



Name :
Roll No. :
Invigilator's Signature :

CS/BCA/SEM-2/BM-201/2013

2013

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

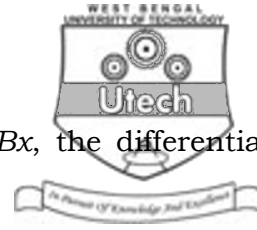
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

- i) A monotonic and bounded sequence is
 - a) convergent
 - b) divergent
 - c) oscillatory
 - d) none of these.

- ii) The sequence $\{r^n\}$ is oscillatory when
 - a) $r > 1$
 - b) $r < 1$
 - c) $-1 < r < 1$
 - d) none of these.



iii) Eliminating A and B from $y = A + Bx$, the differential equation is obtained as

- a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} - y = 0$
 c) $\frac{d^2y}{dx^2} = 0$ d) none of these.

iv) The order and degree of the equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = a \frac{dy}{dx}$ is

- a) 2, 2 b) 2, 3
 c) 3, 2 d) 3, 3.

v) The P.I. of $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = e^x$ is

- a) $\frac{e^x}{3}$ b) $\frac{e^x}{2}$
 c) $\frac{e^x}{6}$ d) none of these.

vi) The series $\sum_{n=1}^{\infty} n^{\frac{1}{p}}$ is convergent if

- a) $p \geq 1$ b) $p < 1$
 c) $p > 1$ d) $p \leq 1$.



vii) If the series $\sum_{n=1}^{\infty} u_n$ is convergent, then

a) $\lim_{n \rightarrow \infty} u_n = 0$

b) $\lim_{n \rightarrow \infty} u_n > 1$

c) $\lim_{n \rightarrow \infty} u_n < 1$

d) none of these.

viii) The series $1 - 1 + 1 - 1 + \dots$ is

a) convergent with sum 0

b) convergent with sum 1

c) divergent

d) oscillatory.

ix) The vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ in V_3 are

a) linearly dependent b) linearly independent

c) both (a) and (b) d) none of these.

x) The basis of a vector space contains

a) linearly independent vectors

b) linearly dependent vectors

c) scalars only

d) none of these.



4. Test the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots, \quad x > 0$$

5. Define monotone sequence. When is a monotone sequence convergent? Is the following sequence convergent?

$$\left\{ \frac{3n+1}{n+2} \right\}$$

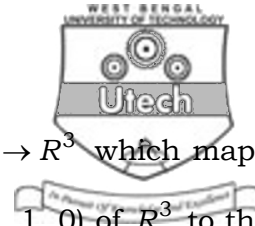
6. Prove that the intersection of two subspaces of a vector space is a subspace.
7. Find the space generated by $(1, 3, 0)$, $(2, 1, -2)$. Examine whether $(4, 7, -2)$ lies in this space.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Find the basis and dimension of the subspace W of R^3 where $W = \{(x, y, z) \in R^3 : x + y + z = 0\}$. 5
- b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}$. 5
- c) Solve $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = x^2 e^{3x}$. 5



9. a) Determine the linear mapping $T : R^3 \rightarrow R^3$ which maps the basis vectors $(0, 1, 1), (1, 0, 1), (1, 1, 0)$ of R^3 to the vectors $(1, 2, 1), (1, 1, 2), (2, 1, 1)$ respectively. Find $\text{Ker} (T)$ and $\text{Im} (T)$. 8

b) Solve : $(x^2D^2 - xD - 3)y = x^2 \log x$. 7

10. a) Define basis and dimension of a vector space. Find a basis and the dimension of $S \cap T$ where S and T are subspaces of R^3 defined by

$$S = \{(x, y, z) \in R^3 : 2x + y + 3z = 0\}$$

$$T = \{(x, y, z) \in R^3 : x + 2y + z = 0\} \quad 2 + 1 + 6$$

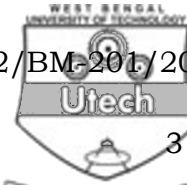
b) Examine whether the vectors $(1, 2, 2), (2, 1, 2), (2, 2, 1)$ are linearly independent in R^3 . 6

11. a) Test the convergence of the following series :

i) $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \dots$

ii) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$ 5 + 5

b) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$ converges conditionally. 5



3 × 5

12. Solve the following :

a) $(xy \sin xy + \cos xy) y dx + (xy \sin xy - \cos xy) x dy = 0$

b) $y = px + \sqrt{a^2 p^2 + b^2}$, $p = \frac{dy}{dx}$

c) $\frac{d^2y}{dx^2} - y = \sin x$

13. a) Solve $(x^3 - 3xy^2) dx + (y^3 - 3x^2y) dy = 0$ 5

b) Find the representative matrix of the linear transformation $T : R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$. 5

c) Show that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$ is a divergent series. 5

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