

2012 MATHEMATICS

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A (Multiple Choice Type Questions)

1. Choose the correct alternatives	for	the	following	
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$$10 \times 1 = 10$$

i) Rank of the matrix
$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}$$
 is

a) 4

b) 3

c) 2

d) 1

ii) The value of
$$t$$
 for which the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is

singular is

a) $\frac{3}{2}$

b) 2

c) 1

d) $\frac{1}{3}$.

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- The equation x + y + z = 0 has iii)
 - infinite number of solutions
 - b) no solution
 - c) unique solution
 - two solutions.
- The value of k for which the vectors (1, 2, 1), (k, 1, 1)iv) and (1, 12) are linearly dependent is
 - a) 2

- 1 c)

- The eigenvalues of the matrix A and B are a and b. then v) The eigenvalues of A^2 are
 - a) ab, b^2

b) a^2 , b

 a^2, b^2 c)

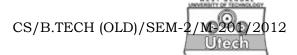
- d) a, b.
- If a linear transformation $T: R^2 \to R^2$ be defined by $T\left(x_{1}, x_{2}\right) = \left(x_{1}, x_{2}\right)$, then Ker $\left(T\right)$ is
- $\{(-1,-1),(1,1)\}$ b) $\{(1,2),(1,\frac{1}{2})\}$
 - $\{(1,0),(0,1)\}$
- d) $\{(0,0)\}.$
- vii) The interpolation formula which can be used to find a polynomial from the following data

X: 0 1 2 4

Y: 3 9 17 22

is

- Newton's forward interpolation formula a)
- b) Gaussian interpolation formula
- Lagrange's interpolation formula c)
- Newton's backward interpolation formula. d)



viii) Which of the following is not true?

a)
$$\Delta = E - 1$$

b)
$$\Delta \cdot \nabla = \Delta - \nabla$$

c)
$$\frac{\Delta}{\nabla} = \Delta + \nabla$$

d)
$$\nabla = 1 - E^{-1}$$
.

The degree and order of the differential equation ix)

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x_2}\right)^{\frac{3}{2}} = x \, \frac{\mathrm{d}y}{\mathrm{d}x} \, \text{are}$$

a)
$$\left(\frac{3}{2}, 2\right)$$
 b) $(2, 3)$

x)
$$L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$$
. Then $L\left\{\frac{\sin at}{t}\right\}$ is

a)
$$\tan^{-1}\left(\frac{1}{s^2}\right)$$
 b) $\tan^{-1}\left(\frac{a}{s}\right)$

b)
$$\tan^{-1}\left(\frac{a}{s}\right)$$

c)
$$\tan^{-1}\left(\frac{1}{as}\right)$$

c)
$$\tan^{-1}\left(\frac{1}{as}\right)$$
 d) $\tan^{-1}\left(\frac{1}{a^2+s^2}\right)$.

GROUP - B

(Short Answer Type Questions)

Answer any three of the following.

$$3 \times 5 = 15$$

Solve the differential equation by Laplace Transformation: 2.

$$\frac{d^2 y}{dt^2} + 9 y = 1, y (0) = 1, y (\frac{\pi}{2}) = -1$$



3. Solve by the method of variation of parameter

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 9y = \sec 3x$$

4. Solve the system of equations if possible:

$$x + y + z = 1$$

$$2x + y + 2z = 2$$

$$3x + 2y + 3z = 5$$

- 5. If $W = \{ (x, y, z) \in \mathbb{R}^3 : x + y + z = 0 \}$, show that W is a subspace of \mathbb{R}^3 , and find a basis of W.
- 6. Examine whether the mapping $T: R^2 \to R^2$ defined by T(x, y) = (2x y, x) is linear.
- 7. Evaluate $\int_{0}^{1} \frac{dx}{1+x}$ using Trapezoidal rule, taking four equal subintervals.

GROUP - C

(Long Answer Type Questions)

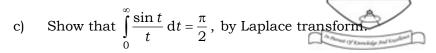
Answer any *three* of the following. $3 \times 15 = 45$

- 8. a) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$.
 - b) Find the missing data in the following table:

х	-2	-1	0	1	2
f(x)	6	0	5	0	6

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- 9. a) Show that (3, 1, -2), (2, 1, 4) and (1, -1, 2) form a basis of \mathbb{R}^3 .
 - b) Apply convolution theorem to prove that $\int_{0}^{t} \sin u \cos (t u) dt = \frac{t}{2} \sin t.$
 - c) Solve $\frac{dy}{dx} \frac{\tan y}{(1+x)} = (1+x) e^x \sec y$.
- 10. a) Solve by Cramer's rule:

$$x + y + z = 7$$

$$x + 2y + 3z = 15$$

$$x - y + z = 3$$

b) Find general solution of $p = \cos(y - px)$, where $p = \frac{dy}{dx}$.

c) Solve
$$\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y (\log y)^2}{x^2}$$

11. a) Solve $(D^2 - 2D)$ $y = e^x \sin x$, where $D = \frac{d}{dx}$.

b) Solve
$$\frac{dx}{dt} - 7x + y = 0$$
 and $\frac{dy}{dt} - 2x - 5y = 0$

c) Solve by Gauss-elimination method:

$$2x + 2y + z = 12$$

$$3x + 2y + 2z = 8$$

$$5x + 10y - 8z = 10$$



- 12. a) Use Lagrange's interpolation formula to find f(x), where f(0) = -18, f(3) = 0, f(5) = -248, f(6) = 0 and f(9) = 13104.
 - b) Apply appropriate interpolation formula to calculate f (2.1), correct up to two significant figures from the following data:

х	0	2	4	6	8	10
f(x)	1	5	17	37	45	51

c) Apply method of variation of parameter to solve the equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = \sec^3 x \cdot \tan x$$

- 13. a) Apply Simpson's $\frac{1}{3}$ rd rule to evaluate $\int_{0}^{6} \frac{dx}{(1+x)^2}$ taking six equal intervals from (0, 6) and correct up to three decimal places. 5+10
 - b) Expand by Laplace method

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^{2}$$

c) Given that

$$L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left(\frac{1}{s}\right)$$
, Find $L\left\{\frac{\sin at}{t}\right\}$.



14. a) Assuming the orthogonal properties of Legend function, prove that

$$\int_{-1}^{+1} [P_n(x)]^2 dx = \frac{2}{2n+1}$$

- b) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
- e) State Cayley-Hamilton theorem and show that the matrix

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$$
 satisfies the above theorem.