Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-2/BM-201/2013

2013

MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) A monotonic and bounded sequence is

a) convergent b) divergent

c) oscillatory d) none of these.

ii) The sequence $\{r^n\}$ is oscillatory when

a) $r > 1$ b) $r < 1$

c) $-1 < r < 1$ d) none of these.

iii) Eliminating A and B from $y = A + Bx$, the differential equation is obtained as

a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} - y = 0$

c) $\frac{d^2y}{dx^2} = 0$ d) none of these.

iv) The order and degree of the equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = a\frac{dy}{dx}$ is

a) 2, 2 b) 2, 3

c) 3, 2 d) 3, 3.

v) The P.I. of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$ is

a) $\frac{e^x}{3}$ b) $\frac{e^x}{2}$

c) $\frac{e^x}{6}$ d) none of these.

vi) The series $\sum_{n=1}^{\infty} n^{\frac{1}{p}}$ is convergent if

a) $p \geq 1$ b) $p < 1$

c) $p > 1$ d) $p \leq 1$.

vii) If the series $\sum_{n=1}^{\infty} u_n$ is convergent, then

a) $\lim_{n \to \infty} u_n = 0$ b) $\lim_{n \to \infty} u_n > 1$

c) $\lim_{n \to \infty} u_n < 1$ d) none of these.

viii) The series 1 – 1 + 1 – 1 + ... is

a) convergent with sum 0

b) convergent with sum 1

c) divergent

d) oscillatory.

ix) The vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) in V_3 are

a) linearly dependent b) linearly independent

c) both (a) and (b) d) none of these.

x) The basis of a vector space contains

a) linearly independent vectors

b) linearly dependent vectors

c) scalars only

d) none of these.

xi) The values of k for which the vectors (1, 2, 1), (k, 1, 1) & (1, 1, 2) in R^3 are linearly independent are

a) $k \neq -\frac{2}{3}$
b) $k \neq \frac{2}{3}$
c) $k \neq -\frac{3}{2}$
d) none of these.

xii) T is a transformation from R^2 to R^3 defined by $T(x_1, x_2) = (x_1,\ x_1^2 + 2,\ -x_1)$. Then the image of (1, 2) is

a) (1, 1, 1)
b) (0, 3, –1)
c) (1, 3, –1)
d) (0,0,0).

xiii) If (3, 1) = x (1, 2)+ y (0, 3) then the values of x and y are respectively

a) (3, –5)
b) (3, 1)
c) $\left(3, -\frac{5}{3}\right)$
d) $\left(3, -\frac{5}{2}\right)$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve $(x + y)\,dy + (x - y)\,dx = 0$.

3. Find the general and singular solutions of

$$y - xp + p^2 = 0, \quad p = \frac{dy}{dx}.$$

4. Test the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots, \; x > 0$$

5. Define monotone sequence. When is a monotone sequence convergent ? Is the following sequence convergent ?

$$\left\{ \frac{3n+1}{n+2} \right\}$$

6. Prove that the intersection of two subspaces of a vector space is a subspace.

7. Find the space generated by (1, 3, 0), (2, 1, –2). Examine whether (4, 7, –2) lies in this space.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) Find the basis and dimension of the subspace W of R^3 where $W = \{ (x, y, z) \in R^3 : x + y + z = 0 \}$. 5

b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n . n!}{n^n}$. 5

c) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 e^{3x}$. 5

9. a) Determine the linear mapping $T: R^3 \to R^3$ which maps the basis vectors (0, 1, 1), (1, 0, 1), (1, 1, 0) of R^3 to the vectors (1, 2, 1), (1, 1, 2), (2, 1, 1) respectively. Find Ker (T) and Im (T). 8

b) Solve : $(x^2D^2 - xD - 3)y = x^2 \log x$. 7

10. a) Define basis and dimension of a vector space. Find a basis and the dimension of $S \cap T$ where S and T are subspaces of R^3 defined by

$S = \{(x, y, z) \in R^3 : 2x + y + 3z = 0\}$

$T = \{(x, y, z) \in R^3 : x + 2y + z = 0\}$ 2 + 1 + 6

b) Examine whether the vectors (1, 2, 2), (2, 1, 2), (2, 2, 1) are linearly independent in R^3. 6

11. a) Test the convergence of the following series :

i) $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots$

ii) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$ 5 + 5

b) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ converges conditionally. 5

12. Solve the following : 3×5

a) $(xy \sin xy + \cos xy)\, y\,dx + (xy \sin xy - \cos xy)\, x\,dy = 0$

b) $y = px + \sqrt{a^2 p^2 + b^2}$, $p = \dfrac{dy}{dx}$

c) $\dfrac{d^2 y}{dx^2} - y = \sin x$

13. a) Solve $(x^3 - 3xy^2)\,dx + (y^3 - 3x^2 y)\,dy = 0$ 5

b) Find the representative matrix of the linear transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$. 5

c) Show that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$ is a divergent series. 5

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