

**BE - 401****B.E. IV Semester Examination, December 2014****Mathematics - III****(Common for all Branches)****Time : Three Hours****Maximum Marks :70**

- Note:** i) Answer five questions. In each question part A, B, C is compulsory and D part has internal choice.  
 ii) All parts of each question are to be attempted at one place.  
 iii) All questions carry equal marks, out of which part A and B (Max.50 words) carry 2 marks, part C (Max.100 words) carry 3 marks, part D (Max.400 words) carry 7 marks.  
 iv) Except numericals, Derivation, Design and Drawing etc.

1. a) Show that  $w=e^z$  is an analytic function and determine  $f'(z)$ .  
 b) Evaluate  $\int_C \frac{z^2 - z + 1}{z - 1} dz$ , where  $C$  is the circle  $|z|=1$ .  
 c) Evaluate  $\int_C \frac{e^{2z}}{(z+1)^4} dz$ , where  $C$  is the circle  $|z|=2$ .  
 d) If  $f(z)$  is a regular function of  $z$  prove that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$ .

OR

If  $f(\xi) = \int_C \frac{3z^2 + 7z + 1}{z - \xi} dz$ , where  $C$  is the circle  $x^2 + y^2 = 4$ , find the values of  $f(3)$ ,  $f'(1-i)$  and  $f''(1-i)$ .

2. a) Define algebraic and transcendental equations.  
 b) Find the smallest positive root of the equation  $x^3 - 2x + 0.5 = 0$  by Newton-Raphson method.  
 c) The equation  $x^6 - x^4 - x^3 - 1 = 0$ , has one real root between 1.4 and 1.5. Find the route to four decimal places by the method of False-Position.  
 d) Solve the following system of equations  $8x - y + z = 18$ ;  $2x + 5y - 2z = 3$ ;  $x + 2y - 3z = -6$  using Gauss-Seidel iterative method.

OR

Apply Gauss-Jordan method to find the solution of the following system of equations:

$$10x + y + z = 12; 2x + 10y + z = 13, x + y + 5z = 7. \quad \text{http://www.rgpvonline.com}$$

3. a) Define interpolation and write the Newton's forward and Backward interpolation formula.  
 b) Find the cubic polynomial which takes the following values:

$$\begin{array}{l} x : \quad 0 \quad 1 \quad 2 \quad 3 \\ f(x) : \quad 1 \quad 2 \quad 1 \quad 10 \end{array}$$

c) Evaluate  $\int_0^6 \frac{dx}{1+x^2}$  by using Weddle's rule.

d) The following table gives the normal weights of babies during the first 12 months of life.

Age in months: 0 2 5 8 10 12

Weights in lbs:  $7\frac{1}{2}$   $10\frac{1}{4}$  15 16 18 21

Estimate the weight of the baby at the age of 7 months.

OR

Find  $f'(x)$  and  $f''(x)$  at  $x = 6$ , given that

$x$  : 4.5 5.0 5.5 6.0 6.5 7.0 7.5

$y = f(x)$  : 9.69 12.90 16.71 21.18 26.37 32.34 39.15

4. a) Use Picards method to approximate  $y$  when  $x = 0.1$  given that  $y = 1$ , when  $x = 0$  and  $\frac{dy}{dx} = \frac{y-x}{y+x}$ .

b) Write the steps of Runge-Kutta method.

c) From the following data, obtain the value of the correlation coefficient:  $n = 10$ ,  $\Sigma x = 140$ ,  $\Sigma y = 150$ ,  $\Sigma (x-10)^2 = 180$ ,  $\Sigma (y-15)^2 = 215$  and  $\Sigma (x-10)(y-15) = 60$ .

d) By the method of least squares, find the curve  $y = ax+bx^2$ , that best fits the following data:

$x$  : 1 2 3 4 5

$y$  : 1.8 5.1 8.9 14.1 19.8

OR

Given  $\frac{dy}{dx} = 1 + \frac{y}{x}$ ,  $y = 2$  at  $x = 1$ . Find approximate value of  $y$  at  $x = 1.4$  by taking step size

$h = 0.2$ , apply modified Euler's method. <http://www.rgpvonline.com>

5. a) Find the mean of the binomial distribution.

b) Given  $A$  and  $B$  two events with  $P(A \cup B) = \frac{7}{8}$ ,  $P(A \cap B) = \frac{1}{4}$  and  $P(A \cap \bar{B}) = \frac{5}{8}$ . Find  $P(A)$  and  $P(B)$ .

c) If the probability of a bad reaction from certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction.

d) Fit a Poisson distribution for the following data and test the goodness of fit, given that  $\chi^2 = 0.05 = 7.815$  for 3 d.f.

$x$  : 0 1 2 3 4

$f$  : 122 60 15 2 1

OR

The life time of a certain kind of battery is a random variable, which as an exponential distribution with a mean of 200 hrs. Find the probability that such a battery will last. (i) At most 100 hrs. and (ii) last any-where from 400 to 600 hrs.

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