CE6303 - MECHANICS OF FLUIDS

(FOR III – SEMESTER)

UNIT – I

PREPARED BY

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TWO MARK QUESTIONS AND ANSWERS

1. Define fluid mechanics.

It is the branch of science, which deals with the behavior of the fluids (liquids or gases) at rest as well as in motion.

2. Define Mass Density.

Mass Density or Density is defined as ratio of mass of the fluid to its volume (V)

Density of water = 1 gm/cm^3 or 1000 kg / m^3.

$$\boxed{p = \frac{Mass\ of\ fluid}{Volume of\ fluid}}$$

3. Define Specific Weight.

It is the ratio between weight of a fluid to its volume.

$$w = \frac{Weight\ of\ fluid}{Volume\ of\ fluid} = \left(\frac{Mass\ of\ fluid}{Volume\ of\ fluid}\right) \times g = p \times g$$

$$\boxed{w = p \times g}$$

Unit: N / m^3

4. Define Viscosity.

Viscosity is defined as the property of fluid, which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

When two layers move one over the other at different velocities, say u and u+ du, the viscosity together with relative velocity causes a shear stress acting between the fluid layers. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity.

$$\boxed{\tau = \mu \frac{du}{dy}}$$

$\mu \Rightarrow$ Coefficient of dynamic viscosity (or) only viscosity
du / dy = rate of shear strain

5. Define Specific Volume.

Volume per unit mass of a fluid is called specific volume

$$Sp.\,volume = \left(\frac{Volume\ of\ a\ fluid}{Mass\ of\ fluid}\right) = \frac{1}{p} = \frac{1}{\left(\frac{mass\ of\ fluid}{volume}\right)}$$

Unit: m^3 / kg.

6. Define Specific Gravity.

Specific gravity is the ratio of the weight density or density of a fluid to the weight density or density of standard fluid. It is also called as relative density.

Unit : Dimension less. Denoted as: ‘S’

$$S(\,for liquid) = \frac{Weight\ density\ of\ liquid}{Weight\ density\ of\ water}$$

$$S\left(for\ gases\right) = \frac{Weight\ density\ of\ gas}{Weight\ density\ of\ air}$$

7. Calculate the specific weight, density and specific gravity of 1 litre of liquid which weighs 7 N.

Solution:

Given $V = 1 ltre = \frac{1}{1000}\, m^3$

$W = 7$ N

i. Sp. Weight (w) $= \frac{weight}{volume} = \frac{7N}{\left(\frac{1}{1000}\right)m^3} = 7000\ N/m^3$

ii Density (p) $= \frac{w}{g} = \frac{7000 \;\; N}{9.81 \, m^3} \, kg / m^3 = 713..5 \, Kg / m^3$

iii. Sp. Gravity (S) $= \frac{Density \; of \;\; liquid}{Density \; of \;\; water} = \frac{713.5}{1000}$ (Density of water = 1000 kg / m$^{3)}$

S = 0.7135

8. State Newton's Law of Viscosity.

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity

$$\boxed{\tau = \mu \frac{du}{dy}}$$

9. Name the Types of fluids.

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non-Newtonian fluid.
5. Ideal plastic fluid

10. Define Kinematic Viscosity.

It is defined as the ratio between the dynamic viscosity and density of fluid.

Represented as γ ; $\upsilon = \frac{Vis\cos ity}{Density} = \frac{\mu}{p}$

Unit: m^2 / sec.

$$1 \, Stoke = \frac{Cm^2}{S} = \left(\frac{1}{100}\right)^2 \frac{m^2}{S} = 10^{-4} \, m^2 \;/ \, s.$$

Centistoke means $= \frac{1}{100}\, stoke$

11. Find the Kinematic viscosity of an oil having density 981 kg/m. The shear stress at a point in oil is 0.2452 N/m^2 and velocity gradient at that point is 0.2 /sec.

Mass density p = 981 kg/m^3, Shear stress $\tau = 0.2452\, N / m^2$

Velocity gradient $\frac{du}{dy} = 0.2$

$$\tau = \mu \frac{du}{dy}$$

$$0.2452 = \mu \times 0.2 \quad \Rightarrow \mu = \frac{0.2452}{0.2} = 1.226 Ns / m^2.$$

$$kinematic\, vis\cos ity(\upsilon) = \frac{\mu}{p} = \frac{1.226}{981}$$

$$= 0.125 \times 10^{-2} m^2 / s.$$

$$= 0.125 \times 10^{-2} \times 10^4 cm^2 / S$$

$$= 12.5 stoke.$$

12. Determine the specific gravity of a fluid having viscosity 0.05 poise and Kinematic viscosity 0.035 stokes.

Given: Viscosity, μ = 0.05 poise = (0.05 / 10) Ns / m^2.

Kinematic viscosity ν = 0.035 stokes = 0.035 cm^2 / s

= 0.035 x 10^{-4} m^2 / s

$$\boxed{\upsilon = \frac{\mu}{p}}$$

$$0.035 \times 10^{-4} = \frac{0.05}{10} \times \frac{1}{p} \Rightarrow p = 1428.5 kg / m^3$$

Specific gravity of liquid = $\frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1428.5}{1000}$ = 1.428 = 1.43

13. Define Compressibility.

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder filled with a piston as shown

V $\rightarrow$ Volume of gas enclosed in the cylinder

P $\rightarrow$ Pressure of gas when volume is $\forall$

Increase in pressure = dp kgf / m^2

Decrease of volume = $d\forall$

$\therefore$ Volumetric strain $= \frac{-d\forall}{\forall}$

- Ve sign $\rightarrow$ Volume decreases with increase in pressure

$\therefore$ Bulk modulus $K = \frac{Increase\ of\ \Pr essure}{Volumetric\ strain} = \frac{d_p}{\frac{-d\forall}{\forall}} = \boxed{-\frac{d_p}{d\forall}\forall}$

$$\boxed{Compressibility = \frac{1}{K}}$$

14. Define Surface Tension.

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrance under tension.

Unit: N / m.

15. Define Capillarity:

Capillary is defined as a phenomenon of rise of a liquid surface is a small tube relative to adjacent general level of liquid when the tube is held vertically in the liquid. The resistance of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid.

16. The Capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension of water in contact with air = 0.0725 N/m

Solution:

Capillary rise, h = 0.2 mm = 0.2 x 10^3 m

Surface tension $\sigma = 0.0725\ N/m$

Let, Diameter of tube = d

Angel θ for water = 0

Density for water = 1000 kg / m^2

$$h = \frac{4\sigma}{p \times g \times d} \Rightarrow 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$d = \frac{4 \times 0.0725}{1000 \times 9.81 \times 0.2 \times 10^{-3}} = 0.148m = 14.8cm$$

Minimum ϕ of the tube = 14.8 cm.

17. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution:

Capillary rise h = 2.0 mm = $2.0 \times 10^{-3} m$

Let, diameter = d

Density of water = 1000 kg / m^3

$\sigma = 0.073575\ N/m$

Angle for water $\theta = 0$

$$h = \frac{4\sigma}{p \times g \times d} \quad \Rightarrow 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

d = 0.015 m = 1.5 cm.

Thus the minimum diameter of the tube should be 1.5 cm.

18. Define Real fluid and Ideal fluid.

Real Fluid:

A fluid, which possesses viscosity, is known as real fluid. All fluids, in actual practice, are real fluids.

Ideal Fluid:

A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

19. Write down the expression for capillary fall.

Height of depression in tube $h = \dfrac{4\sigma Cos\theta}{p \times g \times d}$

Where,

h = height of depression in tube.

d = diameter of the

σ = surface tension

ρ = density of the liquid.

θ = Angle of contact between liquid and gas.

20. Two horizontal plates are placed 1.25 cm apart. The space between them being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.

Solution:

Given:

Distance between the plates, dy = 1.25 cm = 0.0125m.

Viscosity μ = 14 poise = 14 / 10 Ns / m^2

Velocity of upper plate, u = 2.5 m/Sec.

Shear stress is given by equation as $\tau = \mu$ (du / dy).

Where du = change of velocity between the plates = u – 0 = u = 2.5 m/sec.

dy = 0.0125m.

τ = (14 /10) X (2.5 / 0.0125) = 280 N/m^2.

16 MARKS QUESTIONS AND ANSWERS

1. Calculate the capillary effect in millimeters a glass tube of 4mm diameter, when immersed in (a) water (b) mercury. The temperature of the liquid is 20^0 C and the values of the surface tension of water and mercury at 20^0 C in contact with air are 0.073575 and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 130^0. Take specific weight of water as 9790 N / m^3

Given:

Diameter of tube $\Rightarrow$ d = 4 mm = $4 \times 10^{-3} m$

Capillary effect (rise or depression) $\Rightarrow$ $h = \dfrac{4\sigma \cos\theta}{p \times g \times d}$

$\sigma =$ Surface tension in kg f/m

$\theta =$ Angle of contact and p = density

i. **Capillary effect for water**

$$\sigma = 0.073575 \ N/m, \quad \theta = 0^0$$

$$p = 998 \ kg/m^3 \ @ \ 20^0 c$$

$$h = \frac{4 \times 0.73575 \times Cos0^0}{998 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3} m$$

= 7.51 mm.

Capillary effect for mercury:

$$\sigma = 0.51 N/m, \qquad \theta = 130^0$$

$$p = sp \ gr \times 1000 = 13.6 \times 1000 = 13600 \ kg/m^3$$

$$h = \frac{4 \times 0.51 \times Cos130^0}{13600 \times 9.81 \times 4 \times 10^{-3}}$$

$$= -2.46 \times 10^{-3} \text{ m}$$

= - 2.46 mm.

-Ve indicates capillary depression.

2. A cylinder of 0.6 m^3 in volume contains air at 50^{0}C and 0.3 N/ mm^2 absolute pressure. The air is compressed to 0.3 m^3. Find (i) pressure inside the cylinder assuming isothermal process (ii) pressure and temperature assuming adiabatic process. Take K = 1.4

Given:

Initial volume $\forall_1 = 0.36 \, m^3$

Pressure P_1 = 0.3 N/mm^2

$$= 0.3 \times 10^6 \ N/m^2$$

Temperature, t_1 = 50^0 C

T_1 = 273 + 50 = 323 ^{0} K

Final volume, $\forall_2 = 0.3m^3$

K = 1.4

i. Isothermal Process:

$$\frac{P}{p} = Constant \quad (or)\ p\forall = Constant$$

$$p_1\forall_1 = p_2\forall_2$$

$$p_2 = \frac{p_1\forall_1}{\forall_2} = \frac{30\times10^4\times0.6}{0.3} = 0.6\times10^6\ N/m^2$$

= 0.6 N / mm^2

ii. Adiabatic Process:

$$\frac{p}{p^K} = Constant \quad or \qquad \boxed{p\forall^K = constant}$$

$$p_1.\forall_1{}^K = p_2\forall_2{}^K$$

$$p_2 = p_1\frac{\forall_1 K}{\forall_2 K} = 30\times10^4\times\left(\frac{0.6}{0.3}\right)^{1.4} = 30\times10^4\times2^{1.4}$$

$$= 0.791\times10^6\ N/m^2 = 0.791N/mm^2$$

For temperature, $p\forall = RT$**,** $p\forall^k = constant$

$$p = \frac{RT}{\forall} \quad \text{and} \quad \frac{RT}{\forall}\times\forall^k = constant$$

$$RT\forall^{k-1} = Constant$$

$$T\forall^{k-1} = Constant \qquad (\because R\ is\ also\ constant)$$

$$T_1 V_1^{k-1} = T_2 V_2^{k-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{k-1} = 323 \quad \left(\frac{0.6}{0.3} \right)^{1.4-1.0}$$

$$= 323 \times 2^{0.4} = 426.2^0 K$$

$$t_2 = 426.2 - 273 = 153.2^0 C$$

3. **If the velocity profile of a fluid over a plate is a parabolic with the vertex 202 cm from the plate, where the velocity is 120 cm/sec. Calculate the velocity gradients and shear stress at a distance of 0,10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.**

Given,

Distance of vertex from plate = 20 cm.

Velocity at vertex, u = 120 cm / sec.

Viscosity, $\mu = 8.5 poise = \frac{8.5}{10} \frac{Ns}{m^2} = 0.85$

Parabolic velocity profile equation, $u = ay^2 + by + C$ -------------- (1)

Where, a, b and c constants. Their values are determined from boundary conditions.

i) At y = 0, u = 0

ii) At y = 20cm, u = 120 cm/se.

iii) At y = 20 cm, $\frac{du}{dy} = 0$

Substituting (i) in equation (1), C = 0

Substituting (ii) in equation (1), $120 = a(20)^2 + b(2) = 400a + 20b$ -----------(2)

Substituting (iii) in equation (1), $\frac{du}{dy} = 2ay + b$

$$0 = 2 \times a \times 20 + b = 40a + b \text{ -----------(3)}$$

solving 1 and 2, we get,

$$400\,a + 20\,b = 0$$

$$(-)$$

$$800\,a + 20\,b = 0$$

40 a + b = 0

b = - 40 a

$$120 = 400\,a + 20\,b(-40\,a) = 400\,a - 800\,a = -400\,a$$

$$a = \frac{120}{-400} = -\frac{3}{10} = -0.3$$

$$b = -40 \times (-0.3) = 1.2$$

Substituting a, b and c in equation (i) $u = -0.3y^2 + 12y$

$$\frac{du}{dy} = -0.3 \times 2y + 12 = -0.6y + 12$$

Velocity gradient

at y = 0, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=0} = -0.6 \times 0 + 12 = 12/s.$

at y =10 cm, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=10} = -0.6 \times 10 + 12 = -6 + 12 = 6/s.$

at y = 20 cm, Velocity gradient, $\left(\frac{du}{dy}\right)_{y=20} = -0.6 \times 20 + 12 = -12 + 12 = 0$

Shear Stresses:

Shear stresses is given by, $\tau = \mu \frac{du}{dy}$

i. Shear stress at y = 0 , $\tau = \mu \left(\frac{du}{dy} \right)_{y=0} = 0.85 \times 12.0 = 10.2 N/m^2$

ii. Shear stress at y = 10 , $\tau = \mu \left(\frac{du}{dy} \right)_{y=10} = 0.85 \times 6.0 = 5.1 N/m^2$

iii. Shear stress at y = 20 , $\tau = \mu \left(\frac{du}{dy} \right)_{y=20} = 0.85 \times 0 = 0$

4. A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 rpm determine the viscosity of the fluid.

Solution:

Diameter of cylinder = 15 cm = 0.15 m

Diameter of outer cylinder = 15.10 cm = 0.151 m

Length of cylinder ⇒ L = 25 cm = 0.25 m

Torque T= 12 Nm ; N = 100 rpm.

