

Name :

Roll No. :

Invigilator's Signature :

CS/B.Sc(H)BT/GE/MICRO/MOL-BIO/SEM-1/BMT-104/2011-12

2011

BIO-MATHEMATICS - I

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following :

$$10 \times 1 = 10$$

i) The modulus of the complex number $\frac{1+i}{1-i}$ is

- | | |
|---------------|---------------------------|
| a) 2 | b) 1 |
| c) $\sqrt{2}$ | d) $\frac{1}{\sqrt{2}}$. |

ii) The modulus of the complex number $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is

- | | |
|------|-------|
| a) 1 | b) 2 |
| c) 3 | d) 4. |

iii) If $x + iy$ and $-2 + 3i$ are two conjugate complex numbers, then the values of x and y are

- | | |
|--------------------|-----------------------|
| a) $x = 2, y = 3$ | b) $x = -2, y = 3$ |
| c) $x = 2, y = -3$ | d) $x = -2, y = -3$. |



- iv) The value of $\log_{\sqrt{7}} 343$ is
- a) 6
 - b) 5
 - c) 4
 - d) 3.
- v) If $\log_x 81 = 4$, then the value of x is
- a) 2
 - b) 3
 - c) $\sqrt{3}$
 - d) 4.
- vi) If $\log_{10}(7x - 5) = 2$, then the value of x is
- a) 5
 - b) 15
 - c) 10
 - d) 20.
- vii) The number of terms in the expansion $\left(x - \frac{1}{x}\right)^{10}$ is
- a) 11
 - b) 10
 - c) 9
 - d) 8.
- viii) If the coefficients of $(4r + 5)$ th term and $(2r + 1)$ th term in the expansion of $(1 + x)^{10}$ are equal, then the value of r is
- a) 1
 - b) 2
 - c) 3
 - d) 4.
- ix) The value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$ is
- a) 1
 - b) 0
 - c) 2
 - d) 3.
- x) If $\begin{vmatrix} x & 4 & -2 \\ 4 & x & -2 \\ 4 & -2 & x \end{vmatrix} = 0$ then the values of x are
- a) 4, 2
 - b) -4, -2
 - c) 4, -2
 - d) -4, 2.



xi) If $A = \begin{bmatrix} 2 & 3 \\ 6 & 5 \end{bmatrix}$ then A^2 is

- | | |
|---|---|
| a) $\begin{bmatrix} 22 & 21 \\ 42 & 43 \end{bmatrix}$ | b) $\begin{bmatrix} 22 & 42 \\ 21 & 43 \end{bmatrix}$ |
| c) $\begin{bmatrix} 21 & 22 \\ 43 & 42 \end{bmatrix}$ | d) $\begin{bmatrix} 42 & 43 \\ 22 & 21 \end{bmatrix}$. |

xii) If $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ then the values of x and y are

- | | |
|--------------------|-----------------------|
| a) $x = 1, y = -1$ | b) $x = 1, y = 1$ |
| c) $x = -1, y = 1$ | d) $x = -1, y = -1$. |

xiii) The value of $\lim_{x \rightarrow 2} \frac{x - \sqrt{3x-2}}{x^2 - 4}$ is

- | | |
|------------------|--------------------|
| a) $\frac{1}{4}$ | b) $\frac{1}{16}$ |
| c) $\frac{1}{8}$ | d) $\frac{1}{2}$. |

xiv) The value of $\lim_{x \rightarrow 0} \frac{\sqrt{1+ax} - \sqrt{1-ax}}{x}$ is

- | | |
|-------------------|-----------|
| a) a | b) $2a$ |
| c) $\frac{1}{2}a$ | d) $3a$. |

xv) If $y = e^{4x}$, then $\frac{dy}{dx}$ at $x = 0$ is

- | | |
|------|-------|
| a) 0 | b) 4 |
| c) 1 | d) 2. |

xvi) If $y = \sqrt{x}$ then $\frac{dy}{dx}$ at $x = 2$ is

- | | |
|--------------------------|----------------------------|
| a) $\frac{1}{2}$ | b) $\frac{1}{\sqrt{2}}$ |
| c) $\frac{1}{2\sqrt{2}}$ | d) $\frac{1}{4\sqrt{2}}$. |



xvii) The value of $\int_1^2 x^2 dx$ is

a) $\frac{8}{3}$

b) $\frac{1}{3}$

c) $\frac{7}{3}$

d) $\frac{4}{3}$.

