) An infinitely long plane uniform plate is bounded by two parallel edges $x = 0$ and $x = l$, and an end at right angles to them. The breadth of this edge $y = 0$ is l and is maintained at a temperature 100° and all the other three edges are at temperature zero. Find the steady state temperature at any interior point of the plate. (16)

15. (a) i) Find the Z-transform of $\frac{1}{n(n+1)}$, for $n \geq 1$. (8)

ii) Find the inverse Z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$. (8)

(OR)

(b) i) Using convolution theorem, find the inverse Z-transform of $\frac{z^3}{(z-a)^3}$. (8)

ii) Using Z-transforms, solve $u_{n+2} - 3u_{n+1} + 2u_n = 0$ given that $u_0 = 0, u_1 = 1$. (8)

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