Viscosity = μ

Tangential velocity of cylinder $u = \frac{\pi DN}{60} = \frac{\pi \times 0.15 \times 100}{60} = 0.7854 \quad m/s$

Surface area of cylinder $A = \pi D \times L = \pi \times 0.15 \times 0.25$

= 0.1178 m^2

$\tau = \mu \frac{du}{dy}$ $\qquad du = u - 0 = u = 0.7854\ m/s$

$$dy = \frac{0.151 - 0.150}{2} = 0.0005\ m$$

$$\tau = \frac{\mu \times 0.7854}{0.0005}$$

Shear force, $F = Shear\ Stress \times Area = \frac{\mu \times 0.7854}{0.0005} \times 0.1178$

Torque $T = F \times \frac{D}{2}$

$$12.0 = \frac{\mu \times 0.7854}{0.0005} \times 0.1178 \times \frac{0.15}{2}$$

$$\mu = \frac{12.0 \times 0.0005 \times 2}{0.7854 \times 0.1178 \times 0.15} = 0.864 Ns/m^2$$

$$\mu = 0.864 \times 10 = 8.64\ poise.$$

5. The dynamic viscosity of oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90 mm. The thickness of the oil film is 1.5 mm.

Given, $\mu = 6 poise = \frac{6}{10}\frac{Ns}{m^2} = 0.6\frac{Ns}{m^2}$

D = 0.4 m $L = 90mm = 90 \times 10^{-3} m$

N = 190 rpm. $t = 1.5mm = 1.5 \times 10^{-3} m$

$$Power = \frac{2\pi NT}{60} W$$

$$T = force \times \frac{D}{2} Nm.$$

$$F = Shear\ stress \times Area = \tau \times \pi DL$$

$$\tau = \mu \frac{du}{dy} N/m^2$$

$u = \frac{\pi\ DN}{60}\ m/s.$

Tangential Velocity of shaft, $u = \frac{\pi\ DN}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98\ m/s.$

du = change of velocity = u – 0 = u = 3.98 m/s.

$dy = t = 1.5 \times 10^{-3}\ m.$

$$\tau = \mu \frac{du}{dy} \Rightarrow \quad \tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592\ N/m^2$$

Shear force on the shaft F = Shear stress x Area

$$F = 1592 \times \pi D \times L = 1592 \times \pi \times 0.4 \times 90 \times 10^{-3} = 180.05 N$$

Torque on the shaft, $T = Force \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01 \quad Ns.$

$$\text{Power lost} = \frac{2\pi \text{NT}}{60} = \frac{2\pi \times 190 \times 36.01}{60} = 716.48 W$$

6.If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ **in which U is the velocity in m/s at a distance y meter above the plate, determine the shear stress at y = 0 and y = 0.15 m. Take dynamic viscosity of fluid as 8.63 poise.**

Given:

$$u = \frac{2}{3}y - y^2$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3}$$

$$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2\times(0.17) = 0.667 - 0.30$$

$\mu = 8.63\,poise = \frac{8.63}{10}$ SI units $= 0.863$ Ns / m^2

$$\boxed{\tau = \mu \frac{du}{dy}}$$

i. Shear stress at y = 0 is given by

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.863\times0.667 = 0.5756\ N/m^2$$

ii. Shear stress at y = 0.15 m is given by

$$(\tau)_{y=0.15} = \mu\left(\frac{du}{dy}\right)_{y=0.15} = 0.863\times0.367 = 0.3167\ N/m^2$$

7.The diameters of a small piston and a large piston of a hydraulic jack at3cm and 10 cm respectively. A force of 80 N is applied on the small piston Find the load lifted by the large piston when:

a. The pistons are at the same level
b. Small piston in 40 cm above the large piston.

The density of the liquid in the jack in given as 1000 kg/m^3

Given: Dia of small piston d = 3 cm.

$\therefore$ Area of small piston , $a = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times(3)^2 = 7.068cm^2$

Dia of large piston, D = 10 cm

$\therefore$ Area of larger piston, $A = \frac{P}{4} \times (10)^2 = 78.54 cm^2$

Force on small piston, F = 80 N

Let the load lifted = W

a. When the pistons are at the same level

Pressure intensity on small piston

$$P = \frac{F}{a} = \frac{80}{7.068} N / cm^2$$

This is transmitted equally on the large piston.

$\therefore$ Pressure intensity on the large piston $= \frac{80}{7.068}$

$\therefore$ Force on the large piston = Pressure x area

$$= = \frac{80}{7.068} \text{ x } 78.54 \text{ N} = 888.96 \text{ N.}$$

b. when the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} N / cm^2$$

$\therefore$ Pressure intensity of section A – A

$= \frac{F}{a} +$ pressure intensity due of height of 40 cm of liquid. P = pgh.

But pressure intensity due to 40cm. of liquid

$$= p \times g \times h = 1000 \times 9.81 \times 0.4 N / m^2$$

$$= \frac{1000 \times 9.81 \times 0.4}{10^4} N/cm^2 = 0.3924 N/cm^2$$

$\therefore$ Pressure intensity at section

$$A - A = \frac{80}{7.068} + 0.3924$$

$= 11.32 + 0.3924 = 11.71$ N/cm^2

Pressure intensity transmitted to the large piston = 11.71 N/cm^2

Force on the large piston = Pressure x Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54$$

= 919. 7 N.

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(FOR III – SEMESTER)

UNIT – II

FLUID STATICS & KINEMATICS

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UNIT – II

FLUID STATICS & KINEMATICS

Pascal's Law and Hydrostatic equation – Forces on plane and curved surfaces – Buoyancy – Meta centre – Pressure measurement – Fluid mass under relative equilibrium

Fluid Kinematics

Stream, streak and path lines – Classification of flows – Continuity equation (one, two and three dimensional forms) – Stream and potential functions – flow nets – Velocity measurement (Pilot tube, current meter, Hot wire and hot film anemometer, float technique, Laser Doppler velocimetry)

	pipe B for the manometer readings.	
23	A differential manometer is connected at the two points A and B . At B pr is 9.81 N/cm^2 (abs), find the absolute pr at A	11
24	A hydraulic pressure has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic pressure when the force applied at the plunger is 500 N.	11
25	The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe section 1 is 5 m/s. determine also the velocity at section 2	12

S.NO	16 MARKS	PAGE NO
1	A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs U. U tube is 10 cm and the free surface of mercury is in level with over the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m^2, Calculate the new difference in the level of mercury. Sketch the arrangement in both cases	13
2	A differential manometer is connected at the two points A and B of two pipes a shown in figure. The pipe A contains a liquid of sp. Gr = 1.5 while pipe b contains a liquid of sp . gr = 0.9 . The pressures at A and B are 1 kgf / cm^2 respectively. Find the difference in mercury level in the differential manometer	15
3	A vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2m high. On the upstream of the gate, the liquid of sp. Gr 1.45, lies upto a height of 1.5m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate and posiotn of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume the gate is hinged at the bottom	16
4	Find the density of a metallic body which floats at the interface of mercury of sp. Gr 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water	19
5	A solid cylinder of diameter 4.0 m has a height of 3m. Find the meta – centric height of the cylinder when it is floating in water with its axis vertical. The sp gr of the cylinder – 0.6	20
6	A wooden cylinder of sp. Gr = 0.6 and circular in cross – section is required to float in oil (sp.gr = 0.90) . Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameters	21
7	Water flows through a pipe AB 1.2m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter at C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE	2 3
8.	In a two – two dimensional incompressible flow, the fluid velocity components are given by u = x – 4y and v= - y – 4x. show that velocity potential exists and determine its form. Find	25

	also the stream function..	

2 MARKS QUESTIONS AND ANSWERS

1. Define "Pascal's Law":

It stats that the pressure or intensity of pressure at a point in a static fluid is equal in all directions.

2. What is mean by Absolute pressure and Gauge pressure?

Absolute Pressure:

It is defined as the pressure which is measured with the reference to absolute vacuum pressure.

Gauge Pressure:

It is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. Define Manometers.

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing measuring the column of fluid by the same or another column of fluid.

1. Simple M
2. Differential M

4. A differential manometer is connected at the two points A and B . At B pr is 9.81 N/cm^2 (abs), find the absolute pr at A.

Pr above X – X in right limb = $1000 \times 9.81 \times 0.6 + p_B$

Pr above X – X in left limb =
$13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A$

Equating the two pr head

Absolute pr at P_A = 8.887 N/cm^2

5. Define Buoyancy.

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

6. Define META – CENTRE

It is defined as the point about which a body starts oscillating when the body is fitted by a small angle. The meta – centre may also be defined as the poit at which the line of action of the force of buoyancy wil meet the normal axis of the body when the body is given a small angular displacement.

7. Write a short notes on " Differential Manometers".

Differential manometers are the devices used for measuring the difference of pressures between two points in a pipe or in two different pipes/ a differential manometer consists of a U – tube containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly types of differential manometers are:

1. U – tube differential manometer.
2. Inverted U – tube differential manometers.

8. Define Centre of pressure.

Is defined as the point of application of the total pressure on the surface.

The submerged surfaces may be:

1. Vertical plane surface
2. Horizontal plane surface
3. Inclined plane surface
4. Curved surface

9. Write down the types of fluid flow.

The fluid flow is classified as :

1. Steady and Unsteady flows.
2. Uniform and Non – uniform flows.
3. Laminar and turbulent flows.
4. Compressible and incompressible flows.
5. Rotational and irrotational flows
6. One, two and three dimensional flows.

10. Write a short notes on "Laminar flow".

Laminar flow is defined as that type of flow in which the fluid particles move along well – defined paths or stream line and all the stream lines are straight and parallel.

Thus the particles move in laminas or layers gliding over the adjacent layer. This type of flow is also called stream – line flow or viscous flow/

11. Define " Turbulent flow".

Turbulent flow is that type of flow in which the fluid particles move in a zig –zag way. Due to the movement of fluid particles in a zig – zag way.

12. What is mean by Rate flow or Discharge?

It is defined as the quantity of a fluid flowing per second through a section of a pipe or channel. For an incompressible fluid(or liquid) the rate of flow or discharge is expressed as volume of fluid flowing across the section per section. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

The discharge (Q) = A X V

Where, A = Cross – sectional area of pipe.

V = Average velocity of fluid across the section.

13. What do you understand by Continuity Equation?

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.

$$A_1V_1 = A_2V_{2..}$$

14. What is mean by Local acceleration?

Local acceleration is defined as the rate of increase of velocity with respect to time at a given point in a flow field. In equation is given by the expression $(\partial u / \partial t)$, $(\partial v / \partial t)$ or $(\partial w / \partial t)$ is known as local acceleration.

15. What is mean by Convective acceleration?

It is defined as the rate of change of velocity due to the change of position of fluid particles in a fluid flow. The expressions other than $(\partial u / \partial t)$, $(\partial v / \partial t)$ and $(\partial w / \partial t)$ in the equation are known as convective acceleration.

16. Define Velocity potential function.

It is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is defined by Φ (Phi). Mathematically, the velocity, potential is defined as $\Phi = f(x,y,z)$ for steady flow such that.

$u = -(\partial \Phi/\partial x)$

$v = -(\partial \Phi/\partial y)$

$w = -(\partial \Phi/\partial z)$ where, u,v and w are the components of velocity in x.y and z directions respectively.

17. Define Stream function.

It is defined as the scalar function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. It is denoted by ψ (Psi) and only for two dimensional flow. Mathematically. For steady flow is defined as ψ = f(x,y) such that ,

$$(\partial \psi / \partial x) = v$$
$$(\partial \psi / \partial y) = - u.$$

18. What is mean by Flow net?

A grid obtained by drawing a series of equipotential lines and stream lines is called a flow net. The flow net is an important tool in analyzing the two – dimensional irrotational flow problems.

19. Write the properties of stream function.

The properties of stream function (ψ) are:

1. If stream function (ψ) exists, it is possible case of fluid flow which may be rotational or irrotational.
2. If stream function (ψ) satisfies the Laplace equation, it is a possible case of irrotational flow.

20. What are the types of Motion?

1. Linear Translation or Pure Translation.
2. Linear Deformation.
3. Angular deformation.
4. Rotation.

21. Define "Vortex flow".

Vortex flow is defined as the flow of a fluid along a curved path or the flow of a rotating mass of a fluid is known ' Vortex Flow'. The vortex flow is of two types namely:

1. Forced vortex flow, and
2. Free vortex flow.

22. Water is flowing through two different pipes, to which an inverted differential manometer having an oil of sp. Gr 0.8 is connected the pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.