xviii) The value of $\int_2^3 e^{2x} dx$ is

a) $\frac{1}{2}e^4(e^2 - 1)$

b) $e^2 - 1$

c) $\frac{1}{2}(e^2 - 1)$

d) $\frac{1}{2}(e^4 - 1)$.

xix) The order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y = 3x^4$$

a) 2, 2

b) 1, 3

c) 3, 1

d) 2, 1.

xx) The integrating factor of the differential equation

$$\frac{dy}{dx} - \frac{2y}{x} = x^2$$

a) $\frac{1}{x}$

b) $\frac{1}{x^2}$

c) $\frac{2}{x}$

d) $\frac{1}{x^3}$.

xxi) The integrating factor of the differential equation

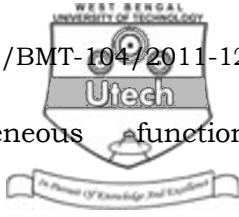
$$\frac{dy}{dx} - y = x$$

a) x

b) e^x

c) e^{-x}

d) $-x$.



xxii) The degree of the homogeneous function

$$f(x, y) = \frac{x+y}{\sqrt{x} + \sqrt{y}} \text{ is}$$

xxiii) If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$, then the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ is

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $z = x + iy$ and $|z + 6| = |2z + 3|$, Show that $x^2 + y^2 = 9$.

3. If n is a positive integer, then prove that

$$(1+i)^n + (1-i)^n = 2^{n+1} \cos \frac{n\pi}{4}.$$

4. If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y}$, show that $x^{y+z} y^{z+x} z^{x+y} = 1$.

5. If the coefficient of x^3 in the expansion $\left(x^2 + \frac{k}{x}\right)^6$ is 160, then find the value of k .

6. Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$



GROUP – C
(Long Answer Type Questions)
 Answer any *three* of the following. $3 \times 15 = 45$

7. a) If $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{bmatrix}$, find A^{-1} .

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 0 & -5 & 6 \\ 3 & -2 & 1 & 2 \\ 5 & -2 & -9 & 14 \\ 4 & -2 & -4 & 8 \end{bmatrix}$ by diagonalisation method.

c) Solve by Cramer's rule :

$$x + 2y + 3z = 6$$

$$2x + 4y + z = 7$$

$$3x + 2y + 9z = 14$$

8. a) A function $f(x)$ is defined by

$$f(x) = x^2, \text{ when } x < 1$$

$$= 2 \cdot 5, \text{ when } x = 1$$

$$= x^2 + 2, \text{ when } x > 1.$$

Is $f(x)$ continuous at $x = 1$?

b) If $y = \sqrt{2x} - \sqrt{\frac{2}{x}} + \frac{x+4}{4-x}$, show that the value of

$$\frac{dy}{dx} \text{ at } x = 2 \text{ is } \frac{11}{4}.$$

c) Find $\frac{dy}{dx}$ if $x^y = y^x$.



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9. a) Test whether Rolle's theorem is applicable or not for the function $f(x) = 1 - |x - 1|$ in $0 \leq x \leq 2$.
- b) In the Mean value theorem $f(b) - f(a) = (b - a)f'(c)$ where $a < c < b$, find the value of c if $f(x) = \sqrt{x}$, $a = 4$, $b = 9$.
- c) Verify Euler's theorem for the function

$$f(x, y) = ax^2 + 2hxy + by^2.$$

10. a) Integrate $\int x^2 \sqrt{a^3 + x^3} dx$
- b) Evaluate $\int_1^{e^2} \frac{dx}{x(1 + \log x)^2}$
- c) Find the area of the segment cut off from $y^2 = 4x$ by the line $y = x$.
11. a) Solve : $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$
- b) Solve : $\frac{dy}{dx} + xy = x$
- c) The population of a country increases at the rate proportional to the number of inhabitants. If the population doubles in 30 years, in how many years will it be triple?

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