## B.C.A.

## Part III: Basic Subjects

## Paper-1 Discrete Mathematics

Time: 3 hours
Max. Marks: 80

## SECTION-A

Answer All Questions. Each Question carry equal marks.

1) a) Find the power sets of (i) $A=\{1,2,3\}$ (ii) $A=\{1,2,3,4\}$ (iii) $A=\{1,2,3,4,5\}$.
b) If $\mathrm{A}=\{1,2\}, \mathrm{B}=\{2,3\}, \mathrm{C}=\{\mathrm{a}, \mathrm{b}\}$, find AXBXC using diagram
c) Write the truth table of $p \leftrightarrow q$.
d) Prove that $p \rightarrow q=-p v q$.
2) a) Solve $2 x+y-z=3, x+y+z=1, x-2 y-3 z=4$ by Cramer's rule.
b) If $u=(5,3,4), v=(3,2,1), w=(1,6,-7)$ verify $(u+v) \cdot w=u . w+v . w$ (Or)
c) Find the independent term of $x$ in $\left(x^{2}+3 a / x\right)^{15}$
d) The probability of solving a problem by $A$ is $2 / 3$, that of $B$ is $4 / 5$ and that of $C$ is $3 / 7$. Find the probability of solving a problem.
3) a) Show that the sum of the degree of the two vertices of a graph is equal to twice the number of edges in $G$.
b) Show that a graph $G$ is connected if and only if it is minimally connected.
(Or)
c) State and prove Lagranges theorem on sets.
d) Explain different types of grammars.
4) a) Show that in a distributive lattice if an element has a complement, then this complement is unique.
b) In any Boolean algebra, if $a^{*} x=a * y$ and $a+x=a+y$, then $x=y$.
(Or)
c) Find the lexicographic ordering of the following n-tuples. (i) $(1,1,2)$ (ii) $(1,2,1)$, $(1,0,1,0,1),(0,1,1,1,0)$.
d) Show that there exists a consistent enumeration for any finite poset S .

## SECTION -B

Answer any FOUR questions.
5) Prove that by means of truth table that (i) $\neg(\mathrm{p} \rightarrow \mathrm{q})=\mathrm{p} \wedge \neg \mathrm{q}$.
(ii) $\neg(\mathrm{p} \leftrightarrow \mathrm{q})=\neg \mathrm{p} \leftrightarrow \mathrm{q}=\mathrm{p} \leftrightarrow \mathrm{q}$
6) Explain Binary addition with example. Draw the machine.
7) Show that a graph is a tree if and only if it is minimally connected.
8) Show that $\mathrm{p}(\mathrm{n})=1^{2}+2^{2}+3^{2}+\ldots$ $\qquad$ $+\mathrm{n}^{2}=\mathrm{n}(\mathrm{n}+1)(2 \mathrm{n}+1) / 6$ by Induction.
9) Define ring homomorphism, isomorphism, kernel and image of homomorphism.
10) Define reflexive, symmetric, transitive, anti-symmetric and equivalence relation.
11) Find g.c.d $(8316,10920)$ and write $d=(8316,10920)$ in the form of $d=m a+n b$. Also find 1.c.m
12) Prove that $\mathrm{C}(12,7)=\mathrm{C}(11,6)+\mathrm{C}(11,7)$.