Pr heat at $A = \frac{p_A}{pg} = 2m$ of water.

$$p_A = p \times g \times 2 = 1000 \times 9.81 \times 2$$

= 19620 N/ m^2

Pr below X – X in left limb = $P_A - p_1 gh_1$ = 19620 - 1000 x 781 x 0.3 = 16677 N/m^2

P_r below X – X in right limb $p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 = P_B - 1922.76$

Equating two pressure, we get,

$$P_B = 16677 + 1922.76 = 18599.76 \quad N/m^2 = 1.8599 \quad N/cm^2$$

23. A differential manometer is connected at the two points A and B . At B pr is 9.81 N/cm^2 (abs), find the absolute pr at A.

Pr above X – X in right limb = $1000 \times 9.81 \times 0.6 + p_B$

Pr above X – X in left limb = $13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + P_A$

Equating the two pr head

Absolute pr at P_A = 8.887 N/cm^2.

24. A hydraulic pressure has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight lifted by the hydraulic pressure when the force applied at the plunger is 500 N.

Given: Dia of ram, D = 30 cm = 0.3 m

Dia of plunger, d = 4.5 cm = 0.045 m

Force on plunger, F = 500 N

To find :

Weight lifted = W

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$

Area of plunger, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = 0.000159 m^2$

Pressure intensity due to plunger $= \dfrac{Force\ on\ plunger}{Area\ of\ plunger} = \dfrac{N}{m^2}$

$$= \frac{F}{a} = \frac{500}{0.00159}\ N/m^2$$

Due to Pascal law, the intensity of pressure will be equally transmitted in all distance . Hence the pressure intensity at

$$ram = \frac{500}{0.00159} = 314465.4 \quad N/m^2$$

Pressure intensity at ram $= \dfrac{weight}{Area\ of\ ram} = \dfrac{W}{A} = \dfrac{W}{0.07068}$

$$\frac{W}{0.07068} = 314465.4$$

Weight $= 314465.4 \times 0.07068 = 22222\,N = 22.222\ KN.$

25. The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe section 1 is 5 m/s. determine also the velocity at section 2.

Solution. Given:

At section 1. D_1 = 10 cm = 0.1 m.

$A_1 = (\pi / 4) X D_1^2 = (\pi / 4) X (0.1)^2 = 0.007854\ m^2$.

V_1 = 5 m/s.

At section 2. D_2 = 15 cm = 0.15 m.

$A_2 = (\pi / 4) X(0.15)^2 = 0.01767\ m^2$.

1. Discharge through pipe is given by equation

$Q = A_1 X V_1$

$= 0.007544 X 5 = 0.03927\ m^3/ s.$

using equation, We have $A_1V_1 = A_2V_2$..

$V_2 = (A_1V_1 / A_1) = (0.007854 /0.01767) X 5 = 2.22$ m/s.

16 MARKS QUESTIONS AND ANSWERS

1. A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs U. U tube is 10 cm and the free surface of mercury is in level with over the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m^2, Calculate the new difference in the level of mercury. Sketch the arrangement in both cases.

Given,

Difference of mercury = 10 cm = 0.1 m.

Let P_A = pr of water in pipe line (ie, at point A)

The point B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C.

But pressure at B = Pressure at A and Pressure due to 10 cm (or) 0.1m of water.

$$= PA + p \times g \times h$$

where , P = 1000kg/m^3 and h = 0.1 m

$$= P_A + 1000 \times 9.81 \times 0.1$$

$$= P_A + 981 N/m^2 \qquad \text{(i)}$$

Pressure at C = Pressure at D + pressure due to 10 cm of mercury

$$0 + P_0 \times g \times h_0$$

where p_o for mercury $= 13.6 \times 1000 kg / m^3$

h_0 = 10 cm = 0.1 m

Pressure at $C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$

= 13341.6 N (ii)

But pressure at B is = to pr @ c. Hence,

equating (i) and (ii)

$$P_A + 981 = 13341.6$$

$$p_A = 13341.6 - 981 = 12360.6 N / m^2$$

II part: Given $p_A = 9810 N / m^2$

In this case the pressure at A is 9810 N/m^2 which is less than the 12360.6 N/m^2. Hence the mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let, x = Ries of mercury in left limb in cm

Then fall of mercury in right limb = x cm.

The points B, C and D show the initial condition.

Whereas points B*, C*, and D* show the final conditions.

The pressure at B * = pressure at C*

Pressure at A + pressure due to (10-x) cm of water.

= pressure at D* + pressure due to (10-2x) cm of mercury.

(or)

$$P_A + p_1 g \times h_1 = pD* + p_2 \times g \times h_2$$

(or)

$$\frac{9810}{9.8} + \frac{1000 \times 9.81}{9.81}\left(\frac{10 - x}{100}\right)$$

$$= 0 + \frac{(13.6 \times 1000) \times 9.81}{9.81} \times \left(\frac{10 - 2x}{100}\right)$$

Dividing by 9.81 , we get,

$$1000 + 100 - 10x = 1360 - 272x$$

$$272x - 10x = 1360 - 1100$$

262 x = 260

$$x = \frac{260}{262} = 0.992cm$$

$\therefore$ New difference of mercury = 10 - 2 x cm

$$= 10 - 2 \times 0.992$$

= 8.016 cm.

2. A differential manometer is connected at the two points A and B of two pipes a shown in figure. The pipe A contains a liquid of sp. Gr = 1.5 while pipe b contains a liquid of sp . gr = 0.9 . The pressures at A and B are 1 kgf / cm^2 respectively. Find the difference in mercury level in the differential manometer.

Given,

Sp. Gr of liquid at A, $S_1 = 1.5$ $\therefore$ $p_1 = 1500$

Sp. Gr of liquid at B, $S_2 = 0.9$ $\therefore$ $p_2 = 900$

Pr at A, P_A = 1kgf / cm^2 = $1 \times 10^4 kgf / m^2$

$$=10^4 \times 9.81 N/m^2 \quad \text{(1 kgf= 9.81 N)}$$

Pr at B, P_B = 1.8 kgf / cm^2

$$=1.8 \times 10^4 \times 9.81 \quad N/m^2$$

Density of mercury $= 13.6 \times 1000 kg/m^3$

Pr above X – X in left limb = $13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2+3) + P_A$

$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 \times 10^4$$

Pr above X – X in the right limb $= 900 \times 9.81 \times (h+2) + P_B$

$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

Equating two pressure, we get,

$$13.6 \times 1000 \times 9.81 h + 7500 \times 93.81 + 9.81 \times 10^4$$

$$= 900 \times 9.81 \times (h+2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h+2.0) \times 0.9 + 18$$

$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

$(13.6 - 0.9)h = 19.8 - 17.5$ or 12.7 h = 2.3

$$h = \frac{2.3}{12.7} = 0.181 m = 18.1 cm$$

3, **A vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2m high. On the upstream of the gate, the liquid of sp. Gr 1.45, lies upto a height of 1.5m above the top of the gate, whereas on the downstream side the water is available upto a height touching the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate and posiotn of centre of pressure. Find also the force acting horizontally at the**

top of the gate which is capable of opening it. Assume the gate is hinged at the bottom.

Given,

b = 2m $$A = b \times d = 2 \times 1.2 = 2.4m^2$$

d = 1.2 m $$p_1 = 1.45 \times 1000 = 1450 kg/m^3$$

F_1 = force exerted by water on the gate

$$F_1 = p_1 g \quad A\overline{h_1}$$

$$p_1 = 1.45 \times 1000 = 1450 kg/m^2$$

$\bar{h} =$ depth of C.G of gate from free surface of liquid.

$$= 1.5 + \frac{1.2}{2} = 2.1m$$

$$F_1 = 1450 \times 9.81 \times 2.4 \times 2.1$$

= 71691 N

$$F_2 = p_2 g A\overline{h_2}$$

Centre of Pressure (h*):

Centre of pressure is calculated by using the "Principle of Moments" which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

$$F_2 = p_2 g A\overline{h_2}$$

p_2 = 1000 kg/m^3

$\overline{h_2} =$ Depth of C.G of gate from free surface of water.

$$= \frac{1}{2} \times 1.2 = 0.6m$$

$$F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126 \ N$$

i. Resultant force on the gate $= F_1 - F_2 = 71691 - 14126$

$= 57565$ N.

ii. Position of centre of pressure of resultant force:

$$h_1{}^* = \frac{I_G}{A\overline{h_1}} + \overline{h_1}$$

$$I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 m^4$$

$$h_1^* = \frac{0.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571 m$$

$\therefore$ Distance of F_1 from hinge $= (1.5 + 1.2) - h_1^*$

$= 2.7 - 2.1571 = 0.5429$ m.

The force F_2 will be acting at a depth of h_2^* from free surface of water nd is given by

$$h_2^* = \frac{I_G}{A\overline{h_2}} + \overline{h_2}$$

where $I_G = 0.288$ m^4, $\overline{h_2} = 0.6m$, A = 2.4 m^2,

$$h_2^* = \frac{0.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8m$$

Distance of F_2 from hinge = 1.2 - 0.8 = 0.4 m.

The resultant force 57565 N will be acting at a distance given by

$$= \frac{71691 \times 0.5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} m$$ above hinge.

= 0.578 m above the hinge.

iii. Force at the top of gate which is capable of opening the gate:

Taking moment of F, F_1 and F_2 about the hing.

$$F \times 1.2 + F_2 \times 0.4 = F_1 \times 0.5429$$

$$F = \frac{F_1 \times 0.5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times 0.5429 - 14126 \times 0.4}{1.2}$$

= 27725.5 N.

4. Find the density of a metallic body which floats at the interface of mercury of sp. Gr 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.

Solution:

Let the volume of the body = V m^3

Then volume of body sub-merged in mercury

$$= \frac{40}{100} V = 0.4Vm^3$$

Volume of body sub-merged I water

$$= \frac{60}{100} \times V = 0.6Vm^3$$

For the equilibrium of the body

Total buoyant force (upward force) = Weight of the body.

But total buoyant force = Force of buoyancy due to water +

Force of buyance due to mercury.

Force of buoyancy due to water = Weight of water displaced by body.

= Density of water x g x Volume of water displaced.

$= 1000 \times g \times$ volume of body in water.

$= 1000 \times g \times 0.6 \times V$ N

Force of buoyancy due to mercury = Weight of mercury displaced by body.

$= g \times Density\ of\ mercury \times Volume\ of\ mercury\ displaced.$

$= g \times 13.6 \times 1000 \times Volume\ of\ body\ in\ mercury$

$= g \times 13.6 \times 1000 \times 0.4V$ N.

Weight of the body = Density x g x volume of body

$= p \times g \times V$

where p is the densityof the body

For equilibrium,

Total buoyant force = Weigt of the body

$1000 \times g \times 0.6 \times V + 13.6 \times 1000 \times g \times 0.4V = p \times g \times V$

$p = 600 + 13600 \times 0.4 = 600 + 54400$

$= 6040.00 \text{ kg / m}^3$

$\therefore$ Density of the body $= 6040.00 \text{ kg / m}^3$

5. A solid cylinder of diameter 4.0 m has a height of 3m. Find the meta – centric height of the cylinder when it is floating in water with its axis vertical. The sp gr of the cylinder – 0.6 .

Given

Dia of cylinder , D = 4.0 m

Height of cyinder, h = 3.0 m

Sp, gr of cylinder = 0.6

Depth of immersion of cylinder = 0.6 x 3.0

$= 1.8 \ m$

$$AB = \frac{1.8}{2} = 0.9m$$

$$AG = \frac{3}{2} = 1.5m$$

BG = AG - AB

$= 1.5 – 0.9 = 0.6$ m.

Now the meta – centric height GM is given by equation $GM = \frac{1}{V} - BG$

I = Moment of Inertia about Y – Y axis of plan of the body

$$= \frac{\pi}{4} D^4 = \frac{\pi}{64} \times (4.0)^4$$

V = volume of cylinder in water.

$$= \frac{\pi}{4} D^4 \times \text{ Depth of immersion}$$

$$= \frac{\pi}{4} (4)^2 \times 1.8m^3$$

$$GM = \frac{\frac{\pi}{64} \times (4.0)^4}{\frac{\pi}{4} \times (4.0)^4 \times 1.8} - 0.6$$

$$= \frac{1}{16} \times \frac{4.0^2}{1.8} - 0.6 = \frac{1}{18} - 0.6 = 0.55 - 0.6$$

= - 0.05 m.

- Ve sign indicates meta- centre (M) below the centre of gravity (G)

6. **A wooden cylinder of sp. Gr = 0.6 and circular in cross – section is required to float in oil (sp.gr = 0.90) . Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameters.**

Solution:

Dia of cylinder = D

Height of cylinder = L

Sp gr of cylinder, $S_1 = 0.6$

Sp gr of oil, $S_2 = 0.9$

Let, depth of cylinder immersed in oil = h

Buoyancy principle

Weight of cylinder = Weight of oil dispersed.

$$\frac{\pi}{4}D^2 \times L \times 0.6 \times 1000 \times 9.81 = \frac{\pi}{4}D^2 \times h \times 0.9 \times 1000 \times 9.81$$

$$L \times 0.6 = h \times 0.9$$

$$h = \frac{0.6 \times L}{0.9} = \frac{2}{3}L$$

Dts of centre of gravity C from A, $AG = \frac{L}{2}$

The distance of centre of buoyancy B from A, $AB = \frac{h}{2} = \frac{1}{2}\left[\frac{2}{3}L\right]$

$$BG = AG - AB = \frac{L}{2} - \frac{L}{3} = \frac{3L - 2L}{6} = \frac{L}{3}$$

$GM = \frac{1}{V} - BG$ $\qquad I = \frac{\frac{\pi}{4}D^4}{64}$ $\qquad$ V = Volume of cylinder in oil.

$$= \frac{\pi}{4} D^2 h$$

$$\frac{1}{V} = \left(\frac{\frac{\pi}{64} D^4}{\frac{\pi}{4} D^2 h} \right) = \frac{1}{16} \frac{D^2}{h} = \frac{D^2}{16 \times \frac{2}{3} L} = \frac{3D^2}{32L}$$

$$GM = \frac{3D^3}{32L} - \frac{L}{6}$$

For stable equilibrium, GM should be +Ve or

$$G_M \rightarrow 0 \qquad \text{(or)} \qquad \frac{3D^2}{32L} - \frac{L}{6} > 0$$

$$\frac{3D^2}{32L} - \frac{L}{6} \qquad \text{(or)} \qquad \frac{3 \times 6}{32} > \frac{L^2}{D^2}$$

$$\frac{L^2}{D^2} < \frac{18}{32} \qquad \text{(or)} \qquad \frac{9}{16}$$

$$\frac{L}{D} < \sqrt{\frac{9}{16}} = \frac{3}{7} \quad \Rightarrow \frac{L}{D} < \frac{3}{4}$$

Experimental Method of Determination of Metacentric Height

W = Weight of vessel including w_1
G = Centre of gravity of the vessel
B = Centre of buoyancy of the vessel

$$\boxed{GM = \frac{\omega_1 x}{\omega Tan\theta}}$$

7. **Water flows through a pipe AB 1.2m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter at C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.**

Solution. Given:

Diameter of Pipe AB,	$D_{AB} = 1.2$ m.
Velocity of flow through AB	$V_{AB} = 3.0$ m/s.
Dia. of Pipe BC,	$D_{BC} = 1.5$m.
Dia. of Branched pipe CD,	$D_{CD} = 0.8$m.
Velocity of flow in pipe CE,	$V_{CE} = 2.5$ m/s.
Let the rate of flow in pipe	AB = Q m^3/s.
Velocity of flow in pipe	BC = V_{BC} m^3/s.
Velocity of flow in pipe	CD = V_{CD} m^3/s.

Diameter of pipe	CE = D_{CE}
Then flow rate through	CD = Q / 3
And flow rate through	CE = Q – Q/3 = 2Q/3

(i). Now the flow rate through AB = Q = V_{AB} X Area of AB

$$= 3 \times (\pi / 4) \times (D_{AB})^2 = 3 \times (\pi / 4) \times (1.2)^2$$

$$= 3.393 \text{ m}^3/\text{s}.$$

(ii). Applying the continuity equation to pipe AB and pipe BC,

$$V_{AB} \times \text{Area of pipe AB} = V_{BC} \times \text{Area of Pipe BC}$$

$$3 \times (\pi / 4) \times (D_{AB})^2 = V_{BC} \times (\pi / 4) \times (D_{BC})^2$$

$$3 \times (1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = (3 \times 1.2^2)/ 1.5^2 = 1.92 \text{ m/s}.$$

(iii). The flow rate through pipe

$$CD = Q_1 = Q/3 = 3.393 /3 = 1.131 \text{ m}^3/\text{s}.$$

$$Q_1 = V_{CD} \times \text{Area of pipe } C_D \times (\pi / 4) (C_{CD})^2$$

$$1.131 = V_{CD} \times (\pi / 4) \times (0.8)^2$$

$$V_{CD} = 1.131 / 0.5026 = 2.25 \text{ m/s}.$$

(iv). Flow through CE,

$Q_2 = Q – Q_1 = 3.393 – 1.131 = 2.262\ m^3/s$

$Q_2 = V_{CE}$ X Area of pipe CE = V_{CE} X $(\pi / 4) (D_{CE})^2$

$2.263 = 2.5 X (\pi / 4) (D_{CE})^2$

$D_{CE} = \sqrt{(2.263 X4)/ (2.5 X \pi)} = 1.0735$ m

Diameter of pipe CE = 1.0735m.

8. In a two – two dimensional incompressible flow, the fluid velocity components are given by u = x – 4y and v= - y – 4x.
show that velocity potential exists and determine its form. Find also the stream function.

Solution. Given:

u = x – 4y and v = - y – 4x

$(\partial u / \partial x) = 1$ & $(\partial v/ \partial y) = -1.$

$(\partial u / \partial x) +(\partial v/ \partial y) = 0$

hence flow is continuous and velocity potential exists.

Let Φ = Velocity potential.

Let the velocity components in terms of velocity potential is given by

$\partial \Phi / \partial x = - u = - (x – 4y) = - x + 4y$ ---------------- (1)

$\partial \Phi / \partial y = - v = - (-y – 4x) = y + 4x$.---------------- (2)

Integrating equation(i), we get $\Phi = - (x^2 / 2) + 4xy + C$---- (3)

Where C is a constant of Integration, which is independent of ‘x’.

This constant can be a function of ‘y’.

Differentiating the above equation, i.e., equation (3) with respect to ‘y’, we get

$\partial \Phi / \partial y = 0 + 4x + \partial C / \partial y$

But from equation (3), we have $\partial \Phi / \partial y = y + 4x$

Equating the two values of $\partial \Phi / \partial y$, we get

$4x + \partial C / \partial y = y + 4x$ or $\partial C / \partial y = y$

Integrating the above equation, we get

$C = (y^2 / 2) + C_1.$

Where C_1 is a constant of integration, which is independent of ‘x’ and ‘y’.

Taking it equal to zero, we get $C = y^2/ 2.$

Substituting the value of C in equation (3), we get.

$$\Phi = - (x^2/2) + 4xy + y^2/2.$$

Value of stream functions

Let $\partial\psi/\partial x = v = -y - 4x.$ ----------------------- (4).

Let $\partial\psi/\partial y = - u = - (x – 4y) = x + 4y$ ------(5)

Integrating equation (4) w.r.t. 'x' we get

$$\Psi = - yx – (4x^2/2) + k \text{ -----------------(6)}$$

Where k is a constant of integration which is independent of 'x' but can be a function 'y'.

Differentiating equation (6) w.r.to. 'y' we get,

$$\partial\psi/\partial x = - x – 0 + \partial k/\partial y$$

But from equation (5), we have $\partial\psi/\partial y = -x + 4y$

Equating the values of $\partial\psi/\partial y$, we get $- x + \partial k/\partial y$ or $\partial k/\partial y = 4y.$

Integrating the above equation, we get $k = 4y^2 /2 = 2y^2.$

Substituting the value of k in equation (6), we get.

$$\Psi = - yx – 2 x^2 + 2 y^2$$

UNIT – III

FLUID DYNAMICS

Euler and Bernoulli's equations – Application of Bernoulli's equation – Discharge measurement – Laminar flows through pipes and between plates – Hagen Poiseuille equation – Turbulent flow – Darcy-Weisbach formula – Moody diagram – Momentum Principle

Assumptions made in the derivation of Bernoulli's equation:

(i). The fluid is ideal, i.e., Viscosity is zero. (ii). The flow is steady

(iii). Te flow is incompressible. (iv). The flow is irrotational.

Bernoulli's theorem for steady flow of an incompressible fluid.

It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are:

Pressure Energy $= p / \rho g$

Kinetic energy $= v^2 / 2g$

Datum Energy $= z$

The mathematically, Bernoulli's theorem is written as

$$(p/w) + (v^2 / 2g) + z = \text{Constant.}$$

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm^2 (gauge) and with mean velocity of 2.0 m/s. find the total head or total energy per unit weight of the water at cross – section, which is 5 cm above the datum line.

Given:

Diameter of the pipe 5 cm = 0.5 m.

Pressure ρ = 29.43 N/cm^2 = 29.23 N/m^2

velocity, v = 2.0 m/s.

datum head z = 5 m

total head = Pressure head + Velocity head + Datum head

pressure head $= (p/ \rho g) = (29.43X10^4/ (2X9.81)) = 30$ m

kinetic head $= (v^2/ 2g) = (2X2/(2X9.81)) = 0.204$ m

Total head $= (p/ (\rho g)) + (v^2 / 2g) + z$

$= 30 + 0.204 + 5 = 35.204$m

Water is flowing through two different pipes, to which an inverted differential manometer having an oil of sp. Gr 0.8 is connected the pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.

Pr head at $A = \frac{p_A}{pg} = 2m$ of water.

$$p_A = p \times g \times 2 = 1000 \times 9.81 \times 2$$

$= 19620$ N/ m^2

Pr below X – X in left limb $= P_A - p_1 gh_1 = 19620 - 1000 \times 781 \times 0.3 =$ 16677 N/m^2

Pr below X – X in right limb

$$p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12 = P_B - 1922.76$$

Equating two pressures, we get,

$$P_B = 16677 + 1922.76 = 18599.76 \quad N/m^2 = 1.8599 \quad N/cm^2$$

The diameters of a pipe at the sections 1 and 2 are 10 cm and 15 cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe section 1 is 5 m/s. determine also the velocity at section 2.

Solution. Given:

At section 1. $D_1 = 10$ cm $= 0.1$ m.

$A_1 = (\pi / 4) X D_1^2 = (\pi / 4) X (0.1)^2 = 0.007854$ m^2.

$V_1 = 5$ m/s.

At section 2. $D_2 = 15$ cm $= 0.15$ m.

$A_2 = (\pi / 4) X(0.15)^2 = 0.01767$ m^2.

1. Discharge through pipe is given by equation

$Q = A_1 X V_1$

$= 0.007544 X 5 = 0.03927$ m^3/ s.

Using equation, We have $A_1V_1 = A_2V_2$..

$V_2 = (A_1V_1 / A_1) = (0.007854 /0.01767) X 5 = 2.22$ m/s.

A pitot – static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6mm and static pressure head is 5m. Calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.

Given:

Stagnation Pressure head, h_s = 6mm.

Static pressure h_t = 5 mm.

$h = 6 – 5 = 1$ m

Velocity of flow $V = C_v \sqrt{2gh} = 0.98 \sqrt{2 X 9.81 X 1} = 4.34$ m/s.

A sub-marine moves horizontally in a sea and has its axis 15 m below the surface of water. A pitot tube properly placed just in front of the sub-marine and along its axis connected to the two limbs of a U – tube containing mercury. The difference of mercury level is found to be 170 mm. find the speed of the sub-marine knowing that the sp.gr. of mercury is 13.6 and that of sea-water is 1.026 with respect fresh water.

Given :

Diff. of mercury level x = 170 mm = 0.17 m

Sp. gr. Of mercury S_g = 13.6

Sp. gr. Of sea-water S_o = 1.026

$$\therefore \quad h = x\,[Sg/So - 1] = 0.17\,[(3.6/1.026) - 1] = 2.0834 \text{ m}$$

$$V = \sqrt{2gh} = \sqrt{2 X 9.81 X 20.834} = 6.393 \text{ m/s.}$$

= (6.393X60X60 / 1000) km/hr = 23.01 km/hr.

Write the equations of motion.

$$F_{\mathbf{x}} = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x$$

If the force due to compressibility, F_c is negligible, the resulting net force.

$$F_{\mathbf{x}} = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

Where F_g = gravity force

F_p = Pressure force

F_v = force due to viscosity

F_t = force due to turbulence

F_c = force due to compressibility

Venturi meter.

Venturimeter is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three pats (i). A short converging part (ii) Throat and (iii) Diverging part.

Pitot – tube.

Pitot tube is a device used for measuring the velocity of flow at any point in a pipe or channel. It is based on the principle that if the velocity of flow at a point becomes zero.

Free liquid jet

Free of liquid jet is defined as the jet of water coming out from the nozzle in atmosphere. The path traveled by the free jet is parabolic.

Write down the formulae for finding the discharge in Venturimeter.

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} X \sqrt{2gh}$$

Where a_1 = area of the inlet Venturi meter.

a_2 = area at the throat

C_d = co-efficient of venture meter.

h = difference of pressure head in terms of fluid head flowing through venture meter

Dynamics of fluid flow

The study of fluid motion with the forces causing flow is called dynamics of fluid flow. The dynamic behavior of the fluid flow is analyzed by the Newton's second law of motion, which relates the acceleration with the forces.

Formula to find the maximum height attained by the jet

$$S = \frac{U^2 \sin^2 \theta}{2g}$$

Where, S = maximum vertical height attained by the particle.

U = velocity of jet of water.

g = acceleration due to gravity.

θ = angle with horizontal direction.

Euler's Equation of motion.

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a streamline in which flow is taking place in s-direction as shown in fig. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are:

a. Pressure force pdA In the direction of flow.

b. Pressure force $\left(p+\frac{\partial p}{\partial s}ds\right)dA$ opposite to the direction of flow.

c. Weight of element $\rho g dA ds$

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element of 's' must be equal to the mass on the fluid element X acceleration in the direction 's'.

$$pdA-\left(p+\frac{\partial p}{\partial s}\right)dA-\rho g dA ds\cos\theta$$
$$=\rho dA ds X as--------(1)$$

Where a_s is the acceleration in the direction of 's'

Now, $a_s = \frac{dv}{dt}$, where v is a function of 's' and 't'.

$$=\frac{\partial v}{\partial s}\frac{dv}{dt}+\frac{\partial v}{\partial t}=\frac{v\partial v}{\partial s}+\frac{\partial v}{\partial t}\left\{\frac{ds}{dt}=v\right\}$$

if the flow is steady, $\frac{dv}{dt}=0$

$$a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of 'a_s' in equation (1) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s}dsdA - \rho gdAds\cos\theta = \rho dAdsX\frac{v\partial v}{\partial s}$$

$$Dividingby\rho dsdA, -\frac{\partial p}{\rho\partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$$

$$or\ \frac{\partial p}{\rho\partial s} + g\cos\theta + v\frac{v\partial v}{\partial s} = 0$$

But from fig, we have cos $\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v\partial v}{\partial s} = o$$

(or) $\frac{\partial p}{\rho} + gdz + vdv = 0$ -------------------------------(2).

Equation 2 is known as Euler's equation of motion

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 lit/sec. the section 1 is 6m above datum. If the pressure at section 2 is 4m above the datum. If the pressure at section 1 is 39.24 N/cm^2, find the intensity of pressure at section 2.

Given:

At section 1, D_1 = 20 cm = 0.2m

$A_1 = \frac{\Pi}{4}(0.2)^2 = 0.314m^2$.

P_1 = 39.24 N/cm^2 = 39.24 X10^4 N/m^2.

Z_1 = 6.0m

At section 2, D_2 = 0.10m

$A_2 = \frac{\Pi}{4}(0.1)^2 = 0.0785m^2$.

P_2 =

Z_1 = 4.0m

Rate of flow Q = 35 lit/sec = 35/1000 = 0.035m^3/s

$Q = A_1V_1 = A_2V_2$

$V_1 = Q / A_1$ = 0.035 / 0.0314 = 1.114 m/s

$V_2 = Q / A_2$ = 0.035 / 0.0785 = 4.456 m/s.

Applying Bernoulli's Equations at sections at 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2$$

Or $(39.24 \times 10^4 / 1000 \times 9.81) + ((1.114)^2/2 \times 9.81) + 6.0$

$= (p_2/1000 \times 9.81)+((4.456)^2/2 \times 9.81)+4.0$

$40+0.063+6.0 = (p_2/9810)+1.012+4.0$

$46.063 = (p_2/9810) + 5.012$

$(p_2/9810) = 46.063 - 5.012 = 41.051$

$p_2 = (41.051 \times 9810/10^4) = 40.27\ N/cm^2$

In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively. A is 2 m above B. the pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U – tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.

Given:

Sp.gr.. of oil, $S_o = 0.8$

Density, $\rho = 0.8 \times 1000 = 800\ kg/m^3$.

Dia at A, $D_A = 16\ cm = 0.16 m$

Area at A, $A_1 = \frac{\Pi}{4} X (0.16)^2 = 0.0201 m^2$.

Dia. At B $D_B = 8\ cm = 0.08 m$

Area at B, $A_B = = \frac{\Pi}{4} X (0.08)^2 = 0.005026\ m^2$

(i). Difference of pressures, $p_B - p_A = 0.981\ N/cm^2$.

$= 0.981 \times 10^4\ N/m^2. = 9810\ N/m^2$.

Difference of pressure head $(p_B - p_A)/\rho g = (9810 / (800 \times 9.81)) = 1.25$

Applying Bernoulli's theorem at A and B and taking reference line passing through section B, we get

$$\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\rho g}+\frac{V_B^2}{2g}+Z_B$$

$$\frac{p_A}{\rho g} - \frac{p_B}{\rho g} + Z_A - Z_B = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$\frac{p_A - p_B}{\rho g} + 2.0 - 0.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g}$$

$$- 1.25 + 2.0 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g} \qquad \frac{p_B - p_A}{\rho g} = 1.25$$

$$0.75 = \frac{V_B^2}{2g} - \frac{V_A^2}{2g} \text{ --------------------- (i)}$$

Now applying continuity equation at A and B, we get

$$V_A X A_1 = V_B X A_2$$

$$V_B = \frac{V_A X A_1}{A_2} = \frac{V_A X \frac{\pi}{4}(.16)^2}{\frac{\pi}{4}(.08)^2} = 4V_A$$

Substituting the Value of V_B in equation (i), we get

$$0.75 = \frac{16V_A^2}{2g} - \frac{V_A^2}{2g} = \frac{15V_A^2}{2g}$$

$$V_A = \sqrt{\frac{0.75 X 2 X 9.81}{15}} = 0.99 m/s.$$

Rate of flow, $Q = V_A X A_1$

$= 0.99 \text{ X } 0.0201 = 0.01989 \text{ m}^3/\text{s}.$

(ii). Difference of mercury in the U –tube.

Let h = difference of mercury level.

Then $h = x\left(\frac{S_g}{S_o} - 1\right)$

Where $h = \left(\frac{p_A}{\rho g} + Z_A\right) - \left(\frac{p_B}{\rho g} + Z_B\right) = \frac{p_A - p_B}{\rho g} + Z_A - Z_B$

$= -1.25+2.0-0 = 0.75.$

$$\therefore 0.75 = x\left[\frac{13.6}{0.8} - 1\right] = x \text{ X } 16$$

x = (0.75 / 16) = 0.04687 cm.

Expression for loss of head due to friction in pipes or Darcy – Weisbach Equation.

Consider a uniform horizontal pipe, having steady flow as shown figure. Let 1 -1 and 2-2 is two sections of pipe.

Let P_1 = pressure intensity at section 1-1.

Let P_2 = Velocity of flow at section 1-1.

L = length of the pipe between the section 1-1 and 2-2

d = diameter off pipe.

f^1 = Frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction.

And P_2,V_2 = are the values of pressure intensity and velocity at section 2-2.

Applying Bernoulli's equation between sections 1-1 & 2-2

Total head 1-1 = total head at 2-2 + loss of head due to friction between 1-1&2-2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_f$ ------------(1)

but $Z_1 = Z_1$ [pipe is horizontal]

$V_1= V_2$ [diameter of pipe is same at 1-1 & 2-2]

(1) becomes,

$(P_1/ \rho g) = (P_2/\rho g)+h_f$

$h_f = (P_1/ \rho g) - (P_2/\rho g)$

frictional resistance = frictional resistance per unit wetted area per unit velocity X wetted area X velocity ^{2}.

$F = f^1 \text{ x } \pi \text{ d }_1 \text{x } V^2$ [Wetted area = π d x L, and Velocity $V = V_1 = V_2$]

$F_1 = f^1 xPxLxV^2$ ----------- (2). [π d = wetted perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are,

1) Pressure force at section 1-1 = P_1X A

2) Pressure force at section 2-2 = P_2 X A

3). Frictional force F_1

Resolving all forces in the horizontal direction.,

$P_1 A – P_2A – F_1 = 0$

$(P_1-P_2)A = F_1 = f^1xPxLxV^2$

$(P_1-P_2) = (f^1xPxLxV^2 / A)$.

But from (1) we get

$P_1 – P_2 = \rho g\ h_f$

Equating the values of $(P_1 - P_2)$ we get

$\rho g\, h_f = (f^1 x P x L x V^2 / A)$.

$h_f = (f^1 / \rho g) X (P/A) X LX V^2$

$(P/A) = (\pi d / (\pi d^2/4)) = (4/d)$

Hence, $h_f = (f^1 / \rho g) x (4/d) x LxV^2$.

Putting $(f^1 / \rho) = (f / 2)$, where f is the co – efficient of friction

$$h_f = \frac{4fLV^2}{2gd}$$

This equation is known as Darcy – Weisbach equation. This equation is commonly used to find loss of head due to friction in pipes.

Expression for rate of flow through Venturimeter.

Venturi meter is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three pats (i). A short converging part (ii) Throat and (iii). Diverging part

Let d_1 = diameter at inlet or at section 1

Let P_1 = pressure at section 1

Let V_1 = velocity of fluid at section 1

Let a_1 = area of section 1 = $\frac{\pi}{4} d_1^2$

And d_2, P_2, V_2, a_2 are the corresponding values at section 2

Applying the Bernoulli's equation at section 1 & 2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2$

since the pipe is horizontal $Z_1 = Z_2$

$(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g)$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

We know that $\frac{P_1 - P_2}{\rho g}$ is the difference or pressure head and is equal to h.

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \text{ ------------- (1).}$$

Now applying, continuity equation at 1 & 2

$a_1V_1 = a_2V_2$ or $V_1 = (a_2V_2/ a_1)$ --------------- (2).

Sub (2) in equation (1) we get

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{a_2 V_2}{a_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right]$$

$V_2^2 = 2gh\ (a_1^2 / (a_1^2 - a_2^2))$

$$V_2 = \sqrt{2gh}.\frac{a_1}{\sqrt{a_1^2 - a_2^2}}$$

Discharge, $Q = a_2 V_2$

$$Q = \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$ theoretical discharge

Actual discharge

$$Q_{act} = C_d \times \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

Where C_d = co – efficient of venturi meter.

Water flows through a pipe AB 1.2m diameter at 3 m/s and then passes through a pipe BC 1.5 m diameter at C, the pipe branches. Branch CD is 0.8m in diameter and carries one third of the flow in AB. The flow velocity in branch CE is 2.5 m/s. Find the volume rate of flow in AB, the velocity in BC, the velocity in CD and the diameter of CE.

Solution. Given:

Diameter of Pipe AB,	D_{AB} = 1.2 m.
Velocity of flow through AB	V_{AB} = 3.0 m/s.
Dia. of Pipe BC,	D_{BC} = 1.5m.
Dia. of Branched pipe CD,	D_{CD} = 0.8m.
Velocity of flow in pipe CE,	V_{CE} = 2.5 m/s.
Let the rate of flow in pipe	AB = Q m^3/s.
Velocity of flow in pipe	BC = V_{BC} m^3/s.
Velocity of flow in pipe	CD = V_{CD} m^3/s.

Diameter of pipe CE = D_{CE}

Then flow rate through CD = Q / 3

And flow rate through CE = Q – Q/3 = 2Q/3

(i). Now the flow rate through AB = Q = V_{AB} X Area of AB

$$= 3 \times (\pi / 4) \times (D_{AB})^2 = 3 \times (\pi / 4) \times (1.2)^2$$

$$= 3.393 \text{ m}^3/\text{s}.$$

(ii). Applying the continuity equation to pipe AB and pipe BC,

V_{AB} X Area of pipe AB = V_{BC} x Area of Pipe BC

$$3 \times (\pi / 4) \times (D_{AB})^2 = V_{BC} \times (\pi / 4) \times (D_{BC})^2$$

$$3 \times (1.2)^2 = V_{BC} \times (1.5)^2$$

$$V_{BC} = (3 \times 1.2^2) / 1.5^2 = 1.92 \text{ m/s}.$$

(iii). The flow rate through pipe

$$CD = Q_1 = Q/3 = 3.393 / 3 = 1.131 \text{ m}^3/\text{s}.$$

$$Q_1 = V_{CD} \times \text{Area of pipe } C_D \times (\pi / 4) (C_{CD})^2$$

$$1.131 = V_{CD} \times (\pi / 4) \times (0.8)^2$$

$$V_{CD} = 1.131 / 0.5026 = 2.25 \text{ m/s}.$$

(iv). Flow through CE,

$$Q_2 = Q - Q_1 = 3.393 - 1.131 = 2.262 \text{ m}^3/\text{s}$$

$$Q_2 = V_{CE} \times \text{Area of pipe CE} = V_{CE} \times (\pi / 4) (D_{CE})^2$$

$$2.263 = 2.5 \times (\pi / 4) (D_{CE})^2$$

$$D_{CE} = \sqrt{(2.263 \times 4) / (2.5 \times \pi)} = 1.0735 \text{ m}$$

Diameter of pipe CE = 1.0735m.

A horizontal Venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take C_d = 0.98.

Given:

d_1 = 30 cm

$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2$

$= 706.85 \text{ cm}^2$

$d_2 = 15$ cm

$a_2 = \frac{\pi}{4} d_2^{\ 2} = \frac{\pi}{4}(15)^2$

$= 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

Difference of pressure head, h = $x\left(\frac{S_h}{S_o} - 1\right)$

= 20 [(13.6 / 1) – 1] = 252.0 cm of mercury.

$$Q_{act} = C_d \text{ x } \frac{a_2 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$= 0.98 \text{ x } \frac{706.85 x 176.7 \sqrt{2 x 9.81 x 252}}{\sqrt{706.85^2 - 176.7^2}}$$

$= 125756 \text{ cm}^3 / \text{s}$

= 125.756 lit / s.

UNIT – IV

BOUNDARY LAYER AND FLOW THROUGH PIPES

Definition of boundary layer – Thickness and classification – Displacement and momentum thickness – Development of laminar and turbulent flows in circular pipes – Major and minor losses of flow in pipes – Pipes in series and in parallel – Pipe network

Hydraulic gradient line.

It is defined as the line which gives the sum of pressure head ($P/\rho g$) and datum head (z) of a flowing fluid in a pipe with respect to some reference line or is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head ($P/\rho g$) of a pipe from the center of the pipe. It is briefly written as H.G.L

Major energy loss and minor energy loss in pipe

The loss of head or energy due to friction in pipe is known as major loss while the loss of energy due to change of velocity of the flowing fluid in magnitude or direction is called minor loss of energy.

Total Energy line

It is defined as the line, which gives sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

Equivalent pipeline

An Equivalent pipe is defined as the pipe of uniform diameter having loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters.

Water Hammer in pipes.

In a long pipe, when the flowing water is suddenly brought to rest by closing the valve or by any similar cause, there will be a sudden rise in pressure due to the momentum of water being destroyed. A pressure wave is transmitted along the pipe. A sudden rise in pressure has the effect of hammering action on the walls of the pipe. This phenomenon of rise in pressure is known as water hammer or hammer blow.

Pipes in series:

Pipes in series or compound pipes is defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line.

Pipes in parallel:

The pipes are said to be parallel, when a main pipe divides into two or more parallel pipes, which again join together downstream and continues as a mainline. The pipes are connected in parallel in order to increase the discharge passing through the main.

Boundary layer.

When a solid body is immersed in a flowing fluid, there is a narrow region of the fluid in neighbourhood of the solid body, where the velocity of fluid varies from zero to free stream velocity. This narrow region of fluid is called boundary layer.

laminar sub layer

In turbulent boundary layer region, adjacent to the solid boundary velocity for a small thickness variation in influenced by various effect. This layer is called as laminar sub layer.

Boundary layer thickness.

It is defined as the distance from the boundary of the solid body measured in the y – direction to the point where the velocity of the fluid is approximately equal to 0.99 times the free stream (v) velocity of the fluid.

momentum thickness.

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of the flowing fluid of boundary

$$\theta = \int_0^\delta u/v(1-u/v)dy$$

Incompressible flow.

It is define as the type of flow in which the density is constant for the fluid flow. Mathematically ρ = constant. Examples: Subsonic, aerodynamics.

Different methods of preventing the separation of boundary layers

1. Suction of slow moving fluid by suction slot
2. Supplying additional energy from a blower
3. Providing a bypass in the slotted wring
4. Rotating boundary in the direction of flow.
5. Providing small divergence in diffuser
6. Providing guide – blades in a bend.

Examples laminar flow / viscous flow

(i) Flow past tiny bodies,

(ii) Underground flow

(iii) Movement of blood in the arteries of human body,

(iv) Flow of oil in measuring instruments,

(v) Rise of water in plants through their roots etc.,

Characteristics of laminar flow

(i) No slip at the boundary

(ii) Due to viscosity, there is a shear between fluid layers, which is given by $\tau = \mu(du / dy)$ for flow in x- direction

(iii) The flow is rotational.

(iv) Due to viscous shear, there is continuous dissipation of energy and for maintaining the flow must be supplied externally.

(v) Loss of energy is proportional to first power of velocity and first power of viscosity.

(vi) No mixing between different fluid layers (except by molecular motion, which is very small)

Differentiate between laminar boundary layer and turbulent boundary layer

The boundary layer is called laminar, if the Renolds number of the flow is defined as $R_e = U \times X / v$ is less than $3X10^5$

If the Renolds number is more than $5X10^5$, the boundary layer is called turbulent boundary.

Where, U = Free stream velocity of flow

X = Distance from leading edge

v = Kinematic viscosity of fluid

Chezy's formula.

Chezy's formula is generally used for the flow through open channel.

$$V = C\sqrt{mi}$$

Where , C = chezy's constant, m = hydraulic mean depth and $i = h_f / L$.

A crude of oil of kinematic viscosity of 0.4 stoke is flowing through a pipe of diameter 300mm at the rate of 300 litres/sec. find the head lost due to friction for a length of 50m of the pipe.

Given :

Kinematic viscosity	$v = 0.4$ stoke $= 0.4\ cm^2/s = 0.4 \times 10^{-4}\ m^2/s$
Dia. Of pipe	d = 300mm = 0.3m
Discharge	$Q = 300$ Lit/S $= 0.3\ m^3/s$
Length of pipe	L = 50m
Velocity	$V = Q/ \text{Area} = 0.3 / (\frac{\pi}{4} (0.3)^2) = 4.24$ m/s
Renold number	$R_e = (V X d)/ v = (4.24 X 0.30) / 0.4X10^{-4} = 3.18 X 10^4$

As R_e lies between 4000 and 100,000, the value of "f" is given by

$$f = \frac{0.079}{(R_e)^{1/4}} = \frac{0.079}{(3.18X10^4)^{1/4}} = 0.00591$$

Head lost due to friction $h_f = 4 f L V^2/ 2 g d$

$$= (4 X 0.00591 X 50X 4.24^2) /(0.3 X 2 X 9.81)$$

$$= 3.61 \text{ m}$$

Find the type of flow of an oil of relative density 0.9 and dynamic viscosity 20 poise, flowing through a pipe of diameter 20 cm and giving a discharge of 10 lps.

Solution :

s = relative density = Specific gravity = 0.9

μ = Dynamic viscosity = 20 poise = 2 Ns/m^2.

Dia of pipe D = 0.2 m; Discharge Q = 10 lps = (10 / 1000) m^3/s; Q = AV.

So V = Q / A = $[10 / (1000 \times (\frac{\pi}{4} (0.2)^2))] = 0.3183$ m/s.

Kinematic viscosity = $\nu = \mu / \rho = [2 / (0.9 \times 1000)] = 2.222 \times 10^{-3}\ m^2/s$.

Reynolds number Re = VD / ν

Re = $[0.3183 \times 0.2 / 2.222 \times 10^{-3}] = 28.647$;

Since Re (28.647) < 2000,

It is **Laminar flow.**

Formula for finding the loss of head due to entrance of pipe h_i

$$h_i = 0.5\ (V^2 / 2g)$$

Formula to find the Efficiency of power transmission through pipes

$$n = (H - h_f) / H$$

where, H = total head at inlet of pipe.

h_f = head lost due to friction

Hydro dynamically smooth pipe carries water at the rate of 300 lit/s at 20ºC (ρ = 1000 kg/m^3, $\nu = 10^{-6}\ m^2/s$) with a head loss of 3m in 100m length of pipe. Determine the pipe diameter. Use $f = 0.0032 + (0.221)/(R_e)^{0.237}$ equation for f where $h_f = (fXLXV^2)/2gd$ and $R_e = (\rho VD/\mu)$

Given:

Discharge, Q = 300 lit/sec = $0.3 m^3/s$

Density $\rho = 1000\ kg/m^3$

Kinematic viscosity $\nu = 10^{-6} m^2/s$

Head loss $h_f = 3m$

Length of pipe, L = 100m

Value of friction factor, $f = 0.0032 + 0.221 / (R_e)^{0.237}$

Renolds number $R_e = (\rho VD/\mu) = (VXD) / \nu$ $(\mu/\rho = \nu)$

$$VXD/ 10^{-6} = VXDX10^6$$

Find diameter of pipe.

Let D = diameter of pipe

Head loss in terms of friction factor is given as

$h_f = (fXLXV^2)/ 2gXD$

$3 = (fX100XV^2)/ 2X9.81XD$

$f = (3XDX2X9.81)/ 100V^2$

$f = 0.5886D / V^2$ ------------------------------(i)

now Q = AXV

$0.3 = \frac{\pi}{4} (D)^2 X V$ or $D^2 X V = (4 X 0.3 / \pi) = 0.382$

$V^2 = 0.382 / D^2$ --------------------------- (ii)

$f = 0.0032 + (0.221)/ (R_e)^{0.237}$

$0.5886/ D^2 = 0.0032 + (0.221)/ (VXDX10^6)^{0.237}$

{ from equation (i), $f = 0.5886D/V^2$ and $R_e = VXDX10^6$}

$$0.5886D / (0.382/D^2)^{2} = 0.0032 + \frac{0.221}{\left(\frac{0.382}{D^2} XDX10^6\right)^{0.237}}$$

{ from Equation (ii), $V = 0.382 / D^2$}

$$0.5886 \text{ x } D^5/ 0.382^2 = 0.0032 + \frac{0.221}{\left(\frac{(0.382X10^6)^{0.237}}{D^{0.237}}\right)}$$

$4.0333 D^5 = 0.0032 + 0.0015 X D^{0.237}$

$4.0333 D^5 – 0.0105 D^{0.237} – 0.0032= 0$ ---------------(iii)

the above equation (iii) will be solved hit trial method

(i). Assume D = 1m, then L.H.S of the equation (iii), becomes as

$L.H.S = 4.033 X 1^5 – 0.0105 X 1^{0.237} – 0.0032$

$= 4.033 – 0.0105 – 0.0032 = 4.0193$

by increasing the value of D more than 1m , the L.H.S. will go on increasing.

Hence decrease the value of D.

(ii) Assume D = 0.3 than L.H.S of equation (iii)

becomes as $L.H.S = 4.033 X 0.3^{0.237} – 0.0032$

$= 0.0098 – 0.00789 – 0.0032 = - 0.00129$

as this value of negative, the values of D will be slightly more than 0.3

(iii) Assume D = 0.306 then L.H.S of equation (iii) becomes as

$$L.H.S = 4.033 \text{ X } 0.306^{0.237} – 0.0105\text{X}0.306^{0.237} – 0.0032$$

$$= 0.0108 – 0.00793 – 0.0032 = - 0.00033$$

This value of L.H.S is approximately equal to equal to zero. Actually the value of D will be slightly more than 0.306m say **0.308m**.

Expression for loss of head due to friction in pipes.

Or

Darcy – Weisbach Equation.

Consider a uniform horizontal pipe, having steady flow as shown figure. Let 1 -1 and 2-2 are two sections of pipe.

Let P_1 = pressure intensity at section 1-1.

Let P_2 = Velocity of flow at section 1-1.

L = length of the pipe between the section 1-1 and 2-2

d = diameter off pipe.

f^1 = Frictional resistance per unit wetted area per unit velocity.

h_f = loss of head due to friction.

And P_2,V_2 = are the values of pressure intensity and velocity at section 2-2.

Applying Bernoulli's equation between sections 1-1 & 2-2

Total head 1-1 = total head at 2-2 + loss of head due to friction between 1-1&2-2

$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_f$ ------------(1)

but $Z_1 = Z_1$ [pipe is horizontal]

$V_1= V_2$ [diameter of pipe is same at 1-1 & 2-2]

(1) becomes,

$(P_1/ \rho g) = (P_2/\rho g)+h_f$

$h_f = (P_1/ \rho g) - (P_2/\rho g)$

frictional resistance = frictional resistance per unit wetted area per unit velocity X wetted area X velocity ^{2}.

$F = f^1 \text{ X } \pi \text{ d l X } V^2$ [Wetted area = π d X L, and Velocity $V = V_1 = V_2$]

$F_1 = f^1 X P X L X V^2$ ----------- (2). [π d = wetted perimeter = p]

The forces acting on the fluid between section 1-1 and 2-2 are,

1) Pressure force at section 1-1 = $P_1 X A$

2) Pressure force at section 2-2 = $P_2 X A$

3). Frictional force F_1

Resolving all forces in the horizontal direction.,

$P_1 A - P_2 A - F_1 = 0$

$(P_1 - P_2)A = F_1 = f^1 X P X L X V^2$

$(P_1 - P_2) = (f^1 X P X L X V^2 / A)$.

But from (1) we get

$P_1 - P_2 = \rho g\, h_f$

Equating the values of $(P_1 - P_2)$ we get

$\rho g\, h_f = (f^1 X P X L X V^2 / A)$.

$h_f = (f^1 / \rho g)\, X\, (P/A)\, X\, L X\, V^2$

$(P/A) = (\pi d / (\pi d^2/4)) = (4/d)$

hence, $h_f = (f^1 / \rho g)\, X\, (4/d)\, X\, L X V^2$.

Putting $(f^1 / \rho) = (f / 2)$, where f is the co – efficient of friction

$$h_f = \frac{4 fLV^2}{2gd}$$

This equation is known as Darcy – Weisbach equation. This equation is commonly used to find loss of head due to friction in pipes

The rate of flow through a horizontal pipe is 0.25 m^3/s. the diameter of the pipe which is 200mm is suddenly enlarged to 400mm. the pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine (i). Loss of head due to s udden enlargement (ii). Pressure intensity in large pipe. (iii). Power lost due to enlargement.

Given:

Discharge $Q = 0.25$ m^3/s.

Dia. Of smaller pipe $D_1 = 200mm = 0.2m$

Area $A_1 = \frac{\pi}{4}(0.2)^2 = 0.03141\ m^2$.

Dia of large pipe $D_2 = 400mm = 0.4m$

Area $A_2 = \frac{\pi}{4}(0.4)^2 = 0.12566\ m^2$.

Pressure in smaller pipe $p_1 = 11.772\ N/cm^2 = 11.772\ X10^4\ N/m^2$.

Now velocity $V_1 = Q / A_1 = 0.25 / 0.03414 = 7.96$ m/s.

Velocity $V_2 = Q / A_2 = 0.25 / 0.12566 = 1.99$ m/s.

(i). Loss of head due to sudden enlargement,

$$h_e = (V_1 - V_2)^2 / 2g = (7.96 - 1.99)^2 / 2X\ 9.81 = \mathbf{1.816\ m}.$$

(ii). Let the pressure intensity in large pipe = p_2.

Then applying Bernoulli's equation before and after the sudden enlargement,

$$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2 + h_e$$

But $Z_1 = Z_2$

$$(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g) + h_e$$

Or $(P_1/\rho g) + (V_1^2 / 2g) = (P_2/\rho g) + (V_2^2 / 2g) + Z_2 + h_f$

$$(P_2/\rho g) = (P_1/\rho g) + (V_1^2 / 2g) - (V_2^2 / 2g) - h_e$$

$$= \frac{11.772X10^4}{1000X9.81} + \frac{7.96^2}{2X9.81} - \frac{1.99^2}{2X9.81} - 1.816$$

$= 12.0+3.229-0.2018-1.8160$

$= 15.229 - 20.178 = 13.21$m of water

p_2 $= 13.21\ X\ \rho g = 13.21\ X\ 100X\ 9.81\ N/m^2$

$= 13.21X1000X9.81X10^{-4}\ N/cm^2$. = $\mathbf{12.96 N/cm^2.}$

(iii). Power lost due to sudden enlargement,

$$P = (\rho g\ \mathbf{Q}\ \mathbf{h_e}) / 1000 = (1000X9.81X0.25X1.816)/1000 = \mathbf{4.453kW.}$$

A horizontal pipeline 40m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25m of its length from the tank, the pipe is 150mm diameter is suddenly enlarged to 300mm. the height of water level in the tank is 8m above the centre of the pipe. Considering all losses of head, which occur. Determine the rate of flow. Take f = 0.01 for both sections of the pipe.

Given:

Total length of pipe, $L = 40m$

Length of 1 st pipe, $L_1 = 25m$

Dia of 1st pipe $d_1 = 150mm = 0.15m$

Length of 2nd pipe $L_2 = 40 – 25 = 15m$

Dia of 2nd pipe $d_2 = 300mm = 0.3m$

Height of water $H = 8m$

Co-effi. Of friction $f = 0.01$

Applying the Bernoulli's theorem to the surface of water in the tank and outlet of pipe as shown in fig. and taking reference line passing through the center of the pipe.

$0+0+8 = (P_2/\rho g) + (V_2^2 / 2g) +0+\text{all losses}$

$8.0 = 0+(V_2^2 / 2g)+h_i+ h_{f1}+ h_e+ h_{f2}$

Where, h_i = loss of head at entrance = $0.5\ V_1^2/\ 2g$

h_{f1} = head lost due to friction in pipe 1 = $\dfrac{4XfXL_1XV_1^2}{d_1X2g}$

h_e = loss of head due to sudden enlargement = $(V_1 - V_2)^2/2g$

h_{f2} = head lost due to friction in pipe 2 = $\dfrac{4XfXL_2XV_2^2}{d_2X2g}$

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$V_1 = (A_2V_2/A_1) = \frac{\frac{\pi}{4}d_2^2XV_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 XV_2 = \left(\frac{0.3}{0.15}\right)^2 XV_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have

$$h_i = 0.5\ V_i^2/\ 2g = (0.5\ X\ (4V_2)^2\ )/\ 2g = 8V_2^2/2g$$

$$h_{f1} = \frac{4X0.01X25X(4V_2^2)}{0.15X2g} = \frac{4X0.01X25X16}{0.15}X\frac{V_2^2}{2g} = 106.67\frac{V_2^2}{2g}$$

$$he = (V_1 - V_2)^2/\ 2g \quad = (4V_2 - V_2)^2/2g = 9V_2^2/2g$$

$$h_{f2} = \frac{4X0.01X15X(V_2^2)}{0.3X2g} = \frac{4X0.01X15}{0.3}X\frac{V_2^2}{2g} = 2.0\frac{V_2^2}{2g}$$

Substituting the values of these losses in equation (i), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2X\frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}[\ 1+8+106.67+9+2] = 126.67\frac{V_2^2}{2g}$$

$$V_2 = \sqrt{\frac{8.0x2xg}{126.67}} = \sqrt{\frac{8.0X2X9.81}{126.67}} = 1.113\text{ m/s}$$

Rate of flow Q = $A_2 X V_2 = \frac{\pi}{4}(0.3)^2 X\ 1.113 = 0.07867\ m^3/s$ = **78.67 litres/sec.**

A pipe line, 300mm in diameter amd 3200m long is used to pump up 50kg per second of an oil whose density is 950n kg/m^3.and whose Kinematic viscosity is 2.1 stokes. The center of the pipe at upper end is 40m above than at the lower end. The discharge at the upper end is atmospheric. Find the pressure at the lower end and draw the hydraulic gradient and the total energy line.

Given:

Dia of pipe d = 300mm = 0.3m

Length of pipe L = 3200m

Mass M = 50kg/s = ρ. Q

Discharge Q = 50/ ρ = 50/950 = 0.0526 m^3/s

Density ρ = 950 kg/m^3

Kinematic viscosity ν = 2.1 stokes = 2.1 cm^2/s = 2.1 $X10^{-4}\ m^2/s$

Height of upper end = 40m

Pressure at upper end = atmospheric = 0

Renolds number, R_e = VXD/ν, where V = Discharge / Area

$$= 0.0526/\left(\frac{\pi}{4}(0.3)^2\right) = 0.744\ m/s$$

$$R_e = (0.744X0.30) / (2.1X10^{-4}) = 1062.8$$

Co – efficient of friction, f = 16/ R_e = 16 / 1062.8 = 0.015

Head lost due to friction,h_f

$$= \frac{4XfXL\ \ XV^2}{d\ \ X2g} = \frac{4X0.015X3200X(0.744)^2}{0.3X2X9.81} = 18.05m \text{ of oil.}$$

Applying the Bernoulli's equation at the lower and upper end of the pipe and taking datum line passing through the lower end, we have

$$(P_1/\rho g) + (V_1^2 / 2g) + Z_1 = (P_2/\rho g) + (V_2^2 / 2g) + Z_2+h_f$$

but $Z_1 = 0$, $Z_2 = 40$m., $V_1 = V_2$ as diameter is same.

$$P_2 = 0,\ h_f = 18.05\text{m}$$

Substituting these values, we have

$$= 5400997 \text{ N/m}^2 = 54.099 \text{ N/cm}^2.$$

H.G.L. AND T.E.L.

$V^2/2g = (0.744)^2/2X9.81 = 0.0282$ m

$p_1/ \rho g = 58.05$ m of oil

$p_2 / \rho g = 0$

Draw a horizontal line AX as shown in fig. From A draw the centerline of the pipe in such way that point C is a distance of 40m above the horizontal line. Draw a vertical line AB through A such that AB = 58.05m. Join B with C. then BC is the hydraulic gradient line.

Draw a line DE parallel to BC at a height of 0.0282m above the hydraulic gradient line. Then DE is the total energy line.

A main pipe divides into two parallel pipes, which again forms one pipe as shown. The length and diameter for the first parallel pipe are 2000m and 1.0m respectively, while the length and diameter of 2nd parallel pipe are 2000m and 0.8m. Find the rate of flow in each parallel pipe, if total flow in main is 3.0 m^3/s. the co-efficient of friction for each parallel pipe is same and equal to 0.005.

Given:

Length of Pipe 1 $L_1 = 2000m$

Dia of pipe1 $d_1 = 1.0m$

Length of pipe 2 $L_2 = 2000m$

Dia of pipe 2 $d_2 = 0.8m$

Total flow $Q = 3.0m^3/s$

$f_1 = f_2 = f = 0.005$

let Q_1 = discharge in pipe 1

let Q_2 = discharge in pipe 2

from equation, $Q = Q_1 + Q_2 = 3.0$ ---------------(i)

using the equation we have

$$\frac{4Xf_1XL_1XV_1^2}{d_1X2g} = \frac{4Xf_2XL_2XV_2^2}{d_2X2g}$$

$$\frac{4X0.005X2000XV_1^2}{1.0X2X9.81} = \frac{4X0.005X2000XV_2^2}{0.8X2X9.81}$$

$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} or V_1^2 = \frac{V_2^2}{0.8}$$

$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{0.894}$$

Now, $Q_1 = \frac{\pi}{4} d_1^2XV_1 = \frac{\pi}{4}(1)^2X(V_2/0.894)$

And $Q_2 = \frac{\pi}{4} d_2^2XV_2 = \frac{\pi}{4}(0.8)^{2X}(V_2) = \frac{\pi}{4}(0.64)X(V_2)$

Substituting the value of Q_1 and Q_2 in equation (i) we get

$\frac{\pi}{4}(1)^2X(V_2/0.894) + \frac{\pi}{4}(0.64)X(V_2) = 3.0$ or $0.8785V_2 + 0.5026V_2 = 3.0$

$V_2[0.8785+0.5026] = 3.0$ or $V = 3.0 / 1.3811 = 2.17$ m/s.

Substituting this value in equation (ii),

$V_1 = V_2 / 0.894 = 2.17 / 0.894$ m/s

Hence $Q_1 = \frac{\pi}{4} d_1^2 X V_1 = \frac{\pi}{4} 1^2 X 2.427 =$ **1.096 m³/s**

$Q_2 = Q - Q_1 = 3.0 - 1.906 =$ **1.904 m³/s**.

Three reservoirs A, B, C are connected by a pipe system shown in fig. Find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 litres / s. find the height of water level in the reservoir C. take f = 0.006 for all pipes.

Given:

Length of pipe AD, $L_1 = 1200$m

Dia of pipe AD, $d_1 = 30$cm $= 0.3$m

Discharge through AD, $Q_1 = 60$lit/s $= 0.06\ m^3/s$

Height of water level in A from reference line , $Z_A = 40$m

For pipe DB, length $L_2 = 600$mm, Dia., $d_2 = 20$cm $= 0.20$m, $Z_B = 38.0$

For pipe DC, length $L_3 = 800$mm, Dia., $d_3 = 30$cm $= 0.30$m,

Applying the Bernoulli's equations to point E and, $Z_A = Z_D + \frac{p_D}{\rho g} + h_f$

Where $h_f = \frac{4 X f_1 X L_1 X V_1^2}{d_1 X 2g}$, where $V_1 = Q_1 / \text{Area} = 0.006 / (\frac{\pi}{4}\ (0.3)^2) = 0.848$ m/s.

$$h_f = \frac{4 X 0.006 X 1200 X 0.848^2}{0.3 X 2 X 9.81} = 3.518 \text{ m.}$$

$$\{Z_D + \frac{p_1}{\rho g}\} = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at D = 36.482m. Hence water flows from B to D.

Applying Bernoulli's equation to point B and D

$Z_B = \{Z_D + \frac{p_D}{\rho g}\} + h_{f2}$ or $38 = 36.482 + h_{f2}$

$h_{f2} = 38 - 36.482 = 1.518$m

But $h_{f2} = \frac{4XfXL_2XV_2^2}{d_2X2g} = \frac{4X0.006X600XV_2^2}{0.2X2X9.81}$

$1.518 = \frac{4X0.006X600XV_2^2}{0.2X2X9.81}$

$V_2 = \sqrt{\frac{1.518X0.2X2X9.81}{4X0.006X600}} = 0.643m/s$

Discharge $Q_2 = V_2X\frac{\pi}{4}(d_2)^2 = 0.643 X \frac{\pi}{2}X(0.2)^2 =$ **$0.0202m^3/s$ = 20.2lit/s.**

Applying Bernoulli's equation to D and C

$\{Z_D + \frac{p_D}{\rho g}\} = Z_C + h_{f3}$

$36.482 = Z_C + \frac{4XfXL_3XV_3^2}{d_3X2g}$ $where, V_3 = \frac{Q_3}{\frac{\pi}{4}d_3^2}$

but from continuity $Q_1 + Q_2 = Q_3$

$Q_3 = Q_1 + Q_2 = 0.006 + 0.0202 = 0.0802\ m^3/s$

$$V_3 = \frac{Q_3}{\frac{\pi}{4}d_3^2} = \frac{Q_3}{\frac{\pi}{4}(0.9)^2} = 1.134m/s$$

$36.482 = Z_C + \frac{4X0.006X800X1.134^2}{0.X2X9.81} = Z_C + 4.194$

$Z_C = 36.482 - 4.194 =$ **32.288m**

A Pipe line of length 2000 m is used for power transmission. If 110.365 kW power is to be transmitted through the pipe in which water having pressure of 490.5 N/cm² at inlet is flowing. Find the diameter of the pipe and efficiency

of transmission if the pressure drop over the length of pipe is 98.1 N/cm^2. Take f = 0.0065.

Given:

Length of pipe L = 2000m.

H.P transmitted = 150

Pressure at inlet, p = 490.5 N/cm^2 = 490.5 X10^4 N/m^2.

Pressure head at inlet, H = p / ρg

Pressure drop = 98.1 N/cm^2 = 98.1X10^4/m^2

Loss of head h_f = 98.1 X10^4 / ρg = 98.1X10^4/ (1000 X 9.81) = 100m

Co – efficient of friction f = 0.0065

Head available at the end of the pipe = H – h_f = 500 – 100 = 400m

Let the diameter of the pipe = d

Now power transmitted is given by,

P = [ρg X Q X (H – h_f)] / 1000 kW.

110.3625 = [1000X9.81XQX400] / 1000

Q = [110.3625 X 1000 / (1000X9.81X400)] = 0.02812m^3/s

But discharge Q = AXV = $\frac{\pi}{4}$ d^2 X V

$\frac{\pi}{4}$ d^2 X V = 0.02812

V = (0.02812 X 4) / 3.14 X d^2 = 0.0358 / d^2----------(1)

Total head lost due to friction,

$$h_f = \frac{4\,fXLXV^2}{dX\,2g}$$

but, h_f = 100m

$$100 = h_f = \frac{4XfXL\ \ XV^2}{d\ \ X\,2g} = \frac{4X\,0.0065X\,2000XV^2}{dX\,2X\,9.81} = \frac{2.65XV^2}{d}$$

= (2.65 / d) X (0.358/d^2)^2 = 0.003396 / d^5

from equation (1),

V = 0.0358 / d^2

$$100 = 0.003396 / d^5$$

$$d = (0.00396 / 100)^{1/5} = 0.1277m = 127.7mm.$$

Efficiency of power transmission is given by equation

$$\eta = \frac{H - h_f}{H} = \frac{500 - 100}{500} = 0.80 = 80\%$$

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UNIT – V

SIMILITUDE AND MODEL STUDY

Dimensional Analysis – Rayleigh's method, Buckingham's Pi-theorem – Similitude and models – Scale effect and distorted models

Dimensional analysis.

Dimensional analysis is defined as a mathematical technique used in research work for design and conducting model tests.

Fundamental dimensions

The fundamental units quantities such as length (L), mass (M), and time (T) are fixed dimensions known as fundamental dimensions.

Units.

Unit is defined as a yardstick to measure physical quantities like distance, area, volume, mass etc.

Derive the dimensions for velocity.

Velocity is the distance (L) travelled per unit time (T)

Velocity = Distance/ Time = [L/T] = LT^{-1}.

Model

Model is nothing but small-scale repetition of the actual structure or machine.

List out the advantages of model analysis.

The advantages of model analysis are:

1. The performance of hydraulic structure or machine can be easily predicted in advance from its model.
2. The merits of alternative design can be predicted with the help of model testing and the most economical and safe design may be finally adopted.

Similitude.

The three types of similarities are,

1. Geometric similarity
2. Kinematic similarity
3. Dynamic similarity

Scale ratio.

Scale ratio is the ratio of linear dimension in the model and prototype which are equal in a geometric similarity. It is denoted by L_r.

$L_r = L_p/L_m = b_p/b_m = D_p/D_m$

Dynamic similarity.

It means the similarity of forces at corresponding points in the model and prototype is equal.

Types of forces in a moving fluid.

The types of forces in a moving fluid are,

1. Inertia force (f_i)
2. Viscous force (f_v)
3. Gravity force (F_g)
4. Pressure force (F_p)
5. surface tension (F_s)
6. Elastic force (F_e)

Dimensionless numbers.

Dimensionless numbers are the numbers obtained by dividing inertia force or gravity force or pressure force or elastic force or surface tension. They are called as non-dimensional parameters.

Surface tension.

Surface tension force is defined as the product of surface tension and length of surface of flowing fluid.

Pressure force.

Pressure force is the product of pressure intensity and cross-sectional area of the flowing fluid in case of pipe flow.

Elastic force.

Elastic force is defined as the product of elastic stress and the area of flowing fluid.

Types of dimensionless numbers.

The types of dimensionless numbers are:

1. Reynold's number
2. Froude's number
3. Euler's number
4. Weber's number
5. Mach's number

Reynold's number.

Reynold's number is defined as the ratio of inertia force of flowing fluid and viscous force of the fluid. It is denoted by (R_e)

$$R_e = V \times d / v = \rho V d / \mu$$

Froude's number

Froude's number is defined as square root of ratio of inertia force of flowing fluid to gravity. It is denoted as $F_e = \sqrt{\frac{F_i}{f_g}} = \frac{V}{\sqrt{Lg}}$

Classification of models.

The classifications of models are

i. Undistorted models

ii. Distorted models

Undistorted model

If the scale ratio for the linear dimensions of the model and prototype is same, then the model is said to be undistorted model.

Distorted model

A distorted model is said to be a distorted model only when it is not geometrically similar to prototype.

Advantages of distorted models.

The advantages of distorted models are,

1. The vertical dimensions of the model can be measured accurately.
2. The cost of model can be reduced.
3. Turbulent flow in the model can be maintained.

List out the types of model laws.

The types of model laws are

1. Reynolds's model laws
2. Froude's model laws
3. Euler's model laws
4. Weber's model laws
5. Mach's model laws

List out the application of Froude's model laws

Froude's model laws is applied in

1. Free surface flow such as flow over spillways, weirs, sluices, channels etc
2. Flow of jet from an orifice or nozzle
3. Where waves are likely to be formed on surface
4. Where fluids of different densities flow over one anther

Weber's model laws

When surface tensile forces alone are predominant a model may be taken to be dynamically similar to the prototype when ratio of inertial to the surface tensile forces is the same in the model and prototype.

$$(We)_{model} = (We)\ prototype$$

Buckingham's ' π ' theorem.

The **Buckingham's ' π ' theorem** is a key theorem in dimensional analysis. The theorem states that if we have a physically meaningful equation involving a certain number, n of physical variables and these variables are expressible in terms of **k** independent fundamental physical qualities, then the original expression is equivalent to an equation involving a set of p = n – k dimensionless variables constructed from the original variables. For the purpose of the experimenter, different systems which share the same description in terms of these dimensionless numbers are equivalent.

In mathematical terms, if we have a physically meaningful equation such as

$$f\ (q_{1,}\ q_{2,} \ldots\ldots. q_n) = 0$$

where the q_i are the n physical variables and they are expressed in terms of k independent physical units, the above equation can be restated as

$$F\ (\pi_{1,}\ \pi_{2} \ldots\ldots\ \pi_p) = 0$$

Where, the π_i are the dimensionless parameters constructed from the q_i by p = n–k equations of the form

$$\pi_i = q_1^{m}, q_{2,}^{m} \ldots\ldots. q_n^{m}$$

Where the exponents m_i are the rational numbers. The use of the π_i the dimensionless parameters was introduced by Edge Buckingham in his original 1914 paper on the subject from which the theorem draws its name.

Most importantly, the Buckingham π theorem provides a method for computing sets of dimensionless parameters is not unique: Buckingham's theorem only provides a way of generating sets of dimensionless parameters and will not choose the most 'physically meaningful'.

Proofs of the **π** theorem often begin by considering the space of fundamental and derived physical unit's vector space, with the fundamental units as basis vectors and with multiplication of physical units as the "Vector addition" operation and raising to powers as the "scalar multiplication" operation.

Making the physical units match across sets of physical equations can then the regarded as imposing linear constraints in the physical units vector space.

The theorem describes how every physical meaningful equation involving **n** variables can be equivalently rewritten as an equation of n – m dimensionless parameters, where **m** is the number of fundamental units used. Furthermore and most importantly it provides a method for computing these dimensionless parameters from the given variables, even if the form of the equation is still unknown.

Two systems for which these parameters coincide are called similar; they are equivalent for the purposes of the experimental list who wants to determine the form of the equation can choose the most convenient one.

The **π** theorem uses linear algebra: the space of all possible physical units can be seen as a vector space over the rational numbers if we represent a unit as the set of exponents needed for the fundamental units (with a power of zero if the particular fundamental unit is not present) Multiplication of physical units is then represented by vector addition within this vector space. The algorithm of the π theorem is essentially a Gauss – Jordan elimination carried out in this vector space.

The resisting force of (R) of a supersonic flight can be considered as dependent upon the length of aircraft "l", velocity 'V', air viscosity 'μ', air density 'ρ', and bulk modulus of air 'k'. Express the functional relationship between these variables and the resisting force. (Nov 2005.)

Solution:

Step 1. $R = f(l, V, \mu, \rho, k)$

$\Phi(R, l, V, \mu, \rho, k) = 0$

Number of variables, n = 6

Number of primary variable, m = 3

Number of **π** terms = n – m = 6 – 3 = 3

$f(\pi_1, \pi_2, \pi_3) = 0$

Step 2. Assume l, V and ρ to be the repeating variables.

$\pi_1 = l^x, V^y, \mu^z, R$

$$M^O L^O T^O = [L]^X [LT^{-1}]^y [ML^{-3}]^z [MLT^{-2}]$$

$z_1+1 = 0$; $x_1+y_1-3z_1+1 = 0$ $-y_1-2 = 0$

$z_1 = -1$ $x_1-2+3+1 = 0$ $y_1 = -2.$

$x_2 = -2.$

$$\pi_1 = l^{-2}V^{-2}\rho^{-1}R = \frac{R}{l^2V^2\rho}$$

Step 3: $\pi_2 = l^x V^y \rho z\, \mu$

$$M^O L^O T^O = [L]^X [LT^{-1}]^y [ML^{-3}]^z [ML^{-1}T^{-1}]$$

$z_1+1 = 0$; $x_2+y_2-3z_2 - 1 = 0$ $-y_2 - 1 = 0$

$z_1 = -1$ $x_2 - 1+3 - 1 = 0$ $y_2 = -1.$

$$x_2 = -1.$$

$$\pi_2 = l^{-1}V^{-1}\rho^{-1}\,\mu = \frac{\mu}{l\ V\ \rho}$$

Step 4. $\pi_2 = l^{x} V^{y} \rho z\ K$

$$M^O L^O T^O = [L]^X [LT^{-1}]^y [ML^{-3}]^z [ML^{-1}T^{-2}]$$

$Z_3 + 1 = 0;$ $x_3 + y_3 - 3z_3 - 1 = 0$ $-y_3 - 2 = 0$

$Z_3 = -1$ $x_3 - 2 + 3 - 1 = 0$ $y_2 = -2.$

$$x_3 = 0.$$

$$\pi_2 = l^{-0}V^{-2}\rho^{-1}\,K = \frac{K}{\rho V^2}$$

Step 5.

$$\Phi\,(\pi_1, \pi_2, \pi_3) = 0 \qquad \Phi\left(\frac{R}{l^2V^2\rho}, \frac{\mu}{lV\rho}, \frac{K}{\rho V^2}\right) = 0$$

$$R = l^2\,V^2\,\rho\,\Phi\left(\frac{\mu}{lV\rho}, \frac{K}{\rho V^2}\right) = 0$$

A Ship is 300m long moves in seawater, whose density is 1030 kg/m^3, A 1:100 model of this to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30m/s and the resistance of the model is 60N. Determine the velocity of ship in seawater and also the resistance of the ship in sea water. The density of air is given as 1.24 kg/m^3. Take the Kinematic viscosity of seawater and air as 0.012 stokes and 0.018 stokes respectively.

Given:

For prototype,

Length L_p = 300m

Fluid = Sea water

Density of water = 1030 kg/m^3

Kineamtic viscosity $v_p = 0.018$ stokes $= 0.018 \text{ X}10^{-4}$ m^2/s

Let velocity of ship $= V_p$

Resitance $= F_p$

For Model

Length $L_m = (1/100)\text{X}300 = 3m$

Velocity $V_m = 30m/s$

Resistance $F_m = 60N$

Density of air $\rho_m = 1.24 \text{ kg}/m^3$

Kinematic viscosity of air $v_m = 0.018$ stokes $= 0.018 \text{ X}10^{-4}$ m^2/s

For dynamic similarity between the prototype and its model, the Reynolds's number for both of them should be equal.

$$(V_p \text{X} L_p / v_p) = (V_m \text{X} L_m / v_m) \text{ or } (v_p / v_m) \text{ X } (L_m/L_p) \text{ X } V_m$$

$$= (0.012\text{X}10^{-4} / 0.018\text{X}10^{-4}) \text{ X } (3/300) \text{ X}30 = 0.2 \text{ m/s.}$$

Resistance = Mass X Acceleration

$$= \rho \, L^3\text{X } (V/t) = \rho \, L^2\text{X } (V/1)\text{X } (L / t) = \rho \, L^2V^2$$

Then $F_p / F_m = (\rho \, L^2V^2)_p / (\rho \, L^2V^2)_m = (\rho_p/ \rho_m) \text{ X } (L_p/Lm)^2\text{X } (V_p/V_m)$

$(\rho_p/ \rho_m) = 1030 / 1.24$

$F_p / F_m = (1030 /1.24) \text{ X } (300/3)^2\text{X } (0.2/30) = 369.17$

$F_p = 369.17 \text{ X } F_m = 369.17 \text{ X } 60 = 22150.2 \text{ N.}$

A 7.2 m height and 15m long spill way discharges 94 m^3/s discharge under a head of 2.0m. If a 1:9 scale model of this spillway is to be constructed, determine model dimensions, head over spillway model and the model discharge. If model experience a force of 7500N (764.53Kgf), determine force on the prototype.

Given:

For prototype: height $h_p = 7.2m$

Length, $L_p = 15m$

Discharge $Q_p = 94 \, m^3/s$

Head, $H_p = 2.0m$

Size of model = 1/9. of the size of prototype

Linear scale ratio, $L_r = 9$

Force experienced by model $F_p = 7500N$

Find : (i) Model dimensions i.e., height and length of model (h_m and L_m)

(ii). Head over model i.e., H_m

(iii). Discharge through model i.e., Q_m

(iv). Force on prototype (i.e., F_p)

(i). Model dimensions (h_m and L_m)

$$h_p/\ h_m = L_p/\ L_m = L_r = 9$$

$$h_m = h_p / 9 = 7.2 / 9 = 0.8 \text{ m}$$

$$L_m = L_p / 9 = 15 / 9 = 1.67 \text{ m.}$$

(ii). Head over model (H_m)

$$h_p/\ H_m = L_r = 9$$

$$H_m = H_p / 9 = 2/9 = 0.222 \text{ m.}$$

(iii). Discharge through model (Q_m)

Using equation we get, $Q_p / Q_m = L_r^{2.5}$

$$Q_m = (Q_p / L_r^{2.5}) = 94 / 9^{2.5} = 0.387 \text{ m}^3/\text{s.}$$

(iv). Force on the prototype(F_p)

Using $Fr = F_p / F_m = L_r^3$

$$F_p = F_m \text{ X } L_r^3\ 7500X9^3 = 5467500N.$$

A quarter scale turbine model is tested under a head of 12m. The full-scale turbine is to work under a head of 30m and to run at 428 rpm. Find N for model. If model develops 100 kW and uses 1100 l/s at this speed, what power will be obtained from full scale turbine assuming its n is 3% better than that of model.

Solution :

$D_m / D_p = 1/4$; $H_m = 30$ m; $N_p = 428$ rpm.

w.k.t.

$(DN/\sqrt{H}\ )_m = (DN/\sqrt{H}\ )_p$

$(D^2N^2)_m/\ (D^2N^2)_P = H_m / H_p)$

$$N_m = \sqrt{\left(\frac{D_p}{D_m}\right)^2 X \frac{H_m}{H_p} X (N_p)^2}$$

$$= \sqrt{16 X \frac{12}{30} X 428^2}$$

= 1082.7638 rpm.

(ii). P_m = 100 kW; Q_m = 1.1 m^3/ s.

$(\eta_0)_m$ = (P/ ρg Q H)$_m$ = 100X10^3 / (9810X1.1X12) = 0.772248 or 77.22%.

It is given in this problem that the efficiency of the actual turbine is 3% better than the model.

$(\eta_0)_p$ = 79.54159%

We know that,

$(Q / D^2 \sqrt{H})_m = (Q / D^2 \sqrt{H})_p$

$Q_p = (Q / \sqrt{H})_m (\sqrt{H})_p (D_p/D_m)^2$.

$Q_p = (1.1 / \sqrt{12}) X (\sqrt{30}) X 16$

= 27.82 m^3/s

$P_p = (\eta_0)_p X (\rho g Q H)_p$

= 0.7954159 X 9810 X 27.82 X 30

= 6514.2917 kW